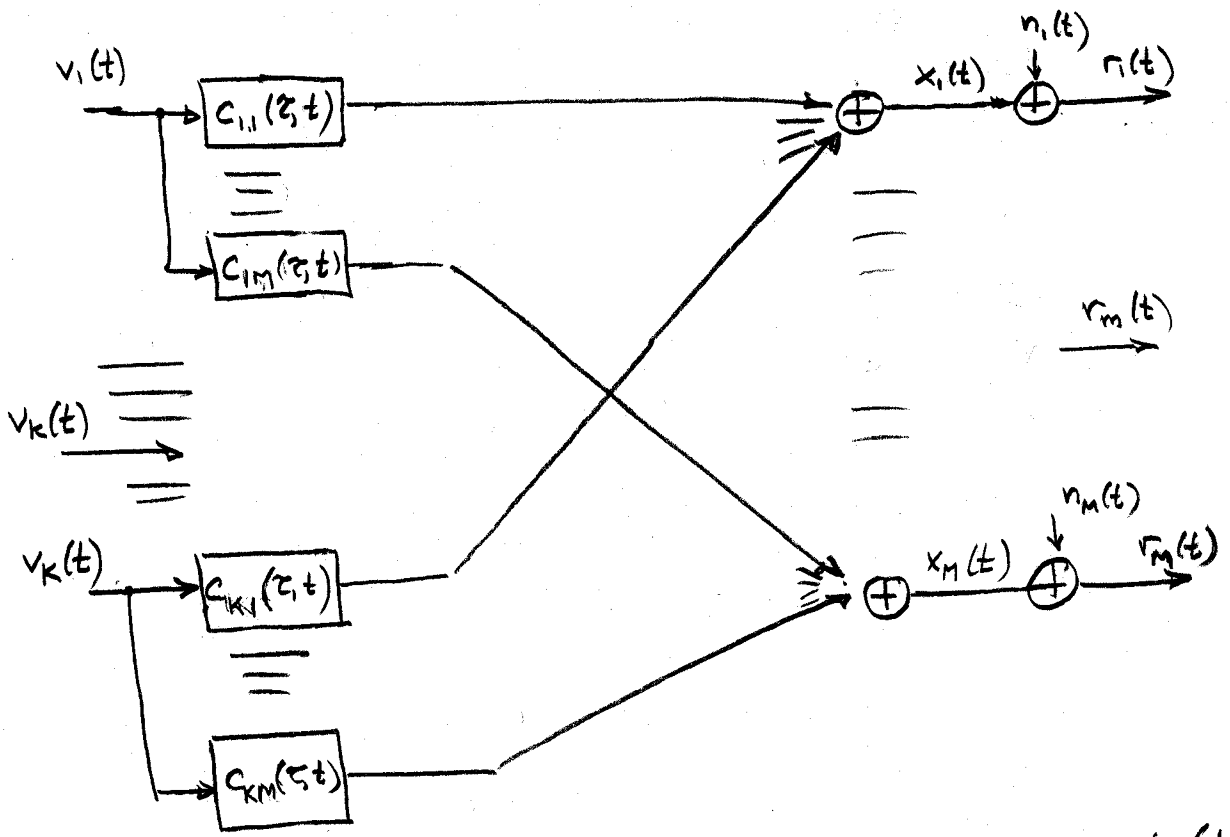
2.2 Received Signals

- All user signals are received at each base station antenna. Each path has its own impulse response (time variant)

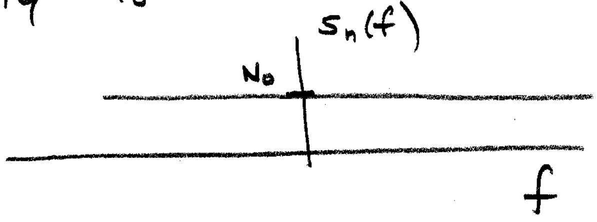
$$c_{k,m}(\tau, t)$$

- user k — antenna m
- echo delay τ — observation time t



- It's obviously linear, so it can't be too hard (!) But we'll spend the next several pages developing a compact, uncluttered notation.

