School of Engineering Science
Simon Fraser University

ENSC-380, Summer 2007
Midterm Test 2
July 19, 2007

Name: ANSWERS

Student No.: _ _ _ _ _ _ - _ _ _ _ _

- Aid allowed: Two double sided A4 formula sheets.
- There are **4 questions** in this exam, please attempt all.
- Time: 1 Hour and 30 Minutes

Potentially useful Formulas:

$$\cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]$$

$$\sin(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2j}[\delta(f-f_0)-\delta(f+f_0)]$$

$$\cos(a)\cos(b) = \frac{1}{2}[\cos(a+b)+\cos(a-b)]$$

$$\sin(a)\sin(b) = \frac{1}{2}[\cos(a-b)-\cos(a+b)]$$

Question	Score
1	/5
2	/5
3	/5
4	/5
Total	/20

1. Consider the periodic signal:

$$x_p(t) = 1 + \cos(10\pi t)$$

- (a) Find the **CTFS coefficients** (harmonic function) of $x_p(t)$, i.e. $X_p[k]$.
- (b) Let

$$x(t) = \begin{cases} x_p(t) & 0 \le t \le 0.2s \\ 0 & \text{otherwise} \end{cases}$$

Show that the expected relationship between the CTFT of $x(t)$, i.e., $X(f)$, and the CTFS of $x_p(t)$ holds.

Hint: You do not need to find $X(f)$ for all f, but only at specific frequencies which relate to the relationship..

(a) $x_p(t) = 1 + \frac{1}{2} e^{j2\pi(1\times 5)t} + \frac{1}{2} e^{j2\pi(-1\times 5)t}$

$T_p = \frac{1}{5}$ (sec) $\Rightarrow f_p = 5$ Hz fundamental period

We can see that $\left(x(t) = \sum X[k] e^{j2\pi(k f_p)t} \right)$: $X_p[0] = 1$, $X_p[\pm 1] = \frac{1}{2}$

$X_p[k] = 0 \quad \forall k \neq 0, \pm 1$

(b) $X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt = \int_0^{0.2} (1 + \cos(10\pi t)) e^{-j2\pi f t} dt$

The expected relation-ship between $X(f)$ and $X_p[k]$ is: $X_p[k] = f_p \, X(k f_p)$

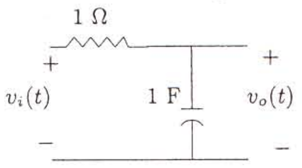
$X(k f_p) = \int_0^{0.2} (1 + \cos(10\pi t)) e^{-j10\pi kt} dt$

$= \underbrace{\int_0^{0.2} e^{-j10\pi kt} dt}_{1} + \underbrace{\int_0^{0.2} \cos(10\pi t)\cos(10\pi kt)\, dt}_{2} - j\underbrace{\int_0 \cos(10\pi t)\sin(10\pi kt)\, dt}_{3}$

1 $= \begin{cases} 0.2 & k = 0 \\ 0 & k \neq 0 \end{cases}$ 2 $= \begin{cases} \frac{1}{10} & k = 1, -1 \\ 0 & k \neq \pm 1 \end{cases}$ 3 $= 0 \quad \forall k$

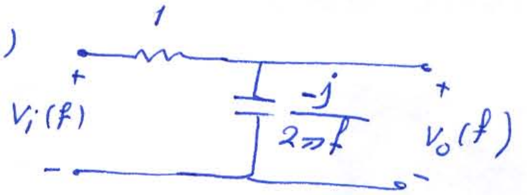
$\Rightarrow X(k f_p) = \begin{cases} 0.2 & k = 0 \\ 0.1 & k = \pm 1 \\ 0 & \text{else} \end{cases} \Rightarrow f_p X(k f_p) = \begin{cases} 1 & k = 0 \\ \frac{1}{2} & k = \pm 1 \\ 0 & \text{else} \end{cases} = X_p[k]$

