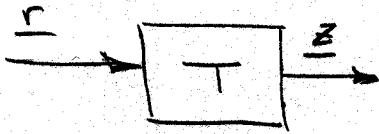


MAXIMUM LIKELIHOOD DETECTION IS UNAFFECTED BY INVERTIBLE TRANSFORMATIONS

- When we process a received signal, we frequently subject it to a transformation (a change of variables) before making the decision on data symbol(s).



Does the transformation affect the decisions? We'll see that invertible transformations have no effect on ML decisions. With non-invertible transformations, it depends on what they are.

- The original ML detector does this:

$$\hat{\underline{b}} = \underset{\underline{b}}{\operatorname{argmax}} p_{r|\underline{b}}(\underline{r}|\underline{b})$$

With a change of variables $\underline{z} = T(\underline{r})$, we do this:

$$\hat{\underline{b}} = \underset{\underline{b}}{\operatorname{argmax}} p_{z|\underline{b}}(\underline{z}|\underline{b})$$

where $T(\cdot)$ is assumed to be independent of \underline{b}

- If $T(\cdot)$ is invertible, then the Jacobian exists uniquely everywhere and

$$P_{z|b}(\underline{z}|\underline{b}) = P_{y|b}(T^{-1}(\underline{z})|\underline{b}) J(\underline{z}) = P_{y|b}(\underline{y}|\underline{b}) J(T(\underline{y}))$$

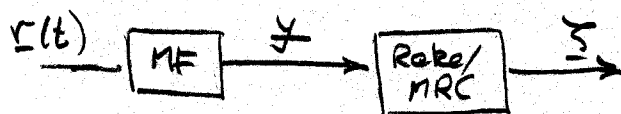
Since the Jacobian is a hypothesis-independent scale factor,

$$\operatorname{argmax}_{\underline{b}} P_{z|b}(\underline{z}|\underline{b}) = \operatorname{argmax}_{\underline{b}} P_{y|b}(\underline{y}|\underline{b})$$

so ML decisions are unaffected. Note that other detectors (e.g. ZF or MMSE) may well be affected, even if T is invertible.

- If T is not invertible, it depends on the situation and on what variable we want to estimate. Some examples:

— Matched filter and Rake/MRC

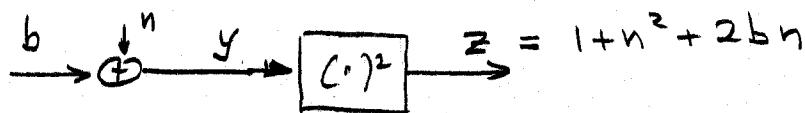


This is an extreme case, since we lose dimensionality at each stage. We have lost information — but only about the noise. In Appendix E, we saw that no information relevant to the \underline{b} decision is lost.

- BPSK and square law.

Suppose $y = b + n$, where $b = \pm 1$, n is real, Gaussian, zero mean.

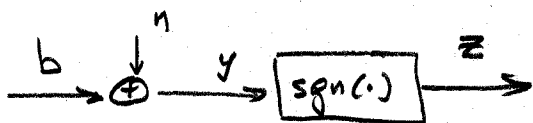
If $T(y) = y^2$, then



We have lost all information that could help with deciding b , T is not invertible.

- BPSK and slicer.

Same as above, but $T(y) = \text{sgn}(y)$. It's not invertible, but it doesn't matter in this case.



The transformation is equivalent to the ML decisions, so no effect.