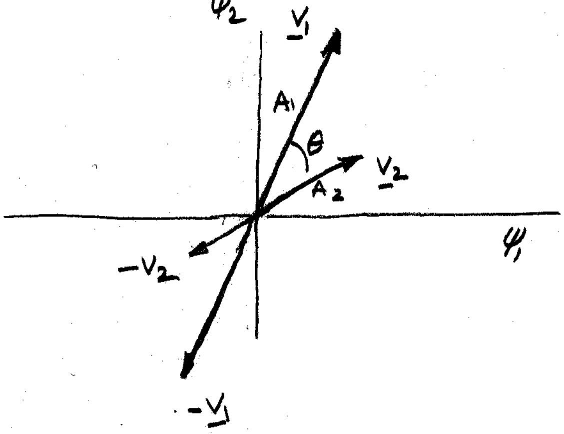
2.5.1 Plausibility of MUD Claims

- Before we launch into Big Math, it is helpful to establish some of the approaches with a very simple example. Two signals in AWGN.



2D

ψ_1, ψ_2 orthonorm. basis

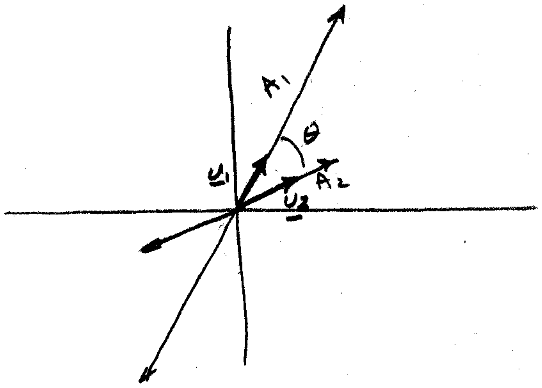
$\underline{v}_1, -\underline{v}_1$ / $\underline{v}_2, -\underline{v}_2$ signal vectors.

A_1, A_2 amplitudes

σ noise std dev.

Receive $\underline{r} = \pm \underline{v}_1 \pm \underline{v}_2 + \underline{n}$, get b_1, b_2, both ± 1.

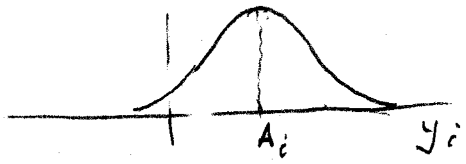
- Single user bound. Correlate against the two pulse shapes (ie inner product with unit vectors $\underline{u}_1, \underline{u}_2$)



$$y_1 = \pm A_1 \pm A_2 \cos\theta + n_1$$

$$y_2 = \pm A_1 \cos\theta \pm A_2 + n_2$$

If no interference



for $A_i > 0$

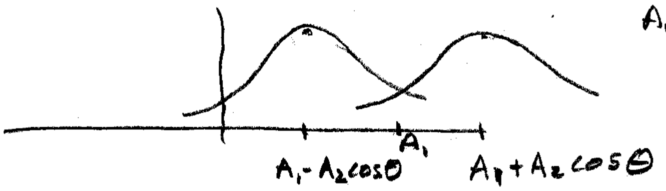
$$P_{e_1} = Q\left(\frac{A_1}{\sigma}\right) = Q\left(\sqrt{\gamma_1}\right) \qquad P_{e_2} = Q\left(\frac{A_2}{\sigma}\right) = Q\left(\sqrt{\gamma_2}\right)$$

for convenience $\gamma = \frac{A^2}{\sigma^2} = 2\frac{E_b}{N_0}$

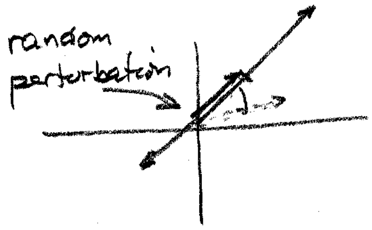
Achievable only if $\theta = \pm\pi/2$.

Define S/I $\lambda = \gamma_1/\gamma_2 = A_1^2/A_2^2$

- Method 1. Ignore the MAI, pretend it is like white noise. Correlate against pulse shapes.



<u>Note</u>
$\cos\theta = \rho$ the pulse cross-correlation coefficient



$$Pe_1 = \frac{1}{2}Q\left(\frac{A_1 - A_2\rho}{\sigma}\right) + \frac{1}{2}Q\left(\frac{A_1 + A_2\rho}{\sigma}\right)$$

$$Pe_2 = \frac{1}{2}Q\left(\frac{A_2 - A_1\rho}{\sigma}\right) + \frac{1}{2}Q\left(\frac{A_2 + A_1\rho}{\sigma}\right)$$

