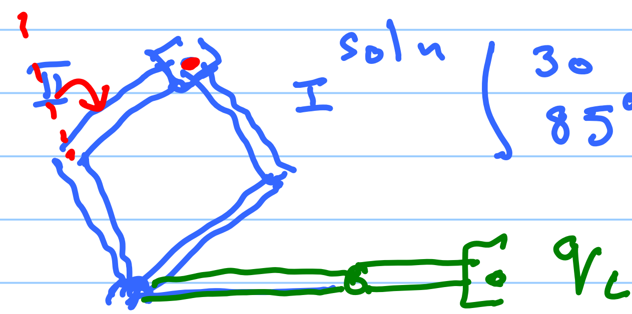
# Lecture 13

Note Title — 10/2/2007

INV KIN. CONTD.

<u>Multiple solns:</u> $\underline{q} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$

Soln II $\begin{pmatrix} 75^\circ \\ -20^\circ \end{pmatrix}$

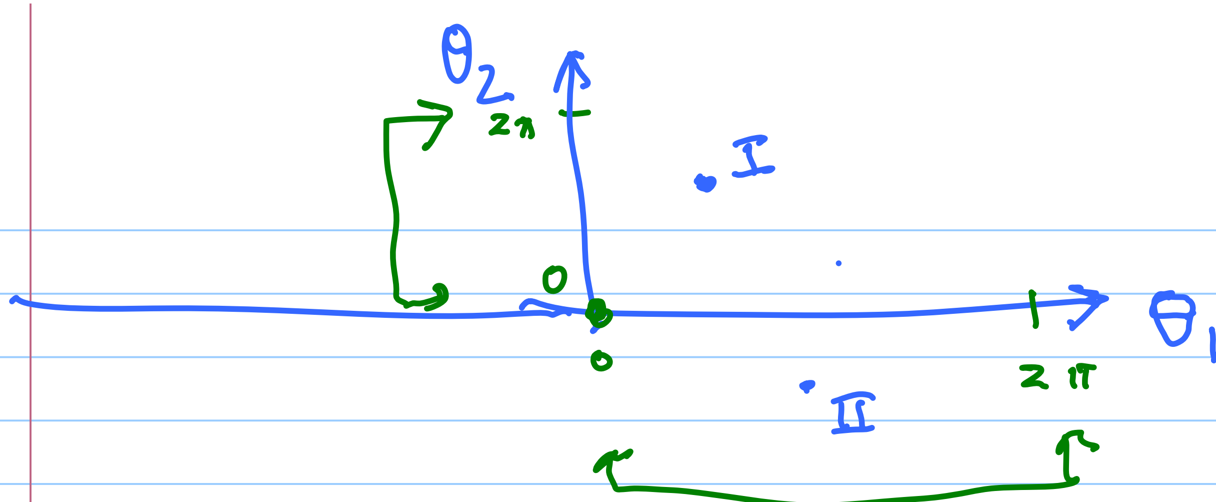



I soln $\begin{pmatrix} 30 \\ 85^\circ \end{pmatrix}$

which soln. will you choose?

"closer" to the current config. $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

"notion of distance" in joint space




$$\min \left| \underline{q}_c - \underline{q}^* \right| \leftarrow \sqrt{(0-30)^2 + (0-85)^2}$$