- Noise processes complex, white, Gaussian, independent across antennas, power spectral density N_0



- At each antenna, received signal is

$$r_m(t) = x_m(t) + n_m(t)$$

where

$$x_m(t) = \sum_k c_{km}(\tau, t) \circledast v_k(t)$$

$$= \sum_k \int c_{k,m}(\tau, t)\, v_k(t-\tau)\, d\tau$$

$$= \sum_k \int c_{k,m}(t-\eta, t)\, v_k(\eta)\, d\eta$$

Channel is frequently modeled by discrete multipath

$$c_{k,m}(\tau, t) = \sum_{l=0}^{L-1} c_{k,l,m}(t)\, \delta(\tau - \tau_{kl})$$

so $$x_m(t) = \sum_k \sum_l c_{k,l,m}(t)\, v_k(t - \tau_{k,l})$$

$C_{user,\,path,\,antenna}(t)$

- If delays are multiples of sample period

$$\tau_{k,\ell} = \ell\, t_s = \ell \frac{T_c}{N_c} = \tau_\ell$$

then

$$x_m(t) = \sum_{k=1}^{K} \sum_{\ell=0}^{L-1} c_{k\ell m}(t)\, v_k(t - \ell t_s)$$

$$= \sum_n \sum_k \sum_\ell A_k b_k(n)\, c_{k,\ell,m}(t)\, s_k(t - nT - \tau_k - \ell t_s)$$

- If fading is slow or nonexistent

$$c_{k\ell m}(t) \longrightarrow c_{k\ell m}(n)$$

$$\longrightarrow c_{k\ell m}$$

$$x_m(t) = \sum_n \sum_k \sum_\ell A_k b_k(n)\, c_{k\ell m}(n)\, s_k(t - nT - \tau_k - \ell t_s)$$

- Received energy from user k at antenna m:

$$E_{skm} = \frac{1}{2}\int_{-\infty}^{\infty} |x_{km}(t)|^2 dt = \frac{1}{2}\int_{-\infty}^{\infty} \left| A_k \sum_n \sum_\ell b_k(n)\, c_{k\ell m}(n)\, s_k(t - nT - \tau_k - \ell t_s)\right|^2 dt$$

$$\text{average } \frac{1}{2}A_k^2 \sum_n \sum_{n'} \sum_\ell \sum_{\ell'} \overline{b_k(n)\, b_k^*(n')}\, c_{k\ell m}(n)\, c_{k\ell' m}^*(n') \cdot r_k\big((n'-n)T + (\ell'-\ell)t_s\big)$$

$$= \frac{1}{2}A_k^2 \sum_n \sum_\ell \sum_{\ell'} c_{k\ell m}(n)\, c_{k\ell m}^*(n)\, r_k\big((\ell'-\ell)t_s\big)$$

where $r_k(\tau) = \int_{-\infty}^{\infty} s_k(t)\, s^*(t-\tau)\, dt$ pulse autocorr'n function

Per sym, this is

$$E_{s_{k,m}}(n) = \tfrac{1}{2} A_k^2 \sum_{\ell} \sum_{\ell'} c_{k\ell m}(n)\, c^*_{k\ell' m}(n)\; r_k\big((\ell-\ell')t_s\big)$$

$$= \tfrac{1}{2} A_k^2\, \underline{c}^{+}(n)\, R_k\, \underline{c}(n)$$

and <u>if</u> path gains are drawn from an ensemble such that phases of $c_{k\ell m}(n)$, $c_{k\ell' m}(n)$ are independent, then a further average gives

$$E_{s\,k,m}(n) = \tfrac{1}{2} A_k^2 \sum_{\ell} |c_{k,\ell,m}(n)|^2\, r_k(0)$$

and if mean square values of path gains are constant

$$E_{s\,km} = \tfrac{1}{2} A_k^2 \underbrace{\sum_{\ell} 2\sigma^2_{c\,k\ell m}}_{\text{normalize to unity if stochastic}} \qquad \text{(assumes zero mean)}$$

$$E_{sk} = \tfrac{1}{2} A_k^2 = \tfrac{1}{2}\left(\sqrt{2E_{sk}}\right)^2 = E_{sk}$$

Why all this fussing with details?

Because correct SNR is important.

- Again, vector-matrix descriptions are common. As much as possible, keep indices k, l, m, n to tag user, path, antenna, symbol time.

