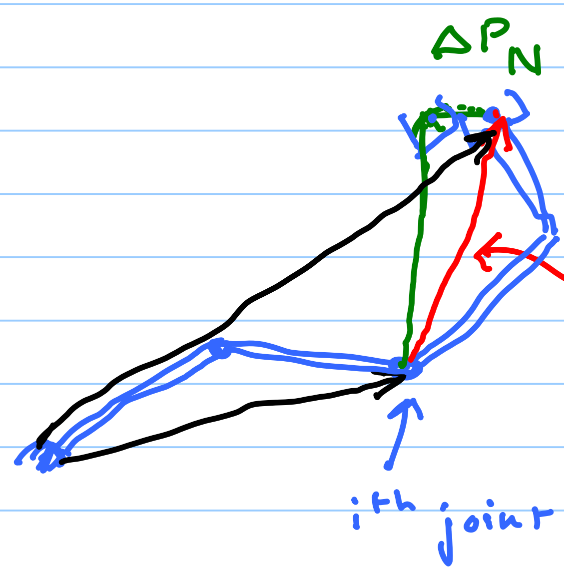
Lecture - 22

J_{iv} :

$v_N = J_i \dot{q}_i + J_{i-1} \dot{q}_{i-1}$



$\frac{\Delta P_N}{\Delta t} = J_i \frac{\Delta q_i}{\Delta t}$

$= {}^0P_N - {}^0P_i$

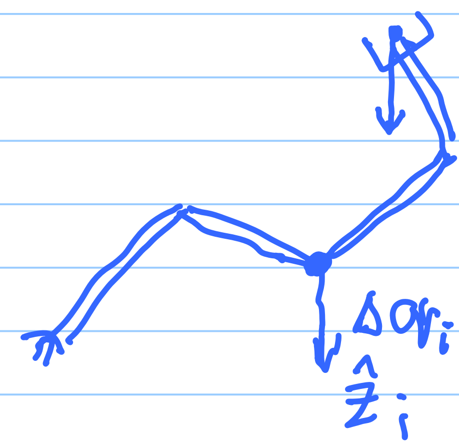
$$\frac{\Delta P_N}{\Delta t} = {}^0\left({}^{i-1}\Omega_i\right) \times \left({}^0P_N - {}^0P_i\right)$$

$$= \left({}^{0}_{i}R \, \frac{\Delta q_i}{\Delta t} \, {}^{i}\hat{Z}_i \right) \times \left({}^{0}P_N - {}^{0}P_i \right)$$

$$\Delta P_N = \Delta q_i \left(\begin{array}{c} \text{3rd col.} \\ \text{of } {}^{0}_{i}R \end{array} \right) \times \left({}^{0}P_N - {}^{0}P_i \right)$$

$$J_{iv} = \frac{\Delta P_N}{\Delta q_i} = \left(\begin{array}{c} \text{3rd col.} \\ \text{of } {}^{0}_{i}R \end{array} \right) \times \left({}^{0}P_N - {}^{0}P_i \right)$$

i^{th} col. of J_v



for prism. joint:

$$\Delta P_N = \Delta q_i \, {}^0_iR \, {}^i\hat{Z}_i$$

$$\frac{\Delta P_N}{\Delta q_i} = \text{3rd col. of } {}^0_iR \rightarrow \text{for prism joint}$$

<u>Exercise</u>: apply this to compute J for 2-link planar arm

i^{th} col. of J

$$= J_i$$

$$= \begin{pmatrix} J_{iv} \\ \hline J_{iw} \end{pmatrix}$$

$$J_{iv} = \begin{cases} \text{3rd col. of } {}^0_iR \times ({}^0P_n - {}^0P_i) & \text{rev joint} \\ \text{or} & \\ \text{3rd col. of } {}^0_iR & \text{prism} \end{cases}$$

$$J_{iw} = \begin{cases} \text{3rd col. of } ({}^0_iR) & \text{rev joint} \\ \text{or} & \\ \underline{0} & \end{cases}$$

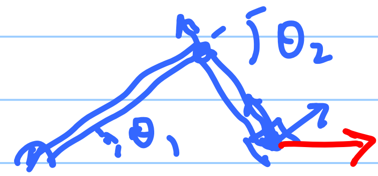
Use above formula to writ "form" of Jacobian for Puma:

$$\begin{array}{c} \left(\begin{array}{c|c|c|c|c|c} J_1 & J_2 & \cdots & J_4 & J_5 & J_6 \\ & & & \underline{0} & \underline{0} & \underline{0} \\ \hline & & & & & \end{array} \right) \begin{array}{l} J_v \\ \\ J_\omega \end{array} \end{array}$$

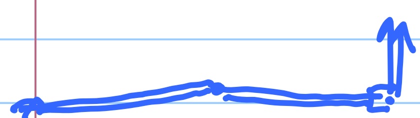
${}^0p_4 = {}^0p_5 = {}^0p_6$

$$\left(\begin{array}{c|c} J_{11} & \underline{0} \\ \hline J_{21} & J_{22} \end{array} \right) \leftarrow \text{form of } J \text{ for Puma arm}$$

|| Interpretation / use / understanding of Jacobian



$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$



rad/sec.

