Note Title 11/20/2007

Lecture - 34

Last lecture: we derived following eqns:

$$\tau_1 = \left[m_1 l_1^2 + m_2 (l_1 + l_2 c_2)^2 \right] \ddot{\theta}_1 \leftarrow \text{dep. on joint acc.}$$

$$= - 2 (l_1 + l_2 c_2) m_2 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 \leftarrow \text{joint vel.}$$

$$\tau_2 = m_2 l_2^2 \ddot{\theta}_2 + (l_1 + l_2 c_2) m_2 l_2 s_2 \dot{\theta}_1^2$$

$$+ m_2 g l_2 c_2 \leftarrow \text{gravity}$$

structure of dynamic eqns:

$\underline{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$ Collate terms based on acc. $(\ddot{\theta}_1, \ddot{\theta}_2)$,

vel. $(\dot{\theta}_1 \dot{\theta}_2, \dot{\theta}_1^2, \dot{\theta}_2^2)$, gravity

$$\underline{\tau} = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \underbrace{\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}}_{M(\underline{\theta})} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} \quad \end{pmatrix} + \begin{pmatrix} \quad \end{pmatrix}$$

$\dot{\theta}_1 \dot{\theta}_2$

$M(\underline{\theta})$ Mass matrix

$\underline{V}(\dot{\underline{\theta}})$ dep. on joint vel.

$\underline{G}(\underline{\theta})$ Gravity

Recursive

N-E

$$\boxed{\underline{\tau} = M(\underline{\theta})\,\ddot{\underline{\theta}} + \underline{V}(\dot{\underline{\theta}}) + \underline{G}(\underline{\theta})}$$

closed form.

eqns.

$$M(\underline{\theta}) = \begin{pmatrix} m_1 l_1^2 + m_2 (l_1 + l_2 c_2)^2 & 0 \\ 0 & m_2 l_2^2 \end{pmatrix}$$

$$\underline{V}(\dot{\underline{\theta}}) = \begin{pmatrix} -2(l_1 + l_2 c_2)\, m_2 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 \\ (l_1 + l_2 c_2)\, m_2 l_2 s_2 \dot{\theta}_1^2 \end{pmatrix}$$

$$G(\underline{\theta}) = \begin{pmatrix} 0 \\ m_2 g l_2 c_2 \end{pmatrix}$$

<u>Mass matrix:</u> 1) func. of $\underline{\theta}$ only

2) symmetric, +ve definit

$\Downarrow$ always invertible $\underline{\underline{M^{-1}}}$

$\downarrow$ $\underline{x^T W x > 0}$

$\underline{V}(\underline{\theta}, \underline{\dot{\theta}})$: dep. on product of velocities $\dot{\theta}_i \dot{\theta}_j$, $\dot{\theta}_i^2$

(coriolis / centripetal force)

$\underline{G}(\underline{\theta})$: dep. only on pos. terms $\underline{\theta}$

Dyn. Eqns :

① closed form derivation ← fairly intricate, complex.