  Sample $x_m(t)$ at N_s times chip rate, $t_s = T_c/N_s$ to produce

$$\underline{x}_m = \begin{bmatrix} x_m(0) \\ x_m(t_s) \\ x_m(2t_s) \\ \vdots \\ x_m(t_{last}) \end{bmatrix}$$

$$t_{last} = (\underbrace{N}_{\text{symbols}}\ \underbrace{N_c}_{\text{chips/sym}}\ \underbrace{N_s}_{\text{samples/chip}} + N_{del} + \underbrace{L}_{\text{max multipath delay}} - 1)\, t_s$$

$$N_{del} = \max_k N_k, \qquad \text{where } \tau_k = N_k t_s$$

- Build up the vector description gradually, since it is quite intricate.

  - sampled signature sequence

$$\underline{s}_k = \begin{bmatrix} s_k(0) \\ s_k(t_s) \\ \vdots \\ s_k((N_c N_s - 1)t_s) \end{bmatrix} \qquad \text{or } \underline{s}_k^{(n)} \text{ if long codes}$$

  - delay-spread signature matrix

$$S_k = \underbrace{\begin{bmatrix} \underline{s}_k & \underline{0} & \underline{0} & \cdots & \underline{0} \\ & \underline{s}_k & \underline{0} & & \underline{0} \\ & & \underline{s}_k & & \\ & & & & \underline{s}_k \\ \underline{0} & \underline{0} & & & \end{bmatrix}}_{L \text{ columns}} \Bigg\} \; N_c N_s + L - 1 \text{ rows} \qquad \text{or } S_k^{(n)} \text{ for long codes}$$

  - contribution to the output from user k, symbol n

$$A_k b_k(n) \sum_{l=0}^{L-1} s_k(t - nT - \tau_k - l t_s)\, c_{klm}(n) \qquad \text{(p. 2.2.3)}$$

n symbol times plus τ_k →

row $nN_cN_s + N_k$ →

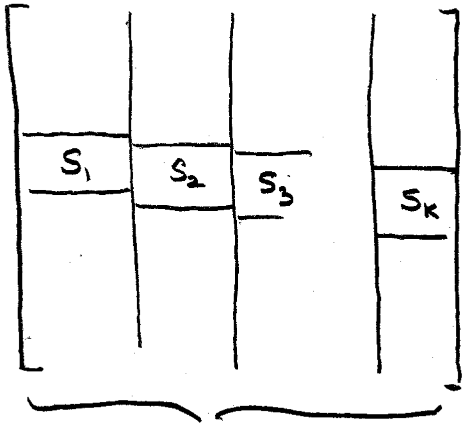
$$\underbrace{\begin{bmatrix} \underline{0} \\ S_k \\ \underline{0} \end{bmatrix}}_{L \text{ cols}} \; \underline{c}_{km}(n)\, A_k\, b_k(n)$$

column vector

$$\underline{c}_{km}(n) = \left(c_{k0m}(n), c_{k1m}(n), \ldots, c_{k,L-1,m}^{(n)}\right)^T$$

$$S_k(n)\, \underline{c}_{km}(n)\, A_k\, b_k(n)$$

- contribution to the output from symbol n of all users:



$\underbrace{}_{KL \text{ cols}}$

$S(n) = [\underline{S}_1(n), \underline{S}_2(n), \cdots, \underline{S}_K(n)]$

$$\underbrace{\begin{bmatrix} c_{1m}(n) & \underline{0}_L & \underline{0}_L \\ \underline{0}_L & \underline{c}_{2m}(n) & \underline{0}_L \\ \underline{0}_L & \underline{0}_L & \vdots \\ \vdots & \vdots & \underline{0}_L \\ \underline{0}_L & \underline{0}_L & \underline{c}_{Km}(n) \end{bmatrix}}_{KL \times K} \quad \underbrace{\begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & \ddots & \\ & & & A_K \end{bmatrix}}_{K \times K} \quad \begin{bmatrix} b_1(n) \\ b_2(n) \\ \vdots \\ b_K(n) \end{bmatrix}$$

$C_m(n) = \text{diag}(c_{1m}(n), \underline{c}_{2m}(n) \cdots \underline{c}_{Km}(n))$ $\quad$ $A = \text{diag}(A_1, \cdots, A_K)$ $\quad$ $\underline{b}(n)$

$$S(n)\, C_m(n)\, A\, \underline{b}(n)$$

This notation adapted from [Junt00b]

