Note Title 10/16/2007

Lecture - 19

<u>Summary:</u>

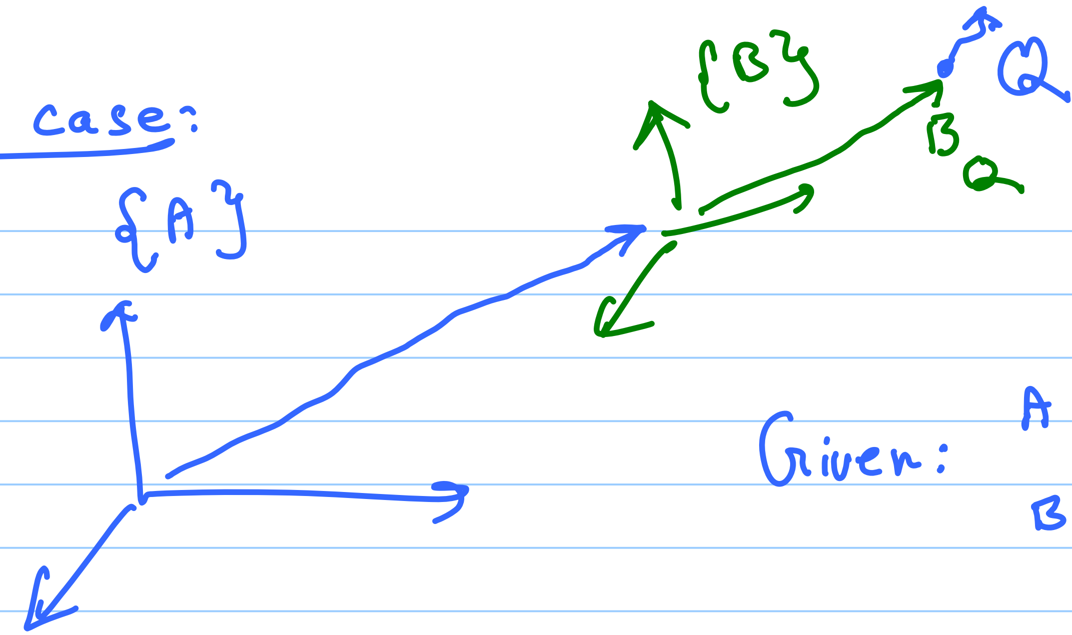
1) Ang. vel. $\Longleftrightarrow$ rigid body or frame

2) $\dot{R} R^T = S(\Omega)$

$$\dot{R} = S(\underbrace{\Omega}_{\Omega}) R$$

$$S(\Omega) \times \vec{p} = \vec{\Omega} \times \vec{p}$$

General case:



Given: ${}^{A}_{B}R$, ${}^{A}P_{BORG}$

$${}^{A}V_{Q} \stackrel{?}{=} {}^{B}V_{Q}$$

$${}^{A}Q = {}^{A}P_{BORG} + {}^{A}_{B}R\,{}^{B}Q$$

$\frac{d}{dt}$ on both sides:

$$
{}^{A}V_{Q} = {}^{A}V_{BORG} + \underbrace{{}^{A}_{B}\dot{R}} \; {}^{B}Q + {}^{A}_{B}R \; {}^{B}\dot{Q}
$$

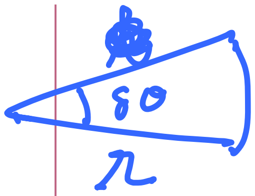
$$
= {}^{A}V_{BORG} + \underbrace{S\left({}^{A}\Omega_{B}\right)} \; \underbrace{{}^{A}_{B}R \; {}^{B}Q} + {}^{A}_{B}R \; {}^{B}V_{Q}
$$

LVE

$$
= {}^{A}V_{BORG} + {}^{A}\Omega_{B} \times \left({}^{A}_{B}R \; {}^{B}Q\right) + {}^{A}_{B}R \; {}^{B}V_{Q}
$$

Alternative derivation: geometric

lin. vel = $\underline{{}^{A}\underline{\Omega}_{B} \times {}^{A}_{B}R \; {}^{B}Q}$