$\underline{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = ?\ \underline{\underline{J}} \begin{pmatrix} \dot\theta_1 \\ \dot\theta_2 \end{pmatrix}$

desired vel.

square

$\begin{pmatrix} \dot\theta_1 \\ \dot\theta_2 \end{pmatrix} = J^{-1} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$

$\rightarrow$ inverse does not exist ||

$|J| \to 0$

$$J = \begin{array}{c} \begin{array}{cc} J_1 & J_2 \end{array} \\ \left(\begin{array}{c|c} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{array} \right) \end{array}$$

$$J^{-1} = \frac{1}{l_1 l_2 \underline{\underline{s_2}}} \left(\begin{array}{c|c} l_2 c_{12} & l_2 s_{12} \\ -(l_1 c_1 + l_2 c_{12}) & -l_1 s_1 - l_2 s_{12} \end{array} \right)$$

$\sin\theta_2 \to 0$

Config at which J becomes singular or (loses rank) are called

$\begin{pmatrix} \theta_1^* \\ \theta_2^* \end{pmatrix}$

singular configuration.

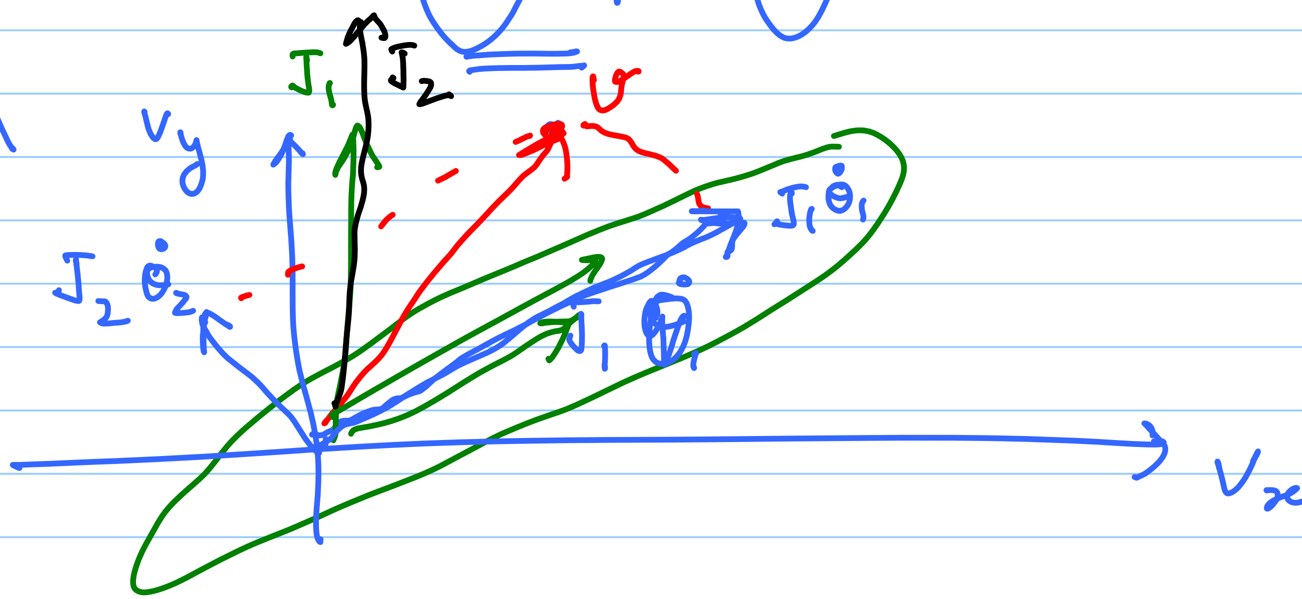
$v = J\,\dot{\underline{q}}$

$\downarrow$

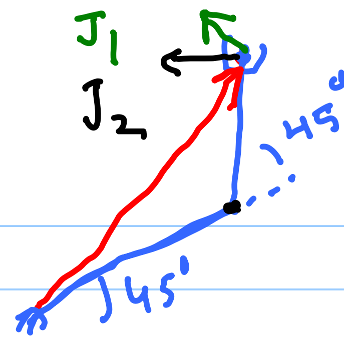
$\begin{pmatrix} v_x \\ v_y \end{pmatrix} \quad \dot{\underline{q}} = \begin{pmatrix} \dot\theta_1 \\ \dot\theta_2 \end{pmatrix}$

