Dynamic Phasors in Modeling, Analysis and Control of Energy Processing Systems

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Research Program Overview

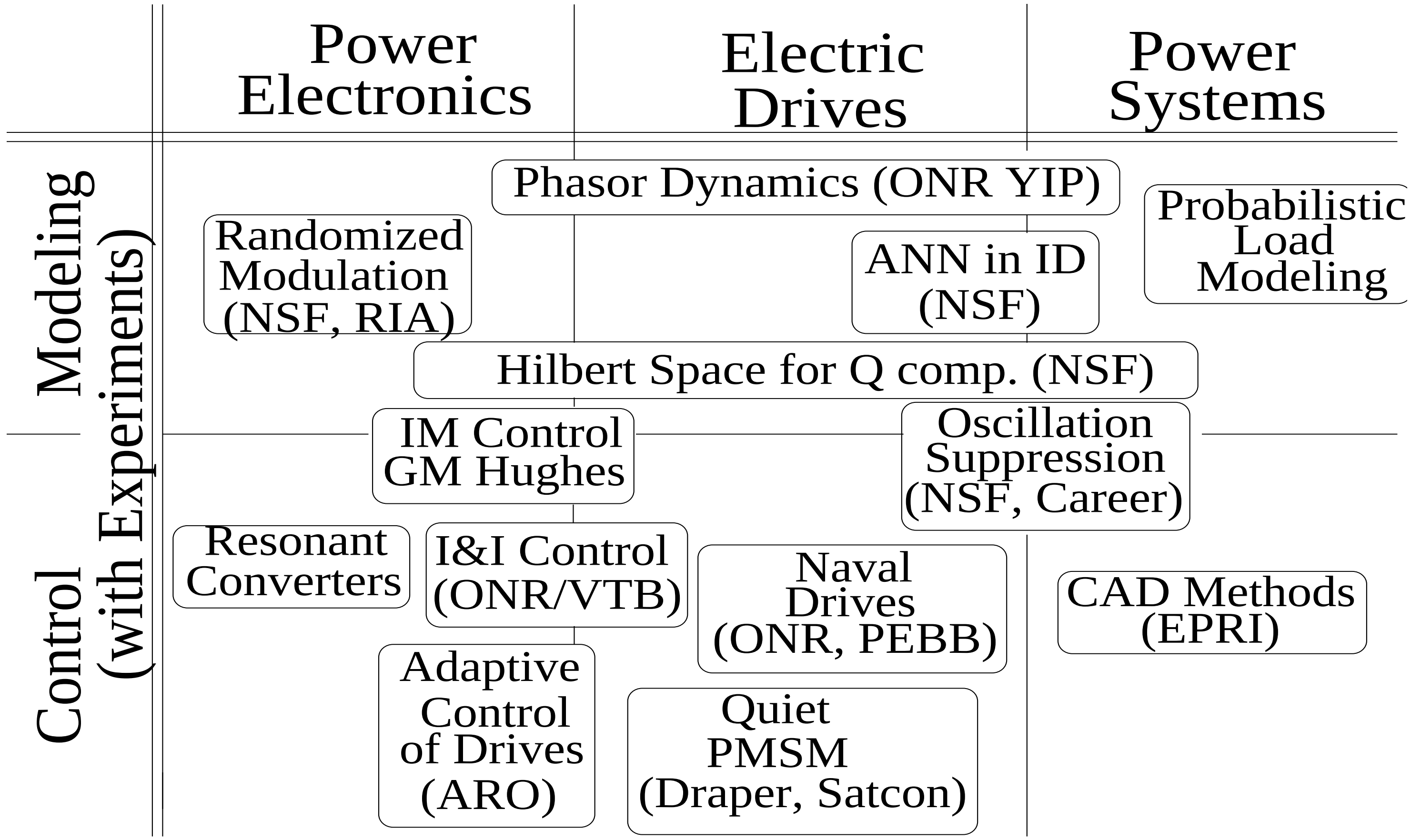
My research program focuses on the interface of control and energy processing.

Emphasis on: **1.** dynamical modeling and **2.** experimental verification.

Emerging energy conversion technologies: **1.** efficiency driven and **2.** require closed–loop control - **nonlinearity** and **uncertainty**.

NEU Energy Processing Laboratory (1994) is a confluence of research and educational efforts:
1. Areas: power electronics, electric drives and power systems,
2. Graduate students,
3. Sponsors in government and industry,
4. Technical collaborations.

Research Program Overview



Presentation Overview

- **Background and definitions,**
- POWER SYSTEMS - Flexible AC Transmission Systems.
- ELECTRIC DRIVES - AC machine modeling,
- An interlude - extension to polyphase systems,
- POWER ELECTRONICS - active filters and switched-mode DC/DC converters.

Background

New challenges in energy processing (power electronics, electric drives, power systems):

- reliance on switching operation for efficiency,
- new dynamic couplings,
- increased performance specifications,
- new problems (e.g., active filtering, pulsed power).

Analytical tools for addressing ("close-to" periodic) system operation:

- "sinusoidal quasi-steady-state" approximation in drives and power systems,
- time-domain simulations (in power electronics and drives),
- systematic exploration of the "middle ground" is largely missing.

Background

Main features of dynamic phasors:

- large signal models,
- nonlinear,
- physical intuition used for simplifications,
- appealing mathematical structure.

Origins and ideas related to our work:

- power electronics,
- power engineering (“space vectors”, “spiral vectors”, polyphasors),
- nonlinear oscillations (classical averaging and recent variants),
- signal processing.

Definitions

A (possibly complex) waveform $x(\cdot)$ can be represented on the interval $(t-T, t]$ using (short-time) Fourier series:

$$x(\tau) = \sum_{k=-\infty}^{\infty} X_k(t) e^{jk\omega_s \tau}$$

where $X_k(t)$ are the complex, slowly time-varying Fourier coefficients, or *dynamic phasors*.

$$X_k(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau) e^{-jk\omega_s \tau} d\tau = < x >_k (t)$$

Our dynamical models describe evolution of $X_k(t)$; for real $x(\cdot)$ we have $X_{-k} = X_k^*$

Definitions

Two useful facts:

Derivative of the k-th dynamic phasor:

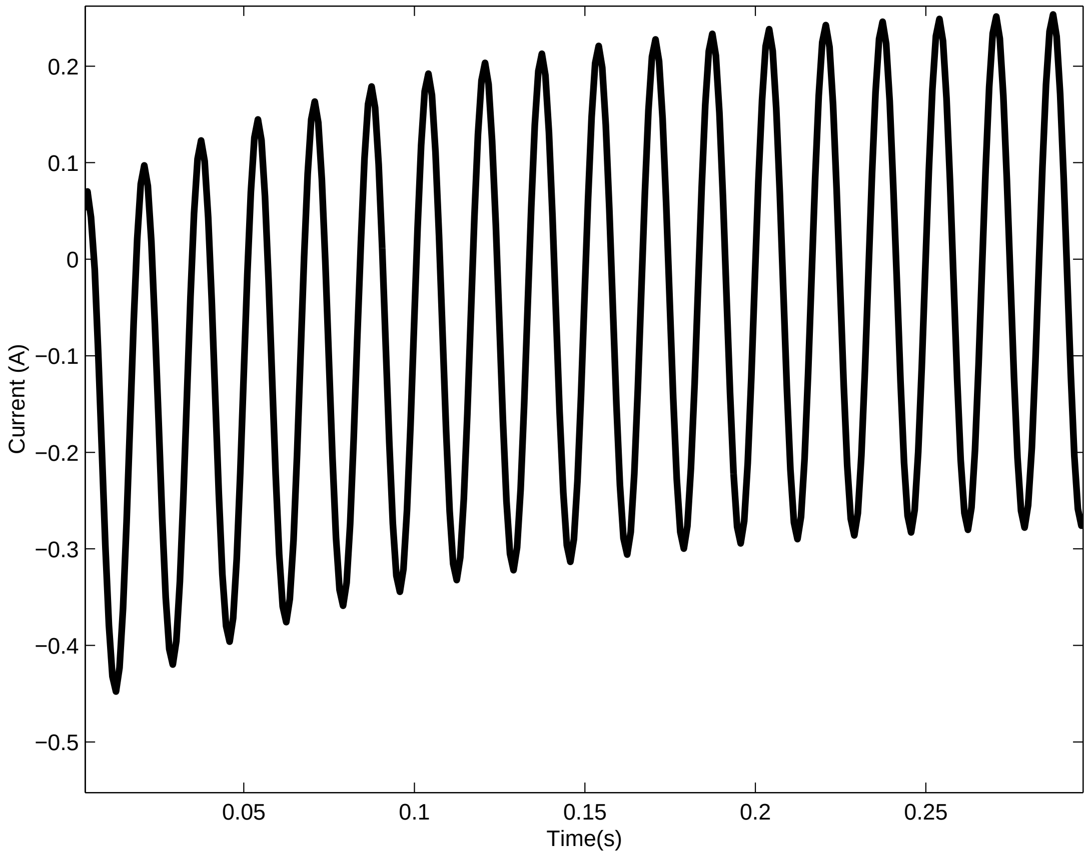
$$\frac{dX_k}{dt} = \left\langle \frac{d}{dt} x \right\rangle_k - j\ k\ \omega_s X_k$$

