School of Engineering Science
Simon Fraser University

ENSC-380, Summer 2007
Midterm Test 1
June 26, 2007

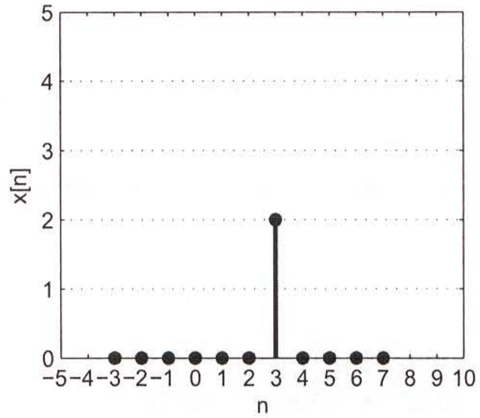
Name: ANSWERS

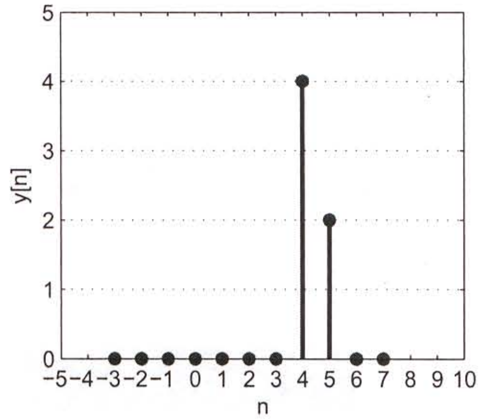
Student No.: _ _ _ _ _ - _ _ _ _

- Aid allowed: One double sided A4 Formula sheet.
- There are **4 questions** in this exam, please attempt all.
- Time: 55 Minutes.

Question	Score
1	/5
2	/5
3	/5
4	/10
Total	/25

1. The input and output of a DT-LTI system are shown below. What is the impulse response of the system, $h[n]$?





$\delta[n] \longrightarrow \boxed{\mathcal{H}} \longrightarrow h[n]$ then

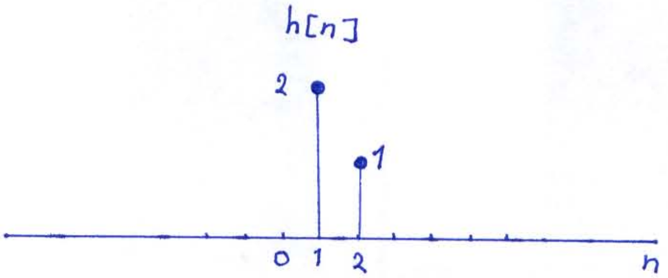
Here we have: $x[n] = 2\,\delta[n-3]$

$\Rightarrow y[n] = 2\,h[n-3]$

$\Rightarrow h[n] = \frac{1}{2}\,y[n+3]$

$y[n] = 4\,\delta[n-4] + 2\,\delta[n-5]$

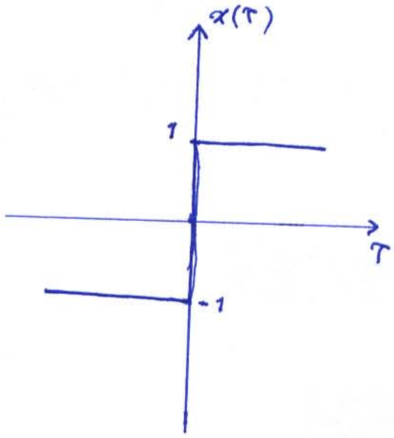
$\Rightarrow h[n] = 2\,\delta[n-1] + \delta[n-2]$

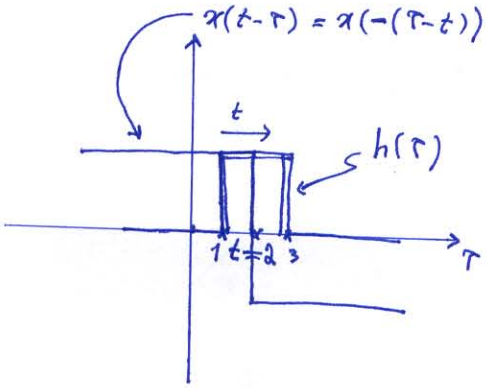


2. The impulse response of a CT-LTI system is given:

$$h(t) = \mathrm{rect}\left(\frac{t-2}{2}\right)$$

If the signal $x(t) = \mathrm{sgn}(t)$ is the input to the system, at what point of time, is the output, $y(t)$ equal to zero?
(**Hint:** It may be easier to show your solution graphically than analytically.)