– Finally, the contribution from all symbol times

$$\underline{x}_m = \begin{bmatrix} S(0) & S(1) & \cdots & S(N) \end{bmatrix} \begin{bmatrix} C_m(0) & 0 & & \\ 0 & C_m(1) & & \\ & & \ddots & 0 \\ & & 0 & C_m(N-1) \end{bmatrix} \begin{bmatrix} A & & & \\ & A & & \\ & & \ddots & \\ & & & A \end{bmatrix} \begin{bmatrix} \underline{b}(0) \\ \underline{b}(1) \\ \vdots \\ \underline{b}(N-1) \end{bmatrix}$$

$S = [S(0) | S(1) | \cdots | S(N-1)]$ $\quad C_m = \mathrm{diag}(C_m(0), \dots, C_m(N-1))$ $\quad \mathcal{A} = \mathrm{diag}(A, \cdots A)$ $\quad \underline{b}$

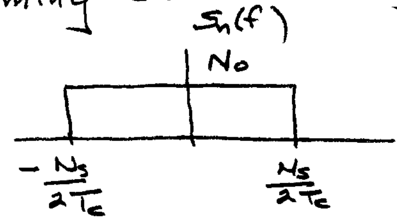
$\underbrace{(NN_cN_s + N_{del} + L)}_{N_t} \times NKL$ $\quad NKL \times NK$ $\quad NK \times NK$ $\quad NK$

large!

$$\underline{x}_m = S\, C_m\, \mathcal{A}\, \underline{b}$$

$$\underline{r}_m = \underline{x}_m + \underline{n}_m$$

assuming band limiting front end, so



$$R_{\underline{n}} = \frac{1}{2} E\left[\underline{n}\,\underline{n}^{+}\right] = \frac{N_o N_s}{T_c} I_{N_t}$$

$$= N_o f_s I_{N_t}$$

- Wait – it gets even better!

Stack the samples from all antennas into a single measurement vector. For $M = 2$, it's

$$\underline{r} = \begin{bmatrix} \underline{r}_1 \\ \hline \underline{r}_2 \end{bmatrix} = \left[\begin{array}{c|c} S & 0 \\ \hline 0 & S \end{array}\right] \begin{bmatrix} C_1 \\ \hline C_2 \end{bmatrix} A\, \underline{b} + \underline{n}$$

Note $A\, \underline{b}$ unchanged.

or

$$\underline{r} = \underline{\underline{S}}\, C\, A\, \underline{b} + \underline{n}$$

linear, but lots of structure.

- Variation 1: Adapt the vector of time functions notation of p. 2.1.2 to obtain

$$r_m(t) = \underline{s}(t)\, C_m\, A\, \underline{b} + n_m(t)$$

$\underline{s}(t)$ a row vec.

$$\underline{r}(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_M(t) \end{bmatrix} = \begin{bmatrix} \underline{s}(t) & & 0 \\ & \underline{s}(t) & \\ 0 & & \ddots \\ & & & \underline{s}(t) \end{bmatrix} C A \underline{b} + \underline{n}$$

See Appendix C for details.

• Variation 2: Reverse roles of c, $\underline{b}$, so that

$$\underline{r}_m = \mathcal{S} \mathcal{A}' \mathcal{B}' \underline{c}_m + \underline{n}_m$$

Now the channel gains are the vector. Good for analysis of channel estimation error.
Definitions and development in Appendix D.

• Summary – we have a compact, but highly structured, notation to represent the vector of samples of received signals on all antennas over a period of time. Linear with data $\underline{b}$ as input, or linear with channel gains $\underline{c}$ as input.

2.2.11

Exercises

1. Simplify $\underline{r}_m = \underline{S} C_m A \underline{b} + \underline{n}_m$ for frequency-flat channels (i.e. no time dispersion, $L=1$). Sketch the resulting S, C_m, A, $\underline{b}$ individually and compare with originals.

2. Complicate it: for long spreading codes, what changes in $\underline{x}_m = \underline{S} C_m A \underline{b}$?

3. In the full version $\underline{x} = \underline{\underline{S}}\, C A \underline{b}$, interpret:

   - the elements of $C A \underline{b}$
   - the columns of $\underline{S} C_m A$
   - the columns of $\underline{\underline{S}} C A$

4. Sketch all arrays for the case of flat channels and synchronous transmission. Long codes? Short codes?

5. Consider a single user in a channel with time dispersion ($L>1$), but a long code. How could you construct an equalizer? Any problems in making it adaptive?

6. What changes in the input/output equations if the channel is time invariant?