Multiplication in time domain:

$$< xy >_k = \sum_{\ell} < x >_{k-\ell} < y >_{\ell}$$

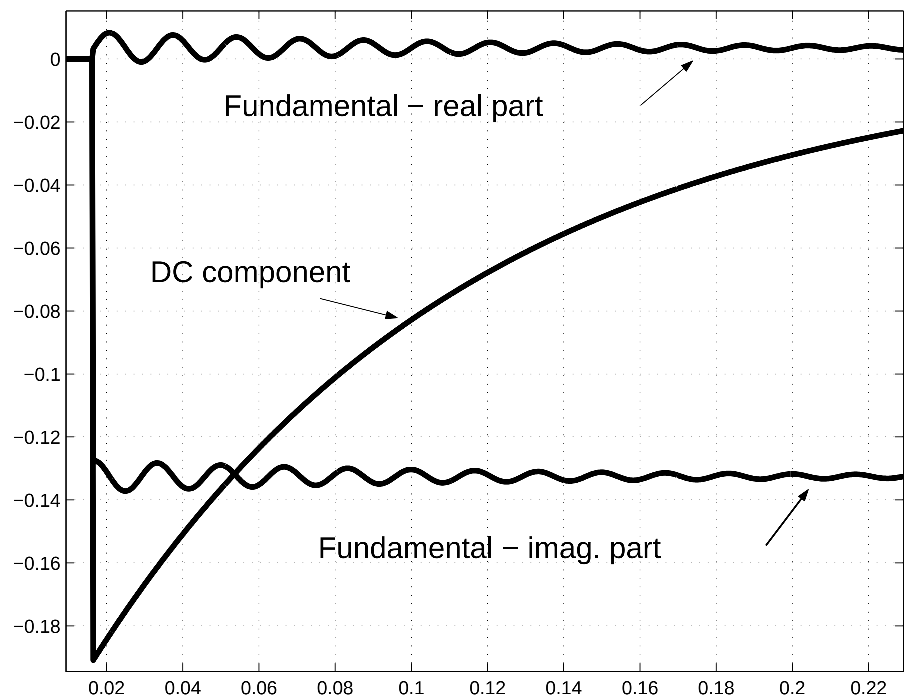
A Simple Example

Consider a simple RL circuit (R=1, L=0.1) with a $\cos$ excitation (V=10), at 60Hz and some initial condition ($i(0) = -0.1$A):



A Simple Example, cont. 1

The dynamic phasors (according to our definition)



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- Definitions,
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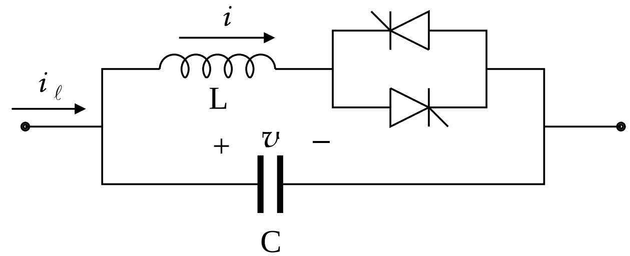
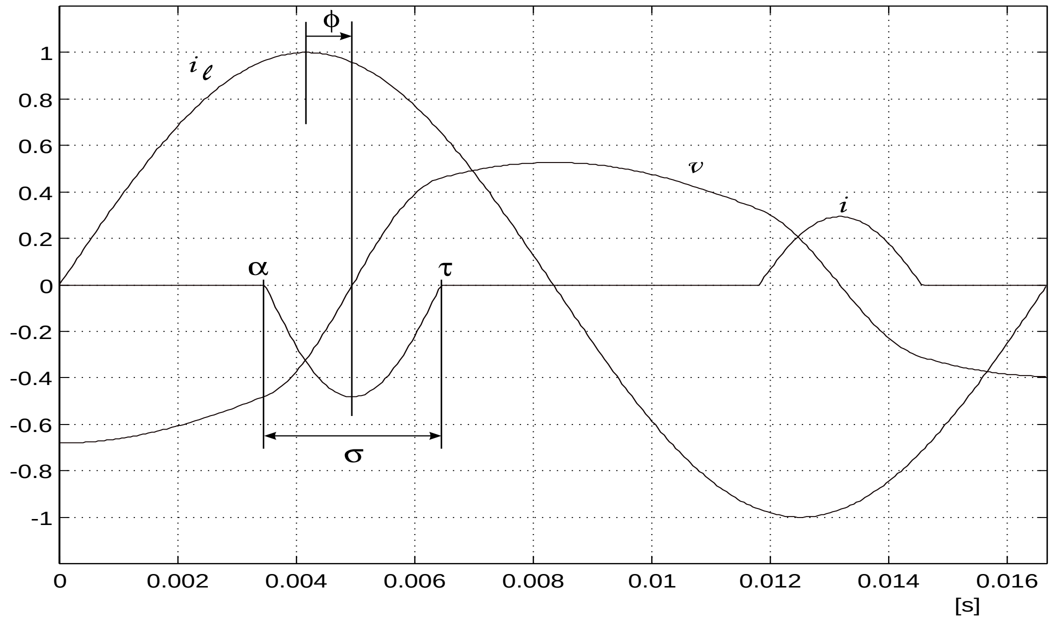
Thyristor Controlled Series Capacitors (TCSCs) are finding increasing application in power systems.





Flexible AC Transmission

Fast and accurate models are needed for simulation and control.

Analytical difficulties stem from the nature of TCSC that amalgamates continuous-time dynamics with discrete events (thyristor firings).

Sampled-data models were the first to offer the needed accuracy, but

- model structure has no clear relation to the system configuration,
- hard to interface with the rest of the system which is usually described with phasor-based continuous-time models.

A state-space model for basic TCSC configuration:

$$C\frac{dv}{dt} = i_\ell - i$$
$$L\frac{di}{dt} = q\ v$$

where q is a $0-1$ switching function.

Evaluating the 1-phasor on both sides of each equation (and assuming i_ℓ is sinusoidal) we obtain a 2nd-order (complex) phasor model:

$$C\ \frac{dV_1}{dt} = I_\ell - I_1 - j\ \omega_s\ C\ V_1$$
$$L\ \frac{dI_1}{dt} = < q\ v >_1 - j\ \omega_s\ L\ I_1$$

where $< q\ v >_1$ is:

$$< q\ v >_1 = \frac{2}{\pi}\int_\alpha^\tau v\ e^{-j\theta} d\theta.$$

Flexible AC Transmission

I_1 has fast dynamics compared to V_1, so we assume

$$I_1 \approx \frac{V_1}{j\omega_s L_{eff}(\sigma)}$$

yielding

$$C\frac{dV_1}{dt} = I_\ell - (j\omega_s C + \frac{1}{j\omega_s L_{eff}(\sigma)})V_1 = I_\ell - j\omega_s C_{eff}(\sigma)V_1$$

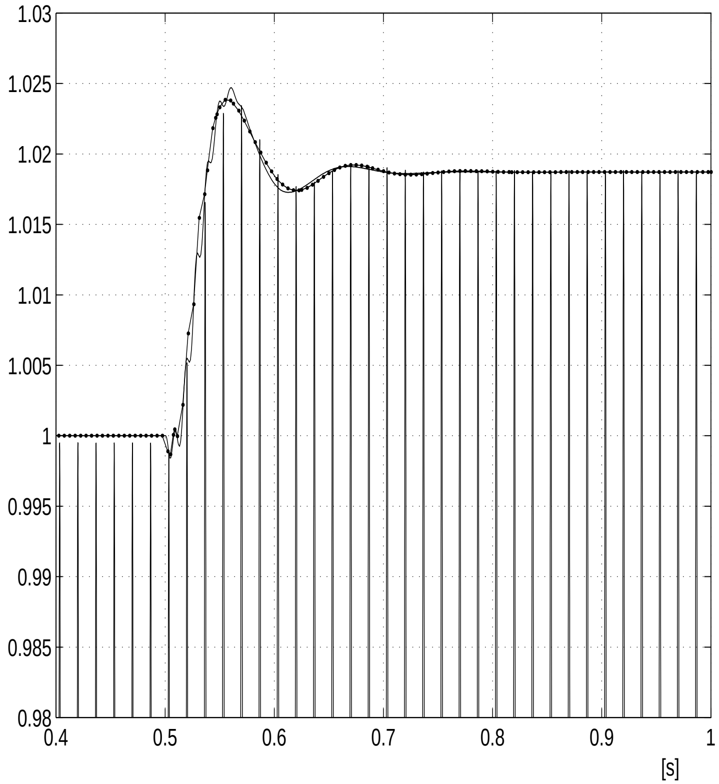
where σ is the prevailing conduction angle

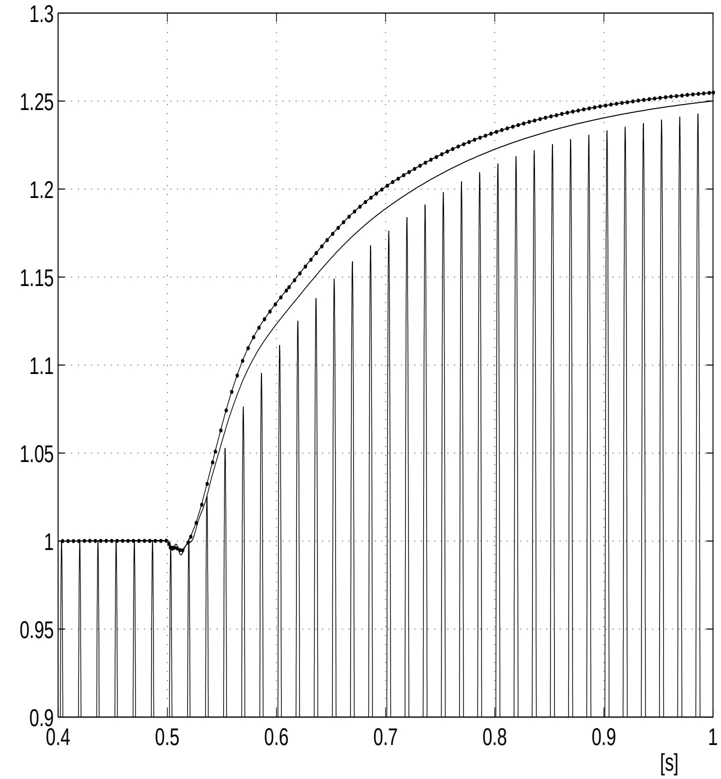
$$\sigma = \sigma^0 + 2\phi \approx \sigma^0 + 2\arg\left[-jI_\ell(V_1)^*\right]$$

and σ^0 is the reference.