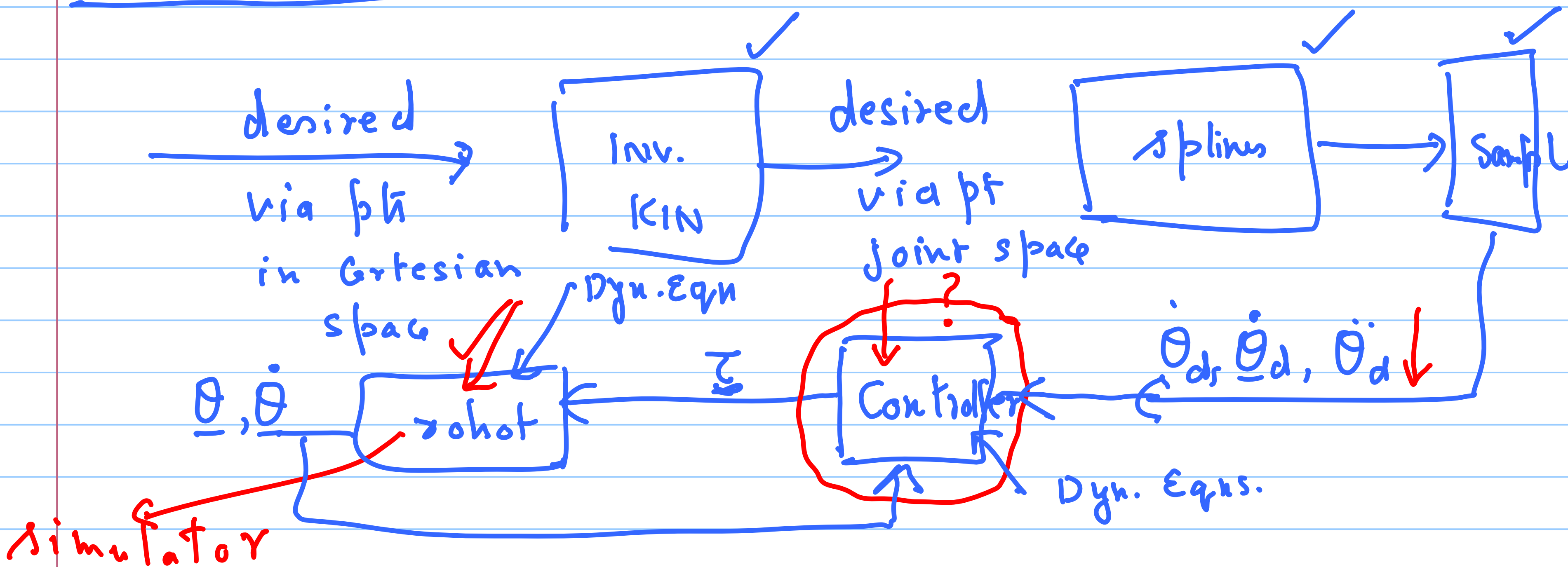
② given desired traj : $\underline{\theta}_d, \dot{\underline{\theta}}_d, \ddot{\underline{\theta}}_d$, use recursive N-E formulation directly to compute $\underline{\tau}$ in numerical form. fairly efficient

$O(N)$ N = # of joints

brute force

non-rec

$O(N^4)$



Dynamic Simulation : for dyn. comput.

$$\underline{\tau} \longrightarrow \boxed{\begin{array}{c}\text{Simulator}\\ \text{D.E. +}\end{array}} \longrightarrow \underline{\theta}, \underline{\dot{\theta}}, \underline{\ddot{\theta}}$$

numerical integration

requires solving coupled, non-linear diff. eqns.

|| numerical integration

KNOWN

$$\underline{\tau} = M(\underline{\theta})\,\underline{\ddot{\theta}} + \underline{V}(\underline{\theta}, \underline{\dot{\theta}}) + G(\underline{\theta}) + \underline{\tau}_{friction}$$

Initial cond. : $\underline{\theta}(0) = \underline{\theta}_0$

$$\dot{\underline{\theta}}(0) = \underline{0}$$

$$\ddot{\underline{\theta}}(0) = \underline{0}$$

2nd order Runge-Kutta:

$$\circledast \rightarrow \quad \ddot{\underline{\theta}} = M^{-1}(\underline{\theta})\left[\underline{\tau} - \underline{V}(\underline{\theta}, \dot{\underline{\theta}}) - \underline{G}(\theta)\right]$$

at t: θ_t, $\dot{\theta}_t$, $\ddot{\theta}_t$: are known

$$\dot{\underline{\theta}}_{t+\Delta t} = \dot{\underline{\theta}} + \Delta t \cdot \ddot{\underline{\theta}}(t)$$

$$\underline{\theta}_{t+\Delta t} = \underline{\theta}(t) + \Delta t \cdot \dot{\underline{\theta}}(t) + \frac{1}{2}\Delta t^2 \ddot{\underline{\theta}}(t)$$

$\ddot{\underline{\theta}}_{t+\Delta t} =$ // derive from (*) by pulugging, $\cancel{\underline{\theta}_t}$ $\dot{\underline{\theta}}_{t+\Delta t}$, $\underline{\theta}_t$.

Comment on D. E. :

A

✓ 1 N-E eqns : Force + Moment / acc. for rigid.

✗ 2 Energy of a system: Lagrangian

B Non-rigid body effects are not included in the D. E.s.

Friction, flexibility - - -

viscous friction

$$\tau_{friction} = b_i \dot{\theta}$$ ← viscous friction

$$\underline{\tau}_{fric.} = \begin{pmatrix} b_1 \dot{\theta}_1 \\ b_2 \dot{\theta}_2 \\ \vdots \\ b_N \dot{\theta}_N \end{pmatrix}$$

Read up to 9.5 : basic control material (383)