2. Consider the simple RC-low pass filter given below, with input $v_i(t)$ and output $v_o(t)$.



- (a) Find the **magnitude and phase** of the frequency response of this system, i.e., $|H(f)|$ and $\angle H(f)$, where $H(f) = \frac{V_o(f)}{V_i(f)}$.
- (b) Plot the **Bode magnitude response** of the filter. Clearly show the asymptotic lines, their slopes, and the cutoff frequency of the filter. Cutoff frequency is the frequency at which the magnitude response of the filter falls 3 dB below it's maximum.
- (c) What is the output of this filter if the input is $v_i(t) = \cos(t)$?

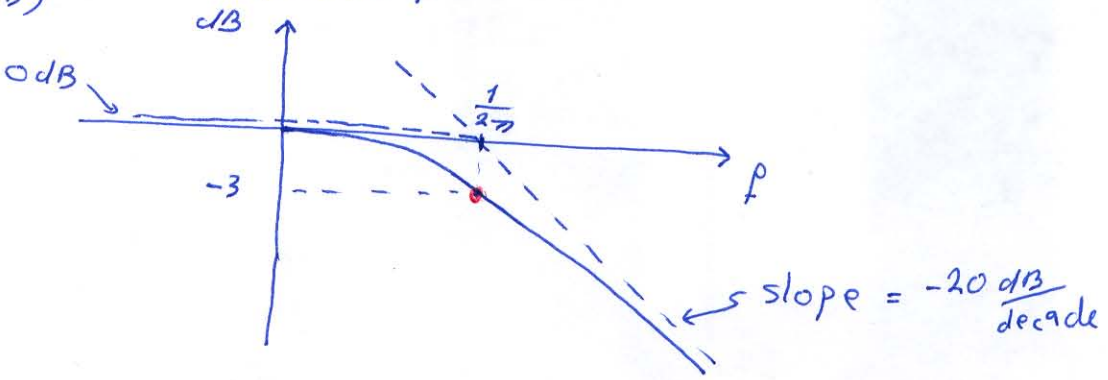
(a)



$$V_o(f) = \frac{V_i(f) \times \frac{-j}{2\pi f}}{1 - \frac{j}{2\pi f}} \Rightarrow H(f) = \frac{V_o(f)}{V_i(f)} = \frac{1}{1 + j2\pi f}$$

$$|H(f)| = \frac{1}{\sqrt{1 + 4\pi^2 f^2}} \qquad \angle H(f) = -\tan^{-1}(2\pi f)$$

(b)



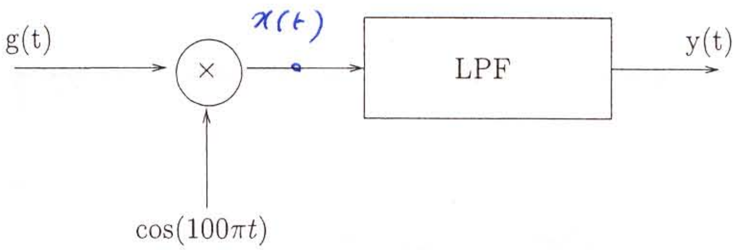
(c) $v_i(t) = \cos(t) \Rightarrow f_0 = \frac{1}{2\pi}$

$$\Rightarrow |H(f)| = -3\,dB = \frac{1}{\sqrt{2}}, \quad \angle H(f) = -\frac{\pi}{4}$$

$$\Rightarrow v_o(t) = \frac{1}{\sqrt{2}} \cos\left(t - \frac{\pi}{4}\right)$$

3. $m(t)$ is a signal with badwidth= 10 Hz. Double sideband modulation is performed to generate the signal $g(t)$ given below. $g(t)$ is then the input to the system shown in the figure, where the ideal low pass filter has a bandwidth of 10 Hz. Find $y(t)$.