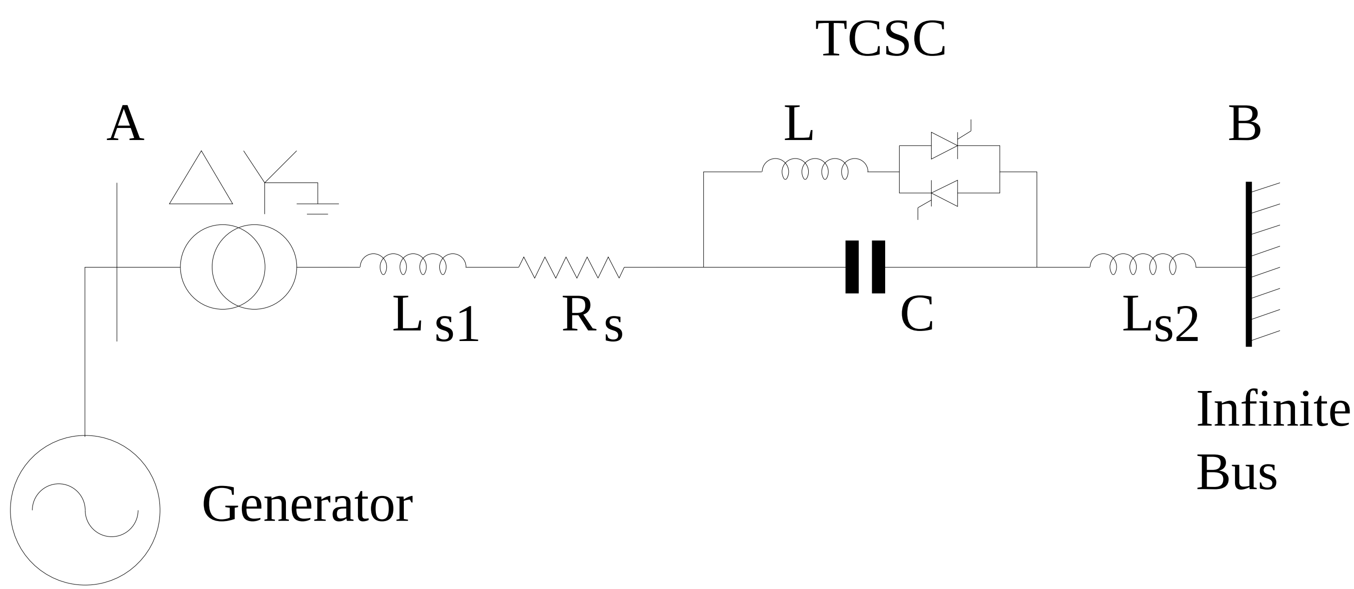
C_{eff} is computed from steady-state assuming sinusoidal line current i_ℓ (in contrast to the conventional approach which assumes v to be sinusoidal).

Line current for 2° step changes of the firing angle:

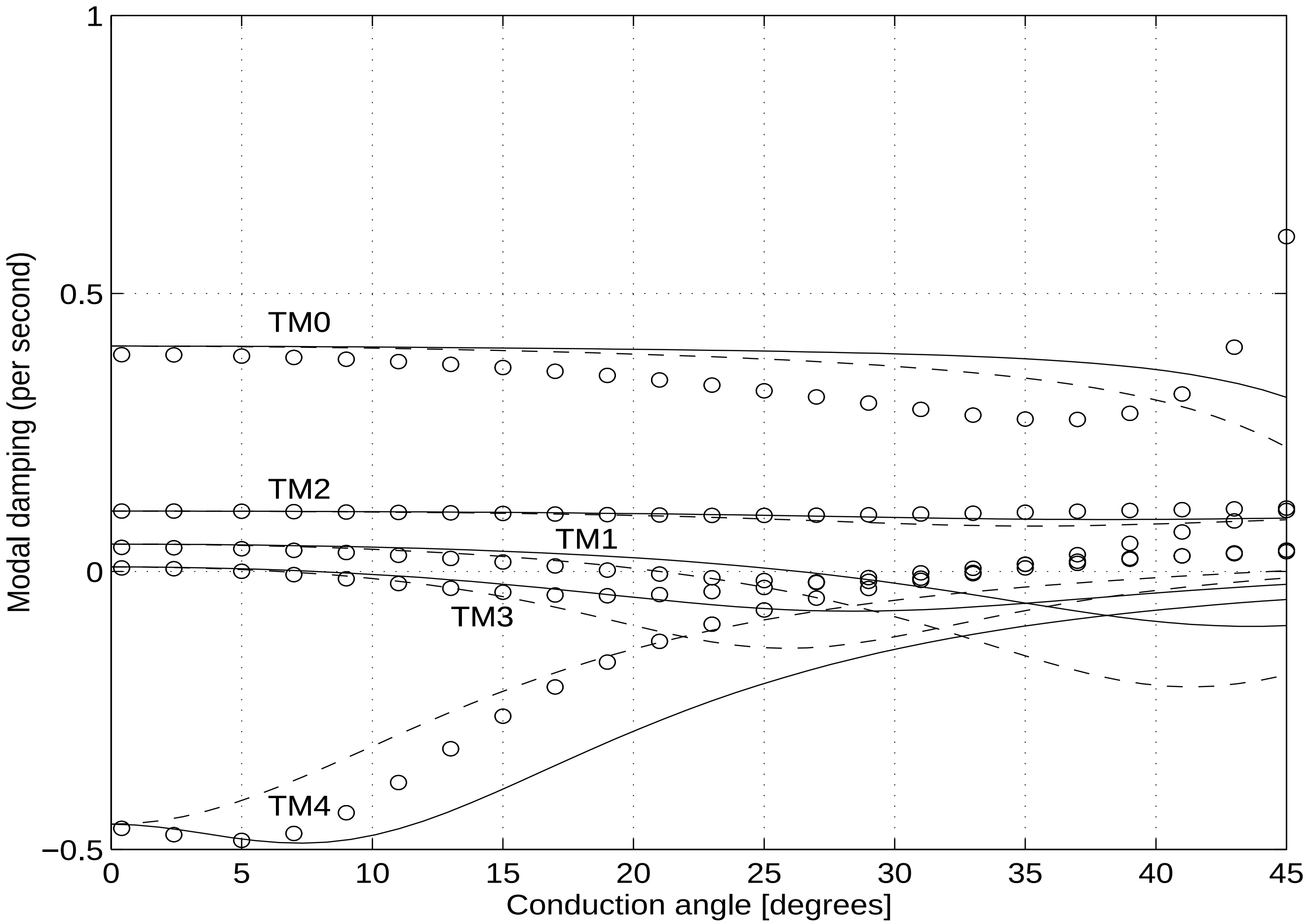




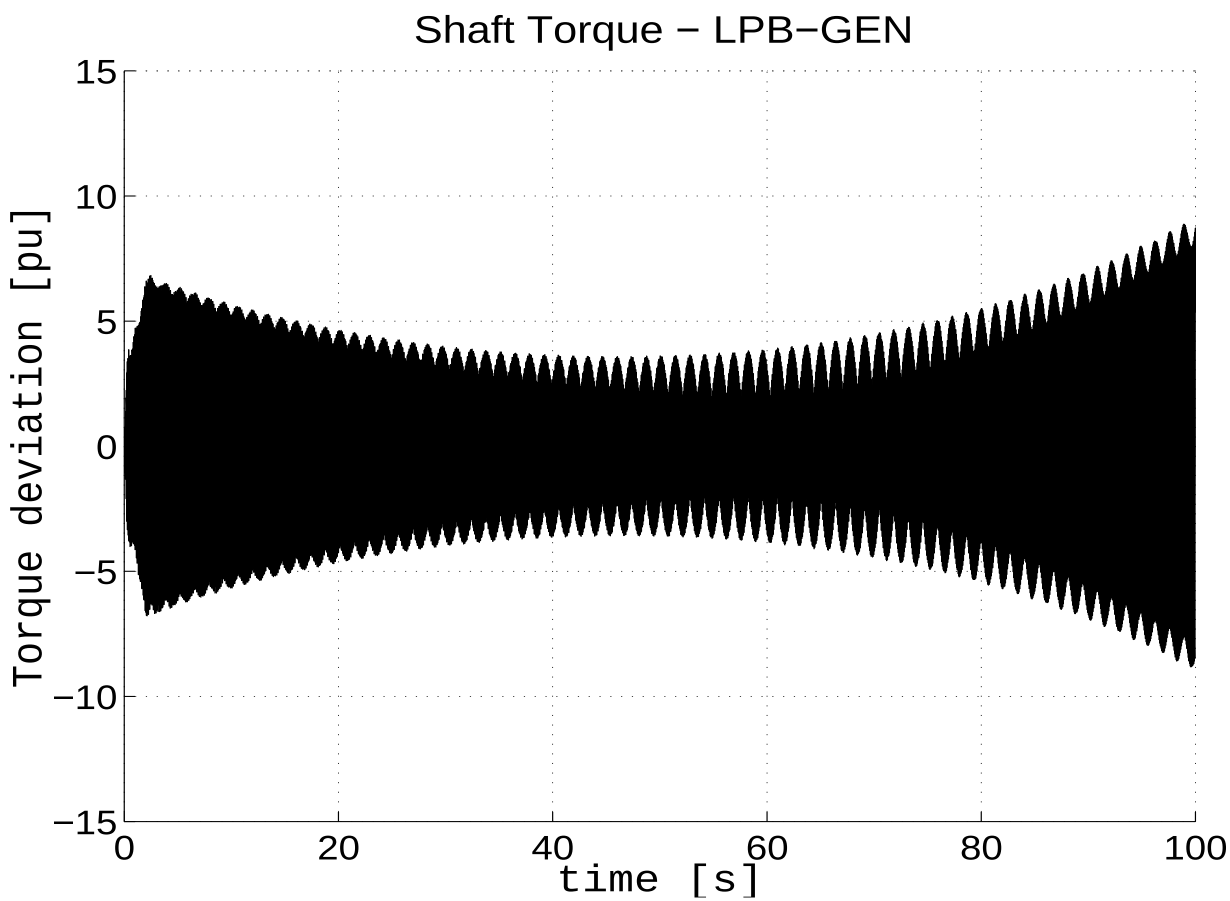
First IEEE benchmark test with TCSC:



Subsynchronous Resonance

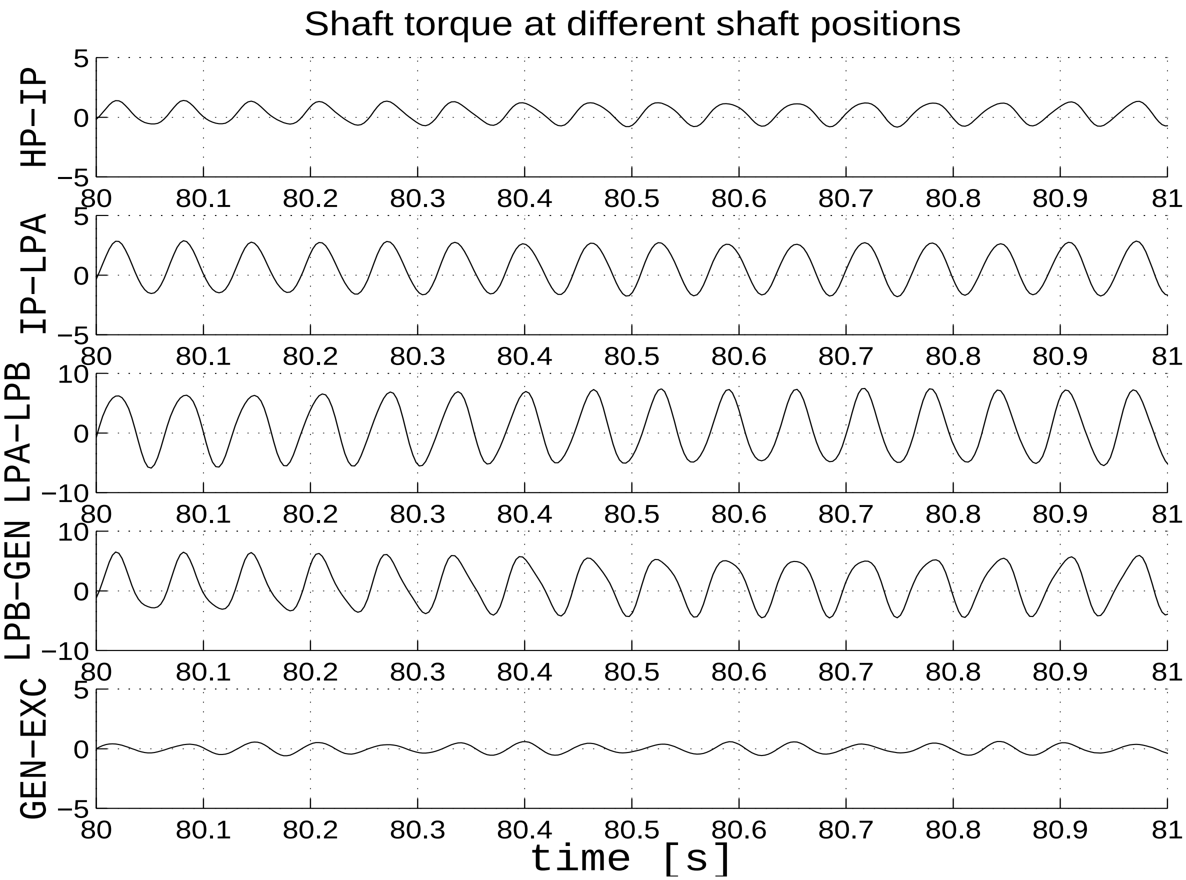


Subsynchronous Resonance



Subsynchronous Resonance

Expanded view (between 80s and 81s) of torque variations



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Dynamic Phasors and Space Vectors

Assuming now that all phase quantities are real; let $\alpha = e^{j2\pi/3}$:

$$\vec{x}(t) = \frac{2}{3}(x_a(t) + \alpha x_b(t) + \alpha^* x_c(t))$$

This complex (scalar) quantity can encode two-dimensional information; for example, $< \vec{x} >_{-k} = < \vec{x}^* >_k^*$.

$$X_{p,1}(t) := < \vec{x} >_1 (t) = < \vec{x}^* >_{-1}^* (t),$$

$$X_{n,1}(t) := < \vec{x} >_{-1}^* (t) = < \vec{x}^* >_1 (t).$$

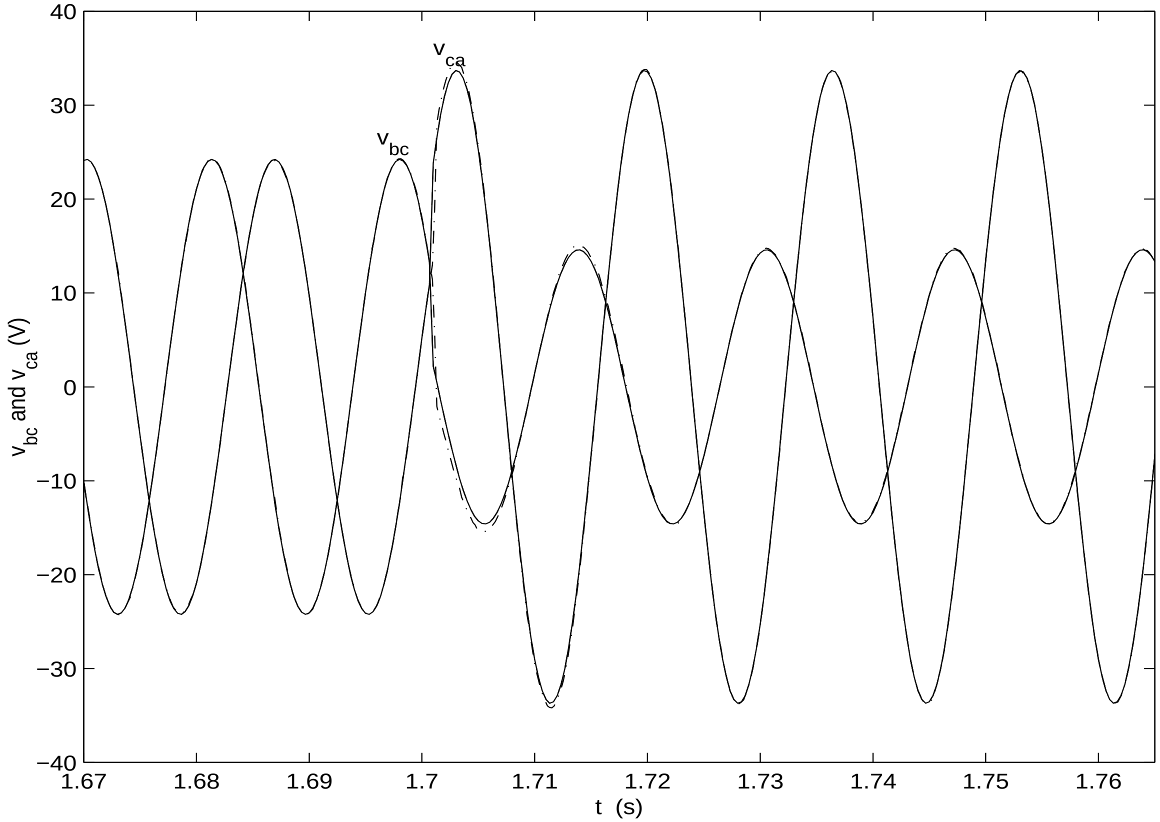
Space vector model of a three-phase induction machine:

$$
\begin{aligned}
\vec{v_s} &= (r_s + L_s \frac{d}{dt})\vec{i_s} + L_m \frac{d}{dt}\vec{i_r} \\
0 &= L_m \frac{d}{dt}\vec{i_s} + (r_r + L_r \frac{d}{dt})\vec{i_r} - j\omega_r \frac{P}{2}(L_m \vec{i_s} + L_r \vec{i_r}) \\
J\frac{d}{dt}\omega_r &= \frac{3P}{4} L_m \Im(\vec{i_s}\vec{i_r}^*) - B\omega_r - T_L
\end{aligned}
$$

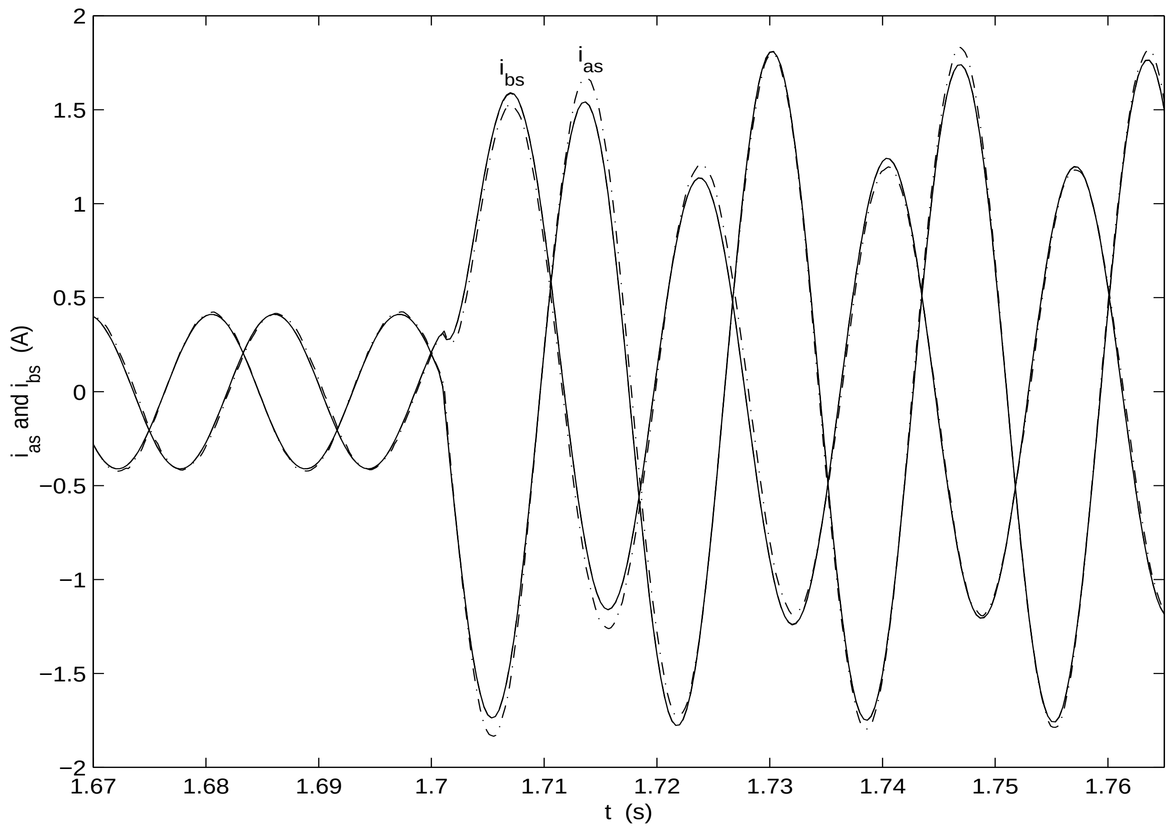
Dynamic phasor model of a three-phase induction machine:

$$
\begin{aligned}
V_p &= (r_s + j\omega_s L_s + L_s\frac{d}{dt})I_{p,s} + (j\omega_s L_m + L_m\frac{d}{dt})I_{p,r} \\
0 &= (j\omega_s L_m + L_m\frac{d}{dt})I_{p,s} + [r_r + (j\omega_s L_r + L_r\frac{d}{dt})]I_{p,r} - j\Omega_{r,0}\frac{P}{2}(L_m I_{p,s} + L_r I_{p,r}) \\
&\quad -j\Omega_{r,2}\frac{P}{2}(L_m I_{n,s}^* + L_r I_{n,r}^*) \\
V_n^* &= (r_s - j\omega_s L_s + L_s\frac{d}{dt})I_{n,s}^* + (-j\omega_s L_m + L_m\frac{d}{dt})I_{n,r}^* \\
0 &= (-j\omega_s L_m + L_m\frac{d}{dt})I_{n,s}^* + [r_r + (-j\omega_s L_r + L_r\frac{d}{dt})]I_{n,r}^* \\
&\quad -j\Omega_{r,0}\frac{P}{2}(L_m I_{n,s}^* + L_r I_{n,r}^*) - j\Omega_{r,2}^*\frac{P}{2}(L_m I_{p,s} + L_r I_{p,r}) \\
J\frac{d}{dt}\Omega_{r,0} &= \frac{3P}{4}L_m \Im(I_{p,s}I_{p,r}^* + I_{n,s}^* I_{n,r}) - B\Omega_{r,0} - T_L \\
J\frac{d}{dt}\Omega_{r,2} &= \frac{3P}{j8}L_m(I_{p,s}I_{n,r} - I_{n,s}I_{p,r}) - (B + j2J\omega_s)\Omega_{r,2}
\end{aligned}
$$

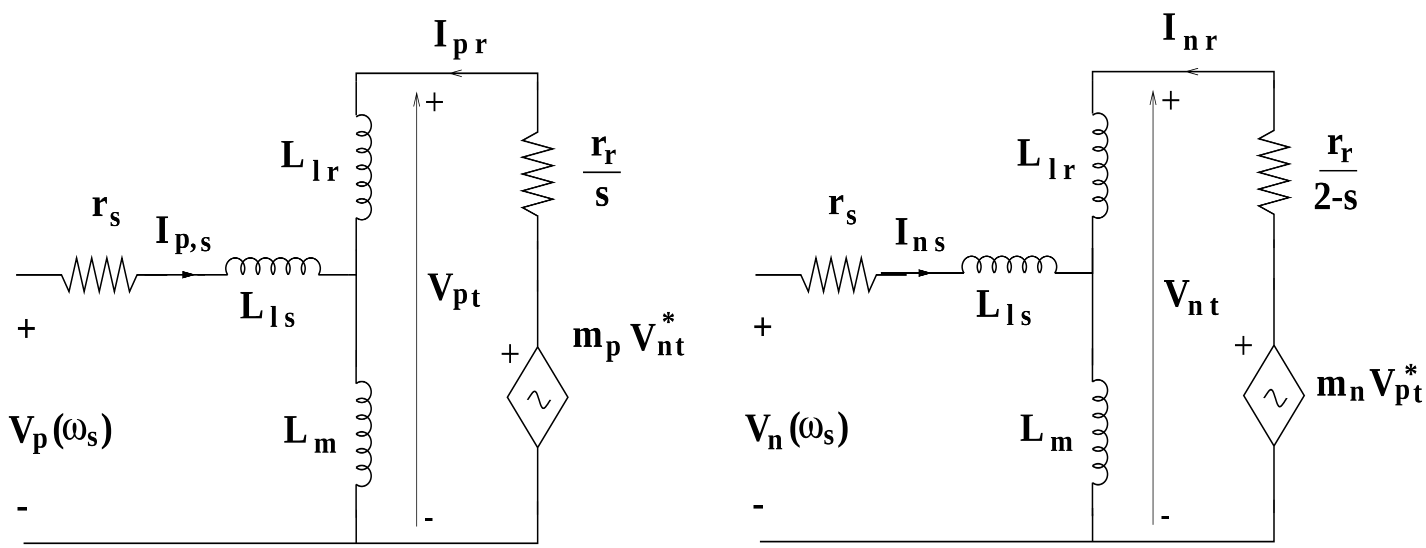
Experimental evaluation:



Experimental evaluation, cont.



Steady-state equivalent circuit ($m_p = \Omega_{r,2}/(\omega_s s)$ and $m_n = -\Omega_{r,2}/[\omega_s(2-s)]$):



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Extension to Polyphase Systems

Dynamical symmetric components - recall $\alpha = e^{j\frac{2\pi}{3}}$,

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}(\tau) = \sum_{\ell=-\infty}^{\infty} e^{j\ell\omega_s\tau} \underbrace{\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ \alpha^* & \alpha & 1 \\ \alpha & \alpha^* & 1 \end{bmatrix}}_{A} \begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t)$$

$$\begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t) = \frac{1}{T}\int_{t-T}^{t} e^{-j\ell\omega_s\tau}\ A^H \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}(\tau)d\tau = \begin{bmatrix} < x >_{p,\ell} \\ < x >_{n,\ell} \\ < x >_{z,\ell} \end{bmatrix}(t).$$

$$\frac{d}{dt}\begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t) = A^H \begin{bmatrix} < {}^{d}\!/\!_{d\tau} x_a(\tau) >_\ell \\ < {}^{d}\!/\!_{d\tau} x_b(\tau) >_\ell \\ < {}^{d}\!/\!_{d\tau} x_c(\tau) >_\ell \end{bmatrix}(t) - j\ell\omega_s \begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t)$$

Properties of Dynamic Symmetric Components

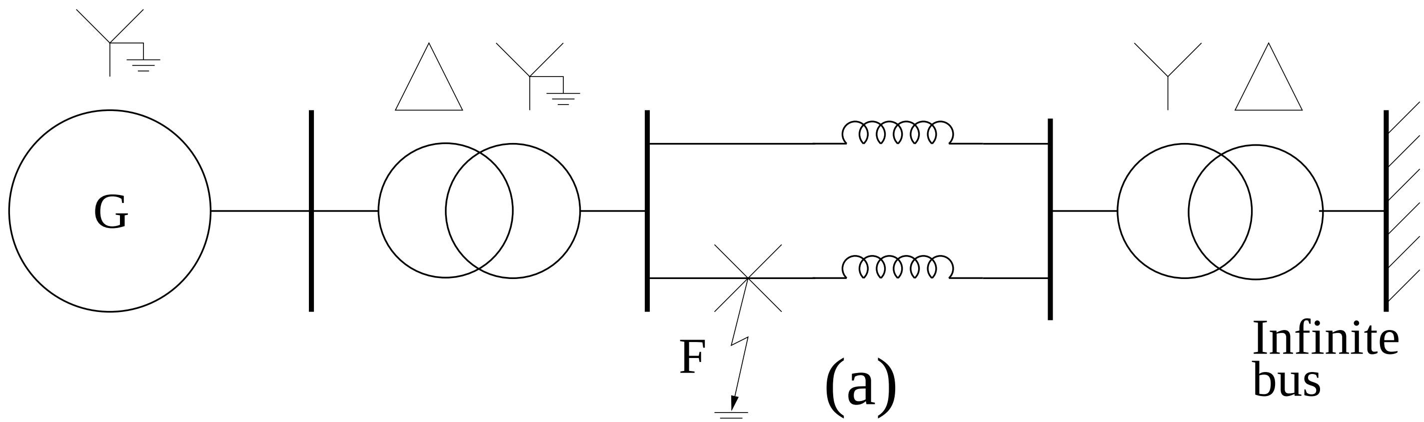
For real waveforms:

$$X_{p,\ell} = X^*_{n,-\ell} \quad X_{n,\ell} = X^*_{p,-\ell} \quad X_{z,\ell} = X^*_{z,-\ell}.$$