$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{1}^{3} h(\tau)\, x(t-\tau)\, d\tau$$

at $t=2$ we have: $y(2) = \int_1^2 \underbrace{h(\tau)}_{=1} \times 1 \, d\tau + \int_2^3 \underbrace{h(\tau)}_{=1} \times (-1)\, d\tau = 0$

$\Rightarrow \boxed{y(2) = 0}$

3. Use the definition of CTFS to show that the Fourier Series of an "even" and real signal, $x(t)$, is a "real" function, $X[k]$.

$$X[k] = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\, e^{-j2\pi k f_0 t}\, dt$$

$$= \frac{1}{T_0}\Big[\underbrace{\int_{-\frac{T_0}{2}}^{0} x(t)\, e^{-j2\pi k f_0 t}\, dt}_{t=-\tau} + \int_{0}^{\frac{T_0}{2}} x(t)\, e^{-j2\pi k f_0 t}\, dt\Big]$$

change of variable

$$= \frac{1}{T_0}\Big[-\int_{0}^{\frac{T_0}{2}} \underbrace{x(-\tau)}_{=x(\tau) \text{ or } x(t)}\, e^{j2\pi k f_0 \tau}\, (-d\tau) + \int_{0}^{\frac{T_0}{2}} x(t)\, e^{-j2\pi k f_0 t}\, dt\Big]$$

$$= \frac{1}{T_0}\int_{0}^{\frac{T_0}{2}} x(t)\cdot\left(e^{j2\pi k f_0 t} + e^{-j2\pi k f_0 t}\right) dt$$

$$= \frac{1}{T_0}\int_{0}^{\frac{T_0}{2}} \underbrace{x(t)}_{\text{real}} \cdot 2 \underbrace{\cos(2\pi k f_0 t)}_{\text{real}}\, dt \Rightarrow X[k] \text{ is real.}$$

Alternative (simpler) approach:

$$X[k] = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\, e^{-j2\pi k f_0 t}\, dt = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\cdot \cos(2\pi k f_0 t)\, dt + \frac{1}{T_0} j \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \underbrace{\underbrace{x(t)}_{\text{even}} \times \underbrace{\sin(2\pi k f_0 t)}_{\text{Odd}}}_{\text{Odd}}\, dt$$

integral over one period $= 0$

$$\Rightarrow \mathrm{Im}\{X[k]\} = 0 \Rightarrow X[k] \text{ is real.}$$

4. The "transfer function" of an LTI system is given:

$$H(f) = \mathcal{F}\{h(t)\} = \frac{1}{(3 + j2\pi f)}$$

- **(a)** Find the linear constant-coefficient differential equation that defines the input-output relationship of this system.
- **(b)** Find the response of this system ($y(t)$), to the input $x(t) = \cos(2\pi f_0 t)$. Use the convolution property of CTFT to first find $Y(f)$ and then $y(t)$. You **do not** need to simplify your answer for this part.
- **Bonus (3Points)** Simplify $y(t)$ as much as you can. For simplicity your final answer can be in terms of $|H(f_0)|$ and $\angle H(f_0)$. (No need to evaluate these two values).

You may find the following FT pairs useful:

$$e^{j2\pi f_0 t} \overset{\mathcal{F}}{\longleftrightarrow} \delta(f - f_0) \qquad \cos(2\pi f_0 t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$$

(a) $H(f) = \frac{Y(f)}{X(f)} \Rightarrow 3Y(f) + (j2\pi f)Y(f) = X(f) \xrightarrow{\mathcal{F}^{-1}} 3y(t) + y'(t) = x(t)$

(b) $Y(f) = H(f)\cdot X(f) = H(f)\cdot\left[\frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)\right] = \frac{1}{2}H(f_0)\,\delta(f-f_0) + \frac{1}{2}H(-f_0)\,\delta(f+f_0)$

$\Rightarrow y(t) = \mathcal{F}^{-1}\{Y(f)\} = \frac{1}{2}H(f_0)\,e^{j2\pi f_0 t} + \frac{1}{2}H(-f_0)\,e^{-j2\pi f_0 t}$

Bonus: Let $H(f) = |H(f)|\,e^{j\angle H(f)}$

For the given $H(f)$ we can show that: $|H(f)| = |H(-f)|$ & $\angle H(f) = -\angle H(-f)$

Let $\theta(f) = \angle H(f)$

$\Rightarrow y(t) = \frac{1}{2}|H(f_0)|\,e^{j\theta(f_0)}\cdot e^{j2\pi f_0 t}$

$+ \frac{1}{2}|H(f_0)|\,e^{-j\theta(f_0)}\cdot e^{-j2\pi f_0 t}$

$= |H(f_0)|\cdot\cos(2\pi f_0 t + \theta(f_0))$