$$g(t) = m(t)\sin(100\pi t)$$



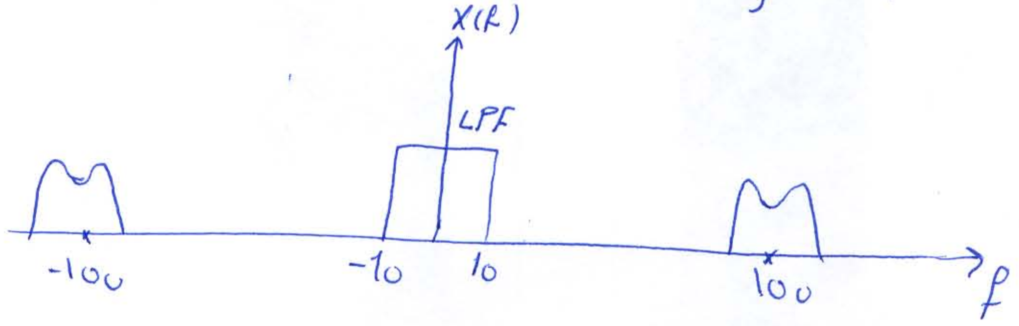
$$G(f) = M(f)\cdot \frac{1}{2j}\left(\delta(f-50) - \delta(f+50)\right)$$

$$= \frac{1}{2j}\left(M(f-50) - M(f+50)\right)$$

$$x(t) = g(t)\cdot \cos(100\pi t)$$

$$\Rightarrow X(f) = \frac{1}{2}\left(G(f-50) + G(f+50)\right)$$

$$= \frac{1}{2}\left[\frac{1}{2j}\left(M(f-100) - \cancel{M(f)}\right) + \frac{1}{2j}\left(\cancel{M(f)} + M(f+100)\right)\right]$$

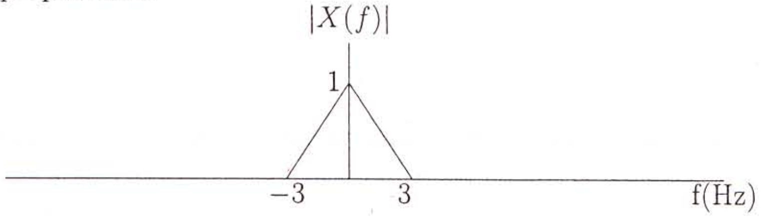


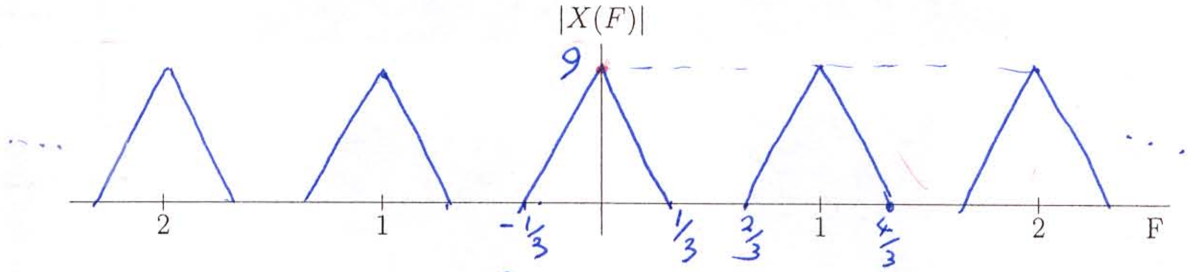
No lowpass signal is present in $X(f)$ $(x(t))$

$\Rightarrow$ $y(t)$ = output of the LPF $\boxed{= 0}$

4. The two parts of this question are independent of each other. For each part, show the important frequency and magnitude values on your graphs.

- (a) The CTFT of a CT signal $x(t)$ with bandwidth $f_m = 3$ (Hz) is given below. The signal is sampled at **1.5 times the Nyquist rate** to result in $x[n]$. Draw the DTFT of $x[n]$ in the graph provided.