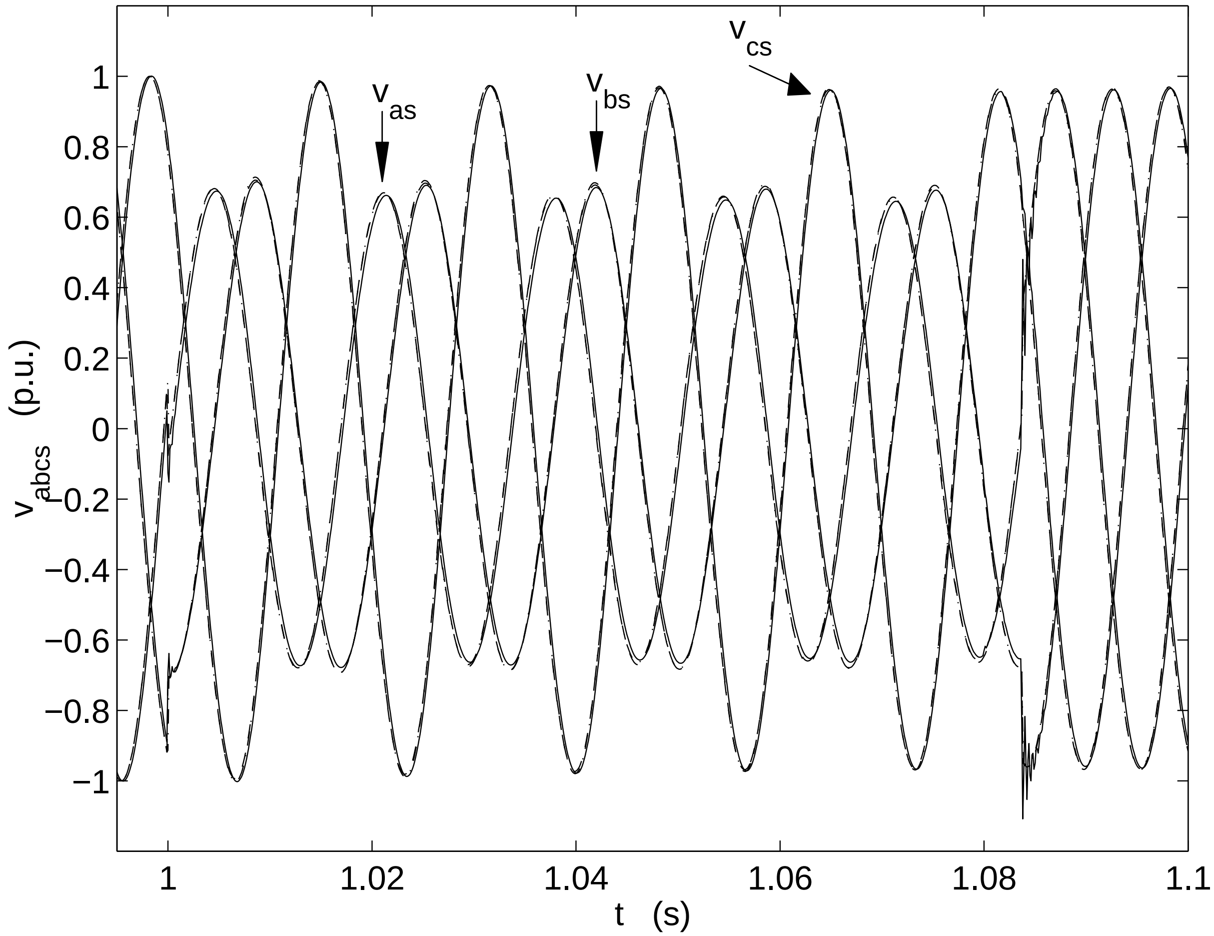
Connection with space vectors:

$$\vec{x}(\tau) = \frac{2}{\sqrt{3}} \sum_{\ell=-\infty}^{\infty} e^{j\ell\omega_s\tau} X_{p,\ell}(\tau).$$

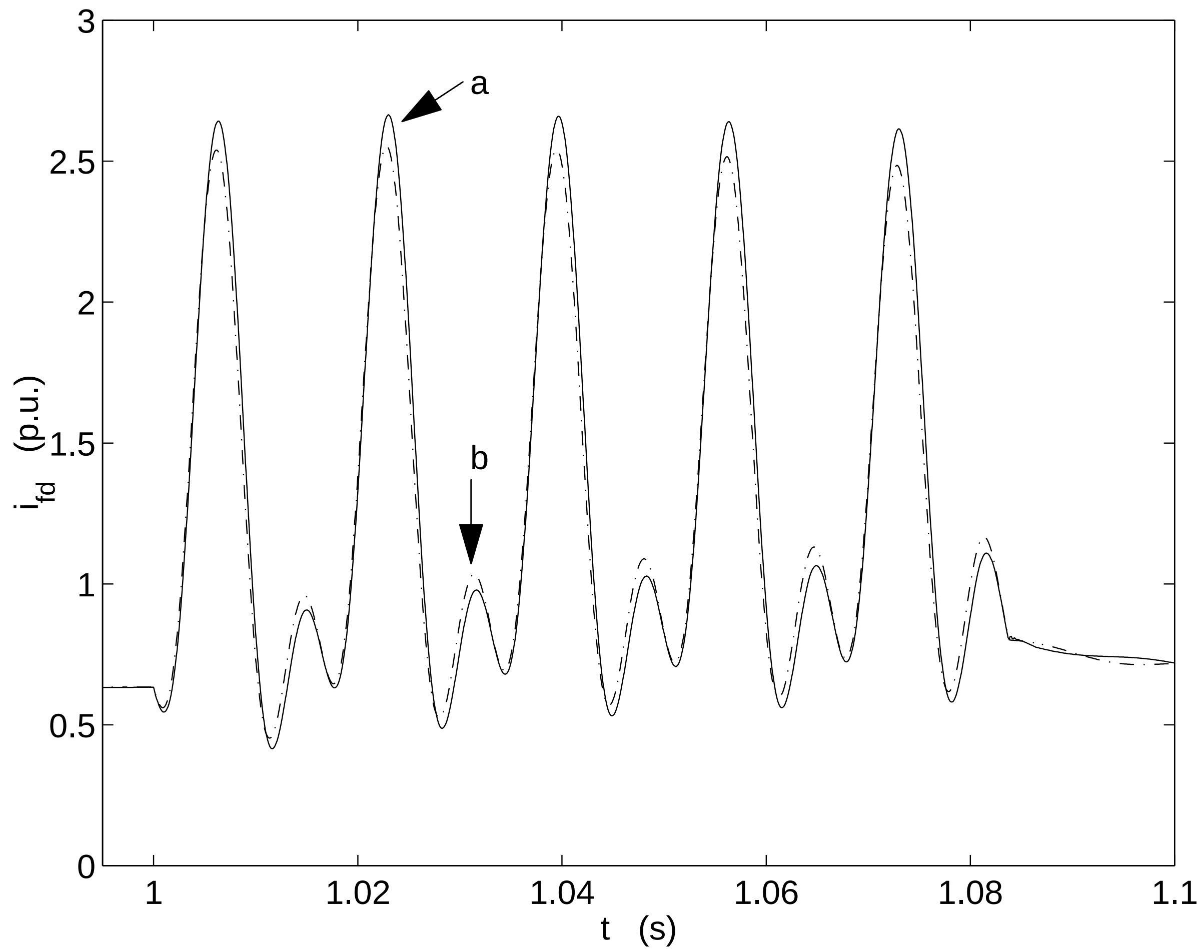
Asymmetric Faults in Power Systems



Line voltages



Field current



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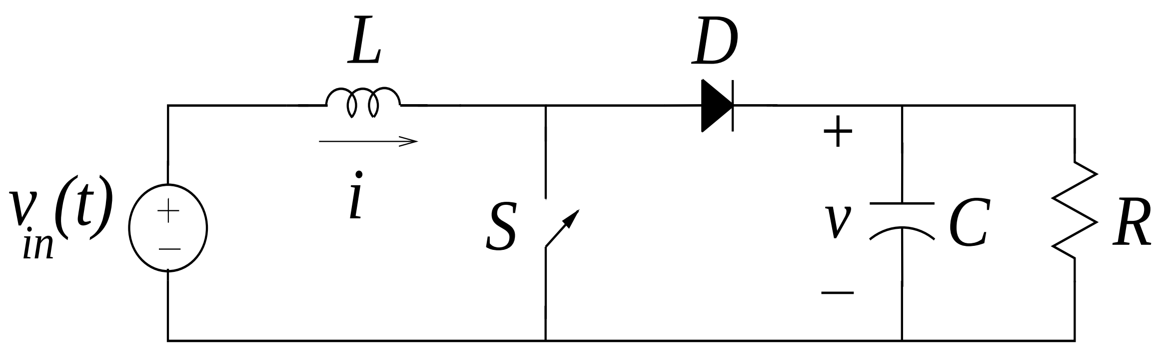
Model Reduction - DC/DC Converters