$(v) = J_1\,\dot\theta_1 + J_2\,\dot\theta_2$

valid for general

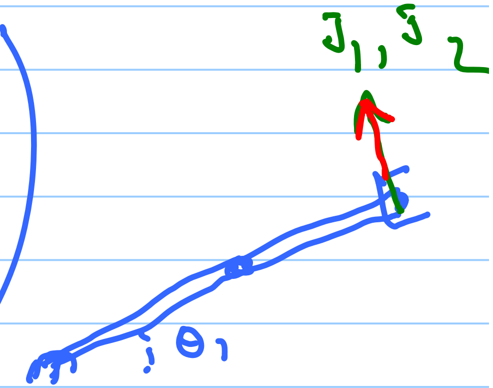


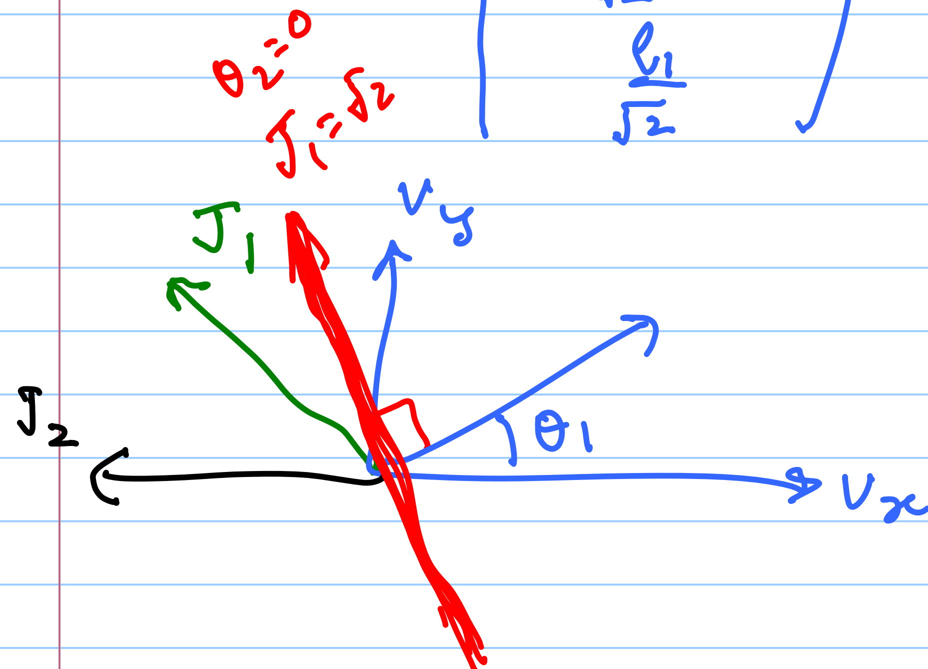
Example:



① $\theta_1 = \theta_2 = 45°$

$$J_1 = \begin{pmatrix} -\frac{l_1}{\sqrt{2}} - l_2 \\ \frac{l_1}{\sqrt{2}} \end{pmatrix} \qquad J_2 = \begin{pmatrix} -l_2 \\ 0 \end{pmatrix}$$





② θ_1

$\theta_2 = 0$

$$J_2 = l_2 \begin{pmatrix} -s_1 \\ c_1 \end{pmatrix}$$

$$J_1 = (l_1 + l_2) \begin{pmatrix} -s_1 \\ c_1 \end{pmatrix}$$

for singular configs:

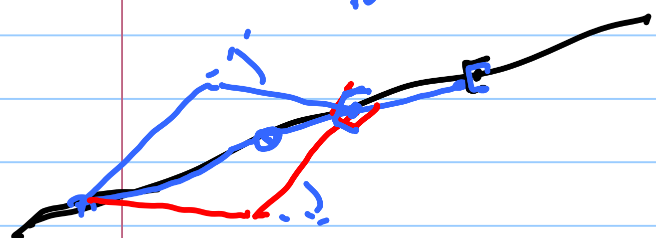
1) ~~J~~ Matrix loses rank $\Rightarrow$

   a) at least two J_i's become linearly dep. (align)

   b) One of J_i's may become $\underline{0}$ (null)

$\Rightarrow$ robot loses a deg. of freedom (instantaneously), thereby it is (end-eff) not able to move in certain directions.

2) # of inv. kin soln. changes



$\theta_1, \theta_2 = 0$

a) may become infinite

b) " be reduced

3) $\dot{\underline{q}} = J^{-1} \underline{v}$ joint rates demanded will go to ∞