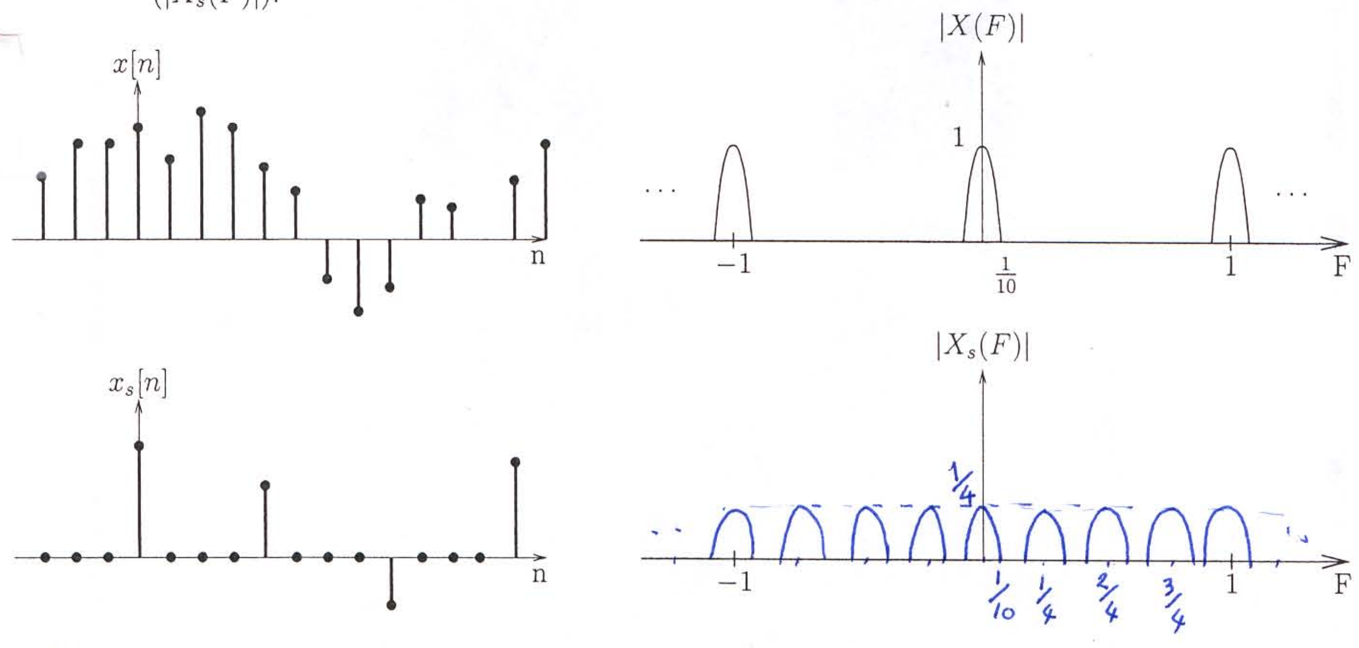




$f_m = 3 \Rightarrow$ Nyquist rate $= 6\,Hz \Rightarrow f_s = 1.5 \times 6 = 9\,Hz$

$\Rightarrow X(F) = f_s \sum_k \underset{CTFT}{X}(f_s(F-k)) = 9 \sum_k X(9(F-k))$

- (b) The DT signal $x[n]$ has been down sampled by a factor of 4 to result in $x_s[n]$, as shown in the figure. The DTFT of $x[n]$ is also given ($|X(F)|$). Draw the DTFT of $x_s[n]$ ($|X_s(F)|$).



$N_s = 4 \Rightarrow X_s(F) = X(F) \circledast \text{comb}(N_s F) = X(F) \circledast \text{Comb}(4F)$

$= X(F) \circledast \sum_{k=-\infty}^{\infty} \frac{1}{4}(F - \frac{k}{4})$