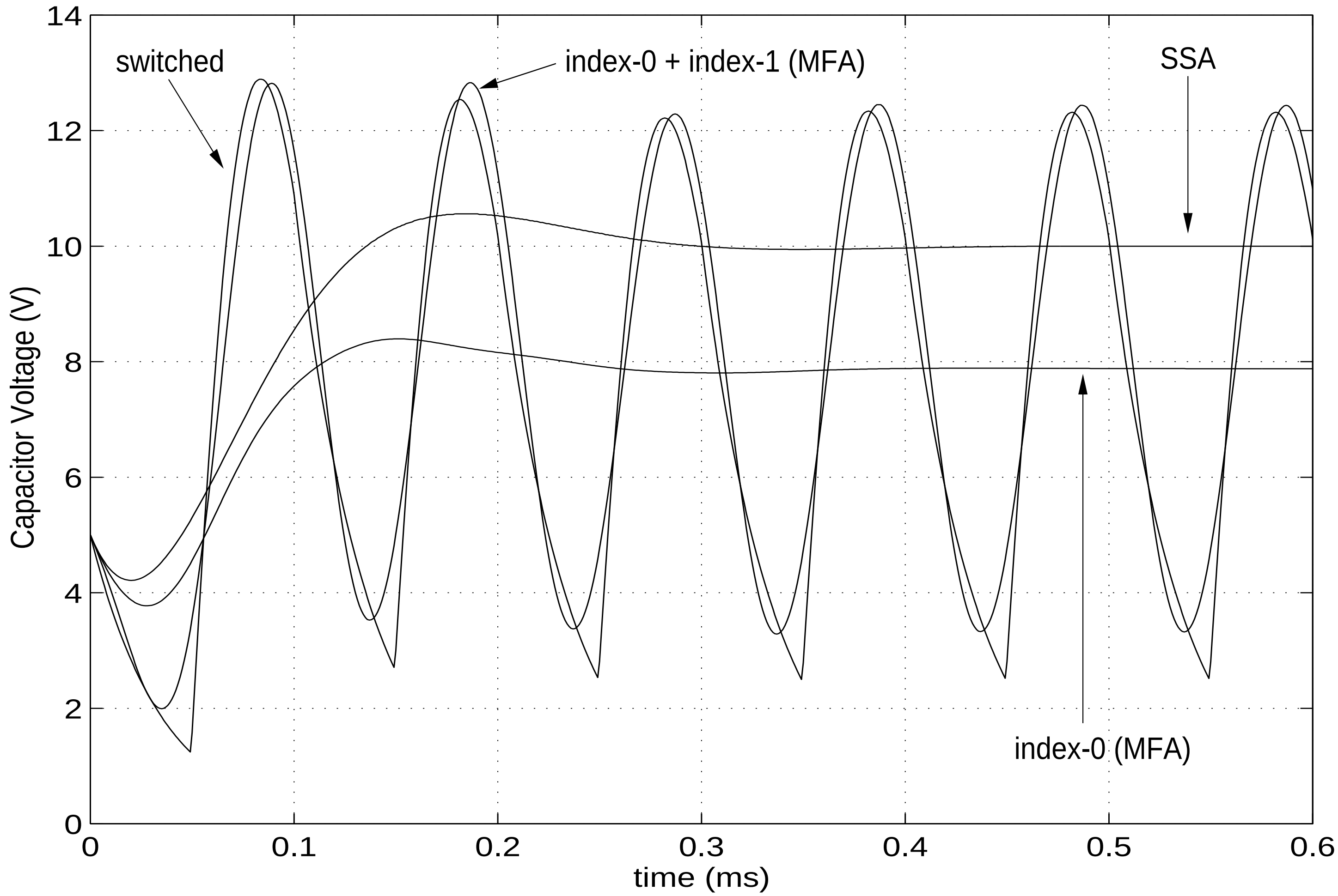
Model in continuous conduction ($q(\cdot) = 1$ when S closed):

$$
\begin{aligned}
L\frac{di}{dt} &= v_{in}(t) - [1 - q(t)]v(t) \\
C\frac{dv}{dt} &= [1 - q(t)]i(t) - \frac{1}{R}v(t)
\end{aligned}
$$

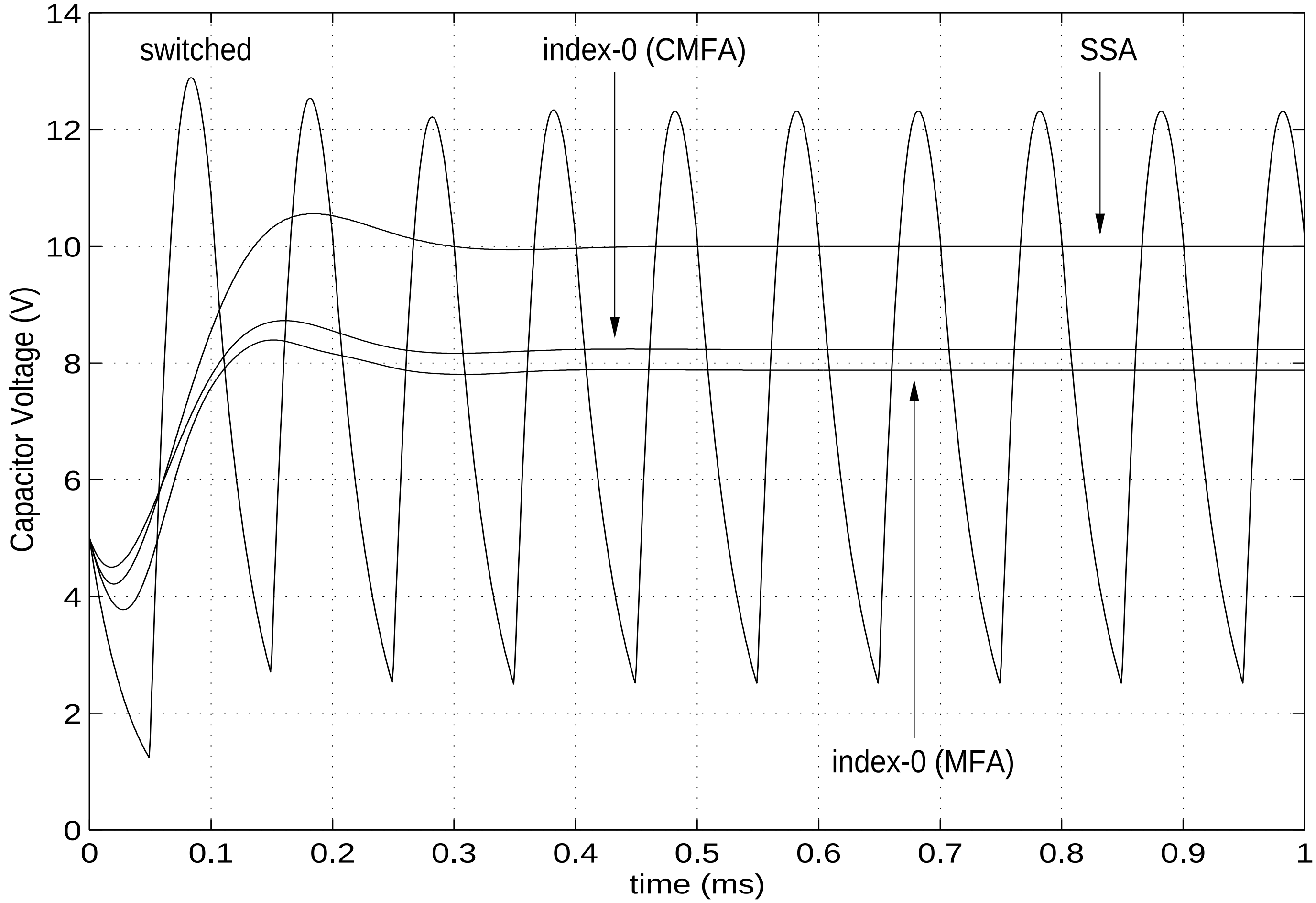
Model Reduction - DC/DC Converters

$$
\begin{aligned}
L\frac{d<i>_0}{dt} &= V_{in} - (1-<q>_0)<v>_0 + 2<q>_1^R<v>_1^R + 2<q>_1^I<v>_1^I \\
C\frac{d<v>_0}{dt} &= (1-<q>_0)<i>_0 - \frac{<v>_0}{R} - 2<q>_1^R<i>_1^R - 2<q>_1^I<i>_1^I \\
L\frac{d<i>_1^R}{dt} &= L\omega_s<i>_1^I + (1-<q>_0)<v>_1^R + <v>_0<q>_1^R \\
L\frac{d<i>_1^I}{dt} &= -L\omega_s<i>_1^R - (1-<q>_0)<v>_1^I + <v>_0<q>_1^I \\
C\frac{d<v>_1^R}{dt} &= C\omega_s<v>_1^I + (1-<q>_0)<i>_1^R - <i>_0<q>_1^R - \frac{<v>_1^R}{R} \\
C\frac{d<v>_1^I}{dt} &= -C\omega_s<v>_1^R + (1-<q>_0)<i>_1^I - <i>_0<q>_1^I - \frac{<v>_1^I}{R}
\end{aligned}
$$

Model Reduction - DC/DC Converters



Model Reduction - DC/DC Converters



Estimation for our Simple Example

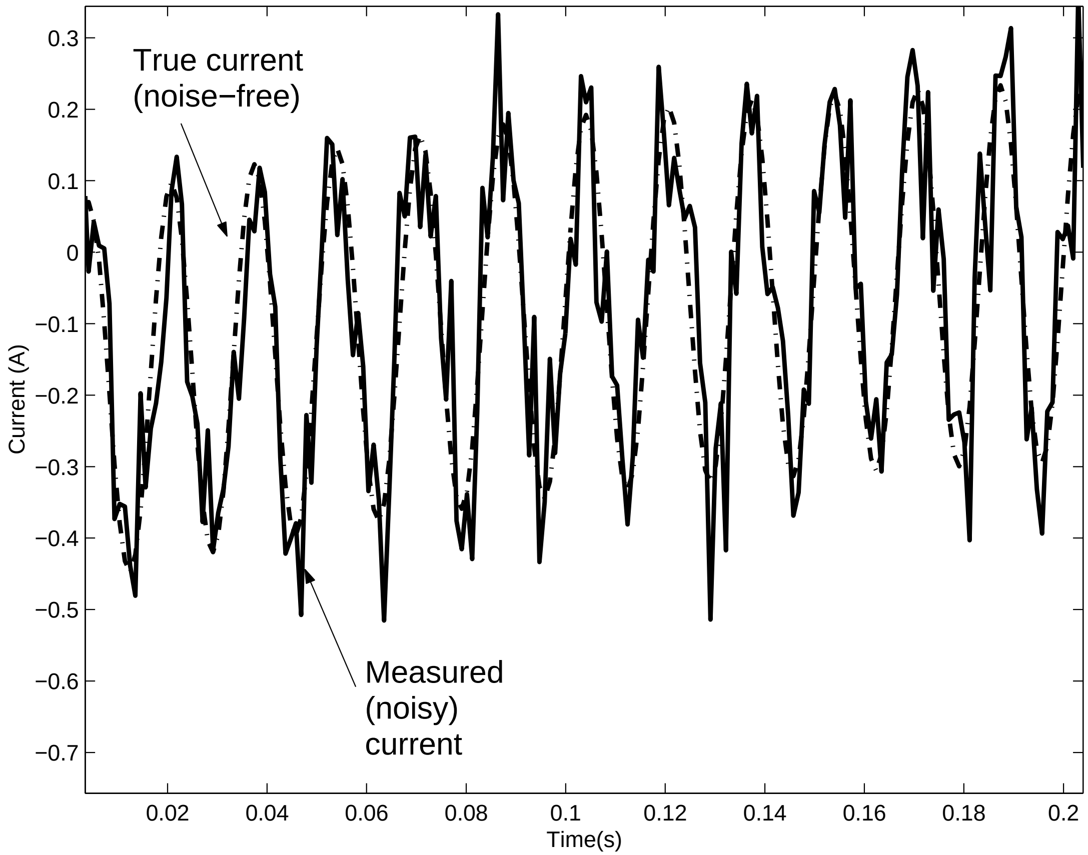
Again we consider a simple RL circuit with $\cos$ excitation:

$$\frac{d}{dt}\begin{bmatrix} I_0 \\ \mu \\ \nu \end{bmatrix} = \begin{bmatrix} -R/L & 0 & 0 \\ 0 & -R/L & \omega_s \\ 0 & -\omega_s & -R/L \end{bmatrix} \begin{bmatrix} I_0 \\ \mu \\ \nu \end{bmatrix} + \begin{bmatrix} 0 \\ 1/2L \\ 0 \end{bmatrix} V$$

$$i(t) = \underbrace{\begin{bmatrix} 1 & 2\cos(\omega_s t) & 2\sin(\omega_s t) \end{bmatrix}}_{C(t)=C(t+T)} \begin{bmatrix} I_0 \\ \mu \\ \nu \end{bmatrix}$$