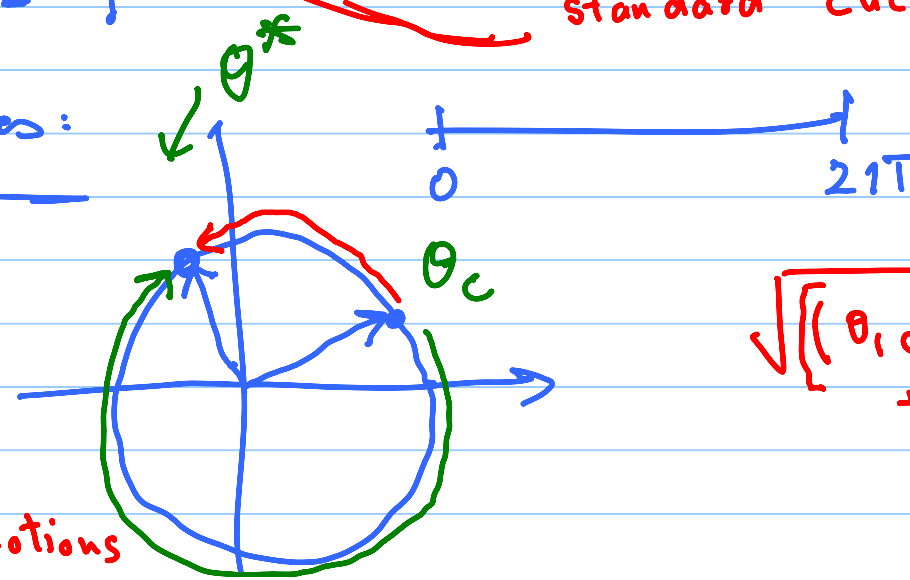
standard "Euclidean Distance"

$$\sqrt{\left[(\theta_{1c} - \theta_1^*)^2 + (\theta_{2c} - \theta_2^*)^2\right]}$$

subtle issues:

topology of rotations

for 1 joint: Two possible motions



"Choose shorter one" (red arrow)

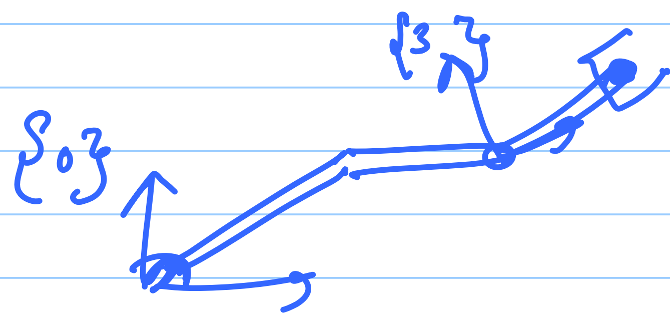
## Manipulator subspace :

6-dof $\begin{pmatrix} R & P \\ 0 & 1 \end{pmatrix}$

$\alpha, \beta, \gamma$ $\quad$ $x, y, z$

for robots with

$N < 6$-dof




set of ind. para. that charact. ${}^0_N T$ matrix, is called the manip. sub-space

$${}^0_3T = \begin{pmatrix} R & P \\ 0 & 1 \end{pmatrix}$$

$$P \rightarrow \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$$R \rightarrow \begin{pmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$(x, y, \phi) \longrightarrow$ manip. sub-space

closed form $\rightarrow$ <u>preferable</u>

<u>Solution Method:</u>

numerical soln. $\rightarrow$ may not converge

may not char. all possible

"Solvable" : A manipulator is solvable if <u>all</u> possible solns. can be enumerated for a given wrist frame

closed form $\rightarrow$ algebraic app.

closed form $\rightarrow$ geometric app.

→ appendix C + in the 4th chap.

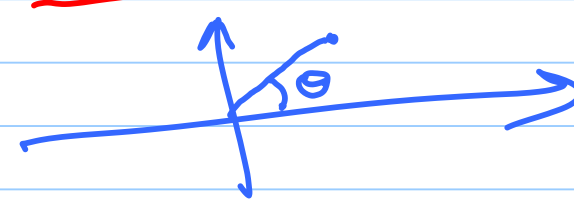
(Trig. Eqns)

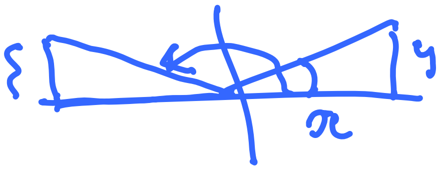
Trig. identities (app. A)

Soln. to some basic trig. eqns:

Errors in text: not consist. in the order of arg. in Atan2

$\theta = \mathrm{Atan2}(y, x)$



1) $\sin\theta = a \implies \theta^*, \pi - \theta^*$ 

$\cos\theta = \pm\sqrt{1-a^2}$

$\implies \theta = \text{Atan2}\left(a, \pm\sqrt{1-a^2}\right)$

making the quadrant explicit.