$l = r \cdot \delta\theta$

$${}^A V_Q = \lim_{\Delta t \to 0} \frac{Q_{t+\Delta t} - Q_t}{\Delta t}$$

ΔQ

$|{}^A Q| \sin\theta$

$|\Omega| \cdot \Delta t$

$\rightarrow \Delta Q = |{}^A Q|\,|\Omega|\,\Delta t \cdot \sin\theta$

$\Rightarrow |{}^A V_Q| = |{}^A Q|\,|\Omega|\,\sin\theta$

${}^A V_Q = \Omega \times {}^A Q$

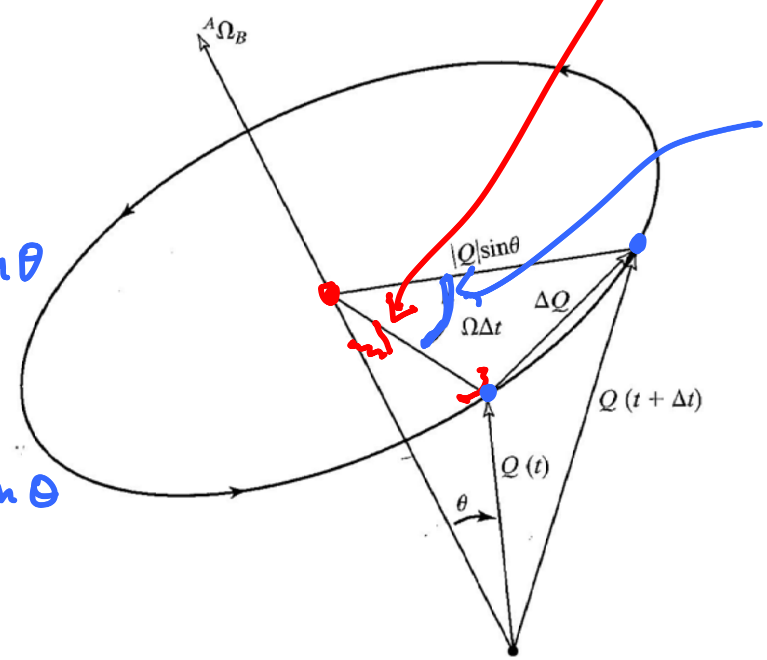
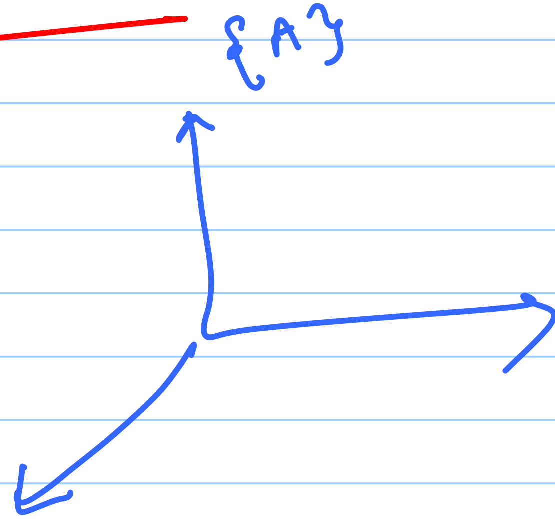
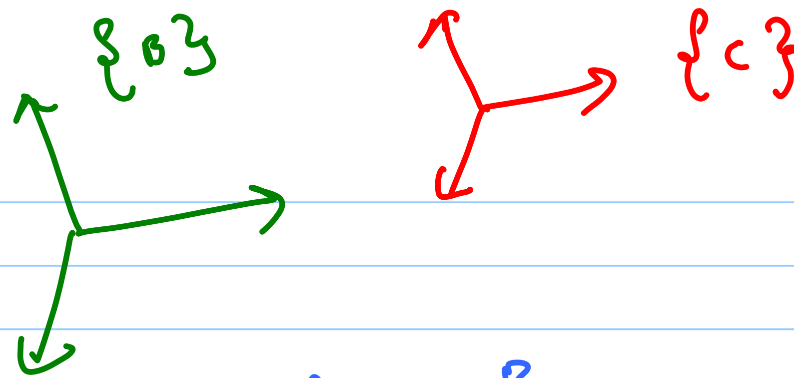


FIGURE 5.5: The velocity of a point due to an angular velocity.

$L\Omega E$: ?





${}^{A}_{B}R$, ${}^{B}_{C}R$ are known

${}^{A}_{C}R = {}^{A}_{B}R \, {}^{B}_{C}R$

${}^{A}\Omega_{B}$, ${}^{B}\Omega_{C}$, ${}^{A}\Omega_{C}$

$${}^{A}\Omega_{C} \overset{?}{=} f\left({}^{A}\Omega_{B}, {}^{B}\Omega_{C}\right)$$

intuition

$$= {}^{A}\Omega_{B} + {}^{A}_{B}R\,{}^{B}\Omega_{C}$$

$${}^{A}_{C}R = {}^{A}_{B}R\,{}^{B}_{C}R \qquad \boxed{\dot{R} = S R}$$

$$\underbrace{{}^{A}_{C}\dot{R}\left({}^{A}_{C}R\right)^{T}} = S\left({}^{A}\Omega_{C}\right) \Rightarrow$$

$${}^{A}_{C}\dot{R} = {}^{A}_{B}\dot{R}\,{}^{B}_{C}R + {}^{A}_{B}R\,{}^{B}_{C}\dot{R}$$

$$S\left({}^{A}\Omega_{C}\right){}^{A}_{C}R = S\left({}^{A}\Omega_{B}\right){}^{A}_{B}R\,{}^{B}_{C}R + {}^{A}_{B}R\,S\left({}^{B}\Omega_{C}\right){}^{B}_{C}R$$

mult. both sides by $\left({}^{A}_{C}R\right)^{T}$

$$S\left({}^{A}\Omega_{C}\right) = S\left({}^{A}\Omega_{B}\right) + {}^{A}_{B}R\, S\left({}^{B}\Omega_{C}\right) \underbrace{{}^{B}_{C}R\, {}^{C}_{A}R}$$

$$= S\left({}^{A}\Omega_{B}\right) + \underbrace{{}^{A}_{B}R\, S\left({}^{B}\Omega_{C}\right) {}^{B}_{A}R}_{S\left({}^{A}_{B}R\, {}^{B}\Omega_{C}\right)}$$

We can show algebraically:

$$\underbrace{R\, S(\Omega)\, R^{T}\, b}_{\text{Can show } R\left[\Omega \times \left(R^{T} b\right)\right]} = \underbrace{S(R\Omega)\, b}_{\left(R\vec{\Omega}\right) \times \vec{b}}$$

Two prop:

1) $S(\Omega)\vec{P} = \vec{\Omega} \times \vec{P}$

2) $R(\vec{a} \times \vec{b}) = R\vec{a} \times R\vec{b}$

$R\Omega \times R R^T b$

$= R\Omega \times b$