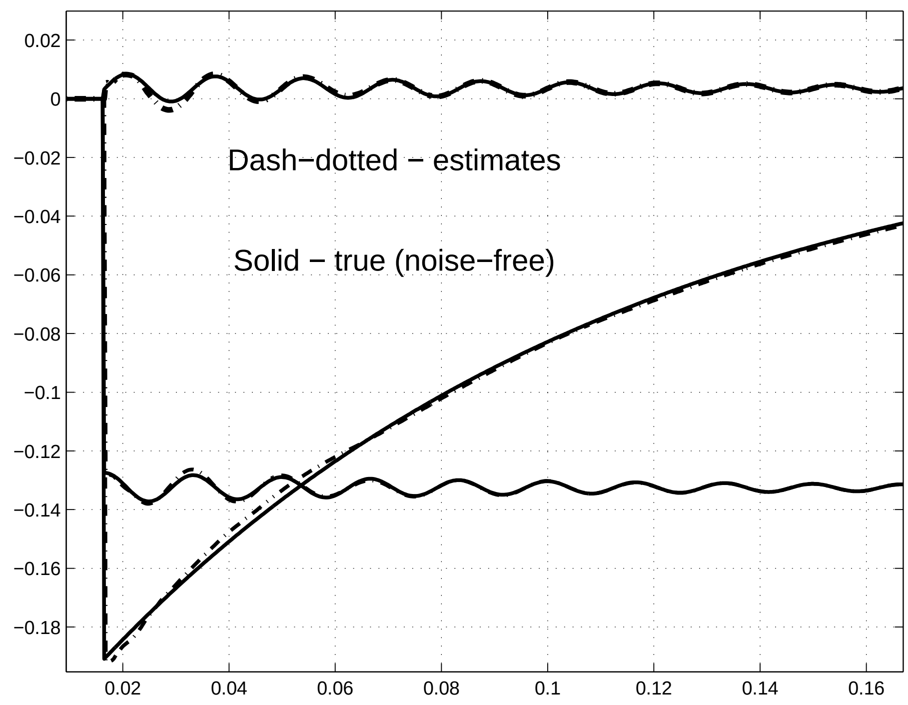
Estimation for our Simple Example, cont. 1

True (dash-dotted) and "measured" (solid line) current:



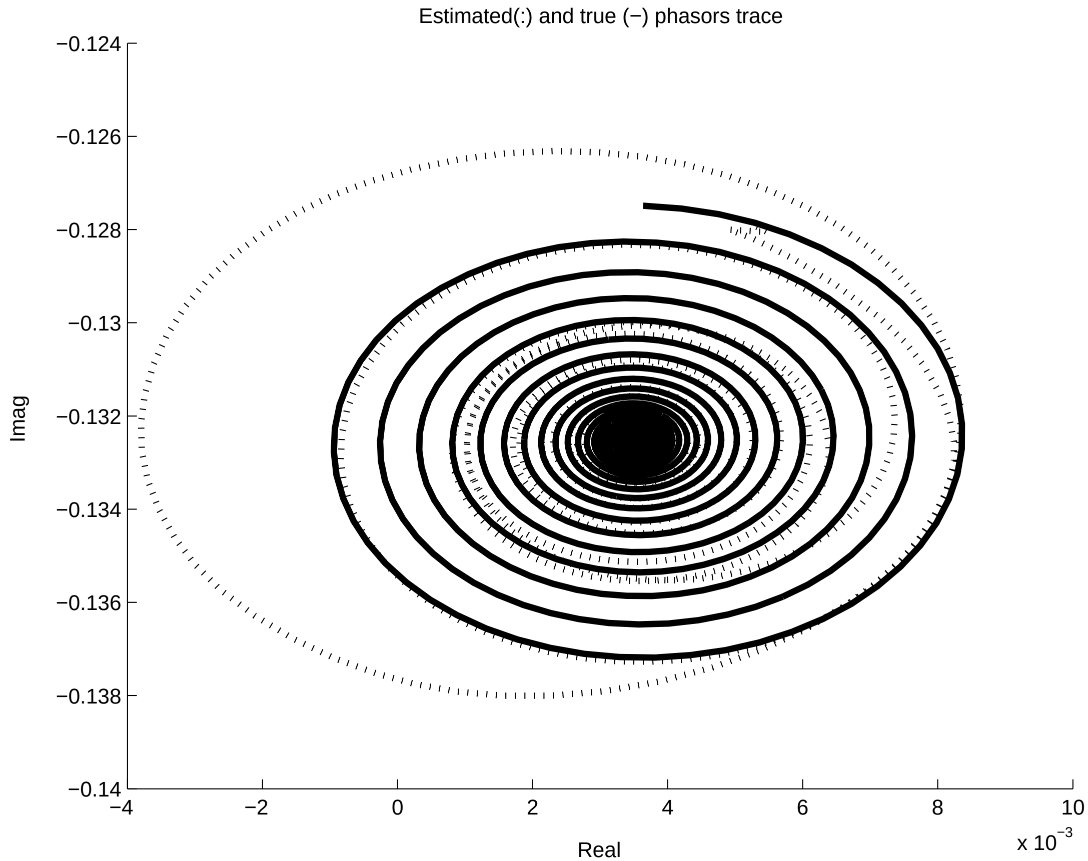
Estimation for our Simple Example, cont. 2

Convergence of estimates:



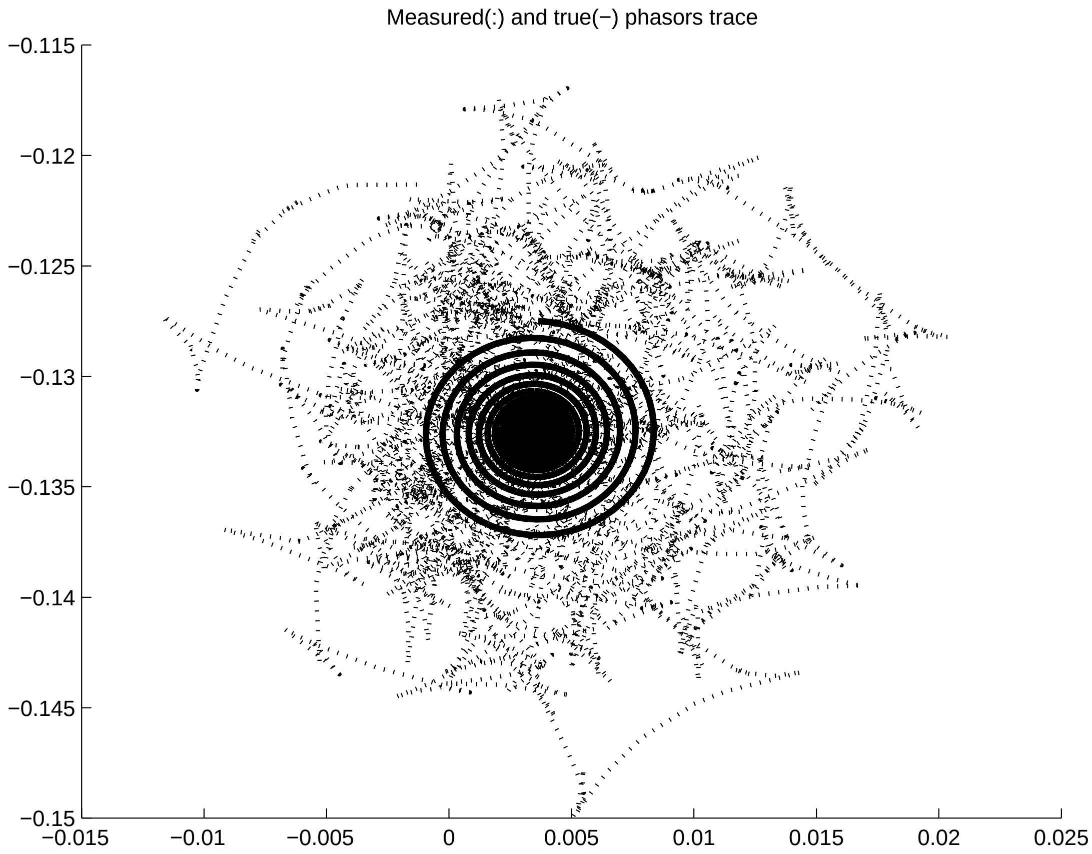
Estimation for our Simple Example, cont. 3

Another view of convergence of the fundamental phasor estimate:



Estimation for our Simple Example, cont. 4

Contrast with a model-free ("running FFT") estimate:



Summary

Dynamic phasors yield simple, but powerful large-signal dynamic models.

A unified approach with additional applications in power electronics (resonant converters), electric drives (torque ripple minimization) and power systems (protection).

Both refinements and simplifications are possible for various accuracy requirements.

Models are modular, and compatible with engineering experience and intuition,

A timely re-examination of analytical tools in energy processing addresses challenges posed by advances in semiconductor and computer technology.