2) $\cos\theta = b$ $\qquad \theta = \text{Atan2}\left(\pm\sqrt{1-b^2}, b\right)$

$\sin\theta = \pm\sqrt{1-b^2}$ $\qquad \theta^*, -\theta^*$

3) $\sin\theta = a, \ \cos\theta = b$

$\theta = \text{Atan2}(a, b)$

4) $\tan\theta = a$

$\theta = \text{Atan2}(\pm a, \pm 1) \to \theta^*, \theta^* + \pi$

5) $a\cos\theta + b\sin\theta = c$

Two diff. ways : I + II

I) : $u = \tan\frac{\theta}{2}$ $\qquad \sin\theta = \frac{2u}{1+u^2}$

$\cos\theta = \frac{1-u^2}{1+u^2}$

sub. in 5) :

$$a\left(\frac{1-u^2}{1+u^2}\right) + b\left(\frac{2u}{1+u^2}\right) = c$$

$$\Leftrightarrow \quad \underline{\underline{(a+c)}}\,u^2 - 2b\cdot u + (c-a) = 0$$

Solve quad. for $u$

$$u = \frac{b \pm \sqrt{b^2 - c^2 + a^2}}{c+a}$$

assump: $c+a \neq 0$

$b^2 - c^2 + a^2 \geq 0$

$$\Rightarrow \quad \tan \theta/_2 = \frac{b \pm \sqrt{b^2 - c^2 + a^2}}{c+a}$$

$$\theta = 2\,\mathrm{Atan2}\left[\frac{b \pm \sqrt{b^2 - c^2 + a^2}}{c+a},\ 1\right]$$

Overall Two solns. in total

<u>Method II :</u> $a\cos\theta + b\sin\theta = c$

$$a = r\cos\phi \qquad \Rightarrow \quad \phi = \mathrm{Atan2}(b, a)$$
$$b = r\sin\phi$$

$$r\cos\phi\cos\theta + r\sin\phi\sin\theta = c$$

$$r\cos(\theta - \phi) = c$$

use eqn. 2 above

$$\theta = \mathrm{Atan2}(b, a) + \mathrm{Atan2}\left(\pm\sqrt{b^2+a^2-c^2},\, c\right)$$

What if $c+a=0$ ?

$\Rightarrow$ try. solve it yourself for this special case.

$$a\cos\theta + b\sin\theta = c$$

$c+a=0 \Rightarrow c=-a$

$$\Rightarrow \quad a\cos\theta + b\sin\theta = -a$$

$$\Rightarrow \quad a(1+\cos\theta) + b\sin\theta = 0$$

$$\Rightarrow \quad a \cdot 2 \cdot \cos^2\frac{\theta}{2} + 2b\sin\theta/_2\cos\theta/_2 = 0$$

$$\Rightarrow \quad \underbrace{2\cos(\theta/_2)}_{= 0}\Big[\underbrace{a\cos\theta/_2 + b\sin\theta/_2}_{= 0}\Big] = 0$$

$$\Downarrow$$

$$\Rightarrow \frac{\theta}{2} = \pm 90^\circ \qquad \tan\theta/_2 = -a/b \Rightarrow \text{\# 4 eqn to solve}$$

#6) $a\cos\theta - b\sin\theta = c$

$a\sin\theta + b\cos\theta = d$

<u>Meth. I:</u> $\left.\begin{aligned} a &= r\cos\phi \\ b &= r\sin\phi \end{aligned}\right\} \Rightarrow$ polar form

$r = \sqrt{a^2+b^2}$

$r\cos(\theta+\phi) = c$

$r\sin(\theta+\phi) = d$

what if $c, d = 0 \Rightarrow$ only way to satisfy eqn is

$a=0, b=0$

$$\Rightarrow \theta = -\phi + \mathrm{Atan2}(d, c)$$

$$= -\mathrm{Atan2}(b, a) + \mathrm{Atan2}(d, c)$$

<u>Method II:</u> use Cramer's rule to

dir. solve for $\sin\theta$ & $\cos\theta$.

<u>Answer:</u> $\Theta = \mathrm{Atan2}(ad - bc,\ ac + bd)$