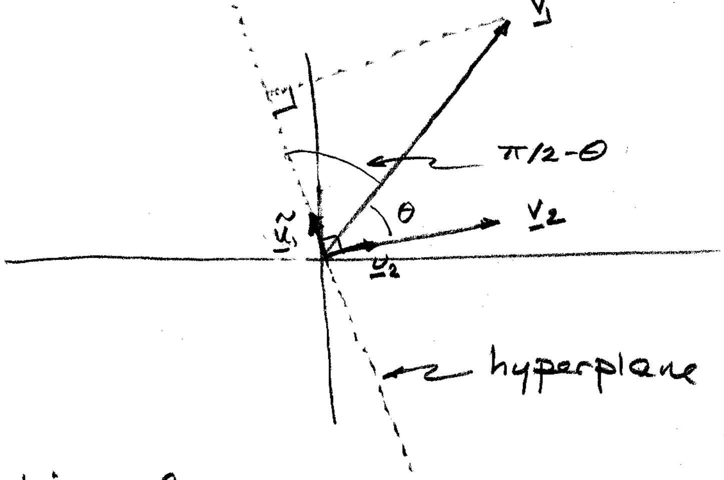
If $\lambda \approx 1$ and $|\rho| \not\ll 1$, this is disastrous for both users

If $\lambda \gg 1$ or $\lambda \ll 1$, and $|\rho| \not\ll 1$, it is disastrous for the weak user and an annoyance to the strong one.

If $|\rho| \ll 1$, for example by spreading code, then close to single user — but additional users cause the MAI to build

This is the conventional CDMA detector.

- Method 2. Zero forcing. Null out the other users by projecting $\underline{r}$ onto a hyperplane orthogonal to all the other users.



- Projection of $\underline{r} = \pm \underline{v}_1 \pm \underline{v}_2 + \underline{n}$ onto the hyperplane does this:

$$\underline{\tilde{u}}_2 \cdot \underline{r} = \underline{\tilde{u}}_2 \cdot (\pm \underline{v}_1 \pm \underline{v}_2 + \underline{n}) = \pm A_1 \cos(\pi/2-\theta) + n$$

$$= \pm A_1 \sqrt{1-\cos^2\theta} + n = \pm A_1 \sqrt{1-\rho^2} + n$$

The unwanted user is eliminated, and

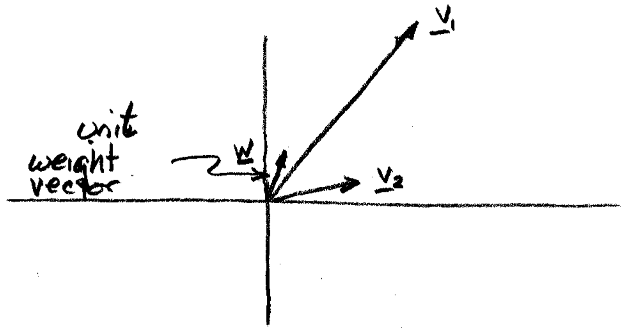
$$P_{e_1} = Q\left(\frac{A_1\sqrt{1-\rho^2}}{\sigma}\right) = Q\left(\sqrt{\gamma_1(1-\rho^2)}\right), \quad P_{e_2} = Q\left(\sqrt{\gamma_2(1-\rho^2)}\right)$$

irrespective of other user power.

- By retaining only the component of $\underline{v}_1$ that is orthogonal to $\underline{v}_2$ (& vice versa), we have weakened the effective signal — "noise enhancement". If $|\rho| \approx 1$, it doesn't work, even for the strong user; a big sacrifice if $\lambda << 1$.
- Insensitive to near-far disparities

- In 2-space we can't add another signal, so dimensionality of measurements must at least equal the number of signals.
- Again, if spreading makes $|\rho| \ll 1$, then not much cost — if only one user. Every additional user and orthogonality constraint chips away at desired user.

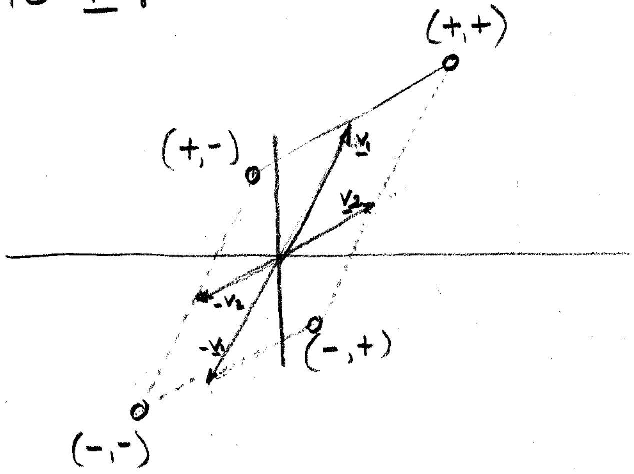
• Method 3. MMSE. Why should the strong user sacrifice power to null a weak user. Why not simply reduce that user to the noise level?



This allows some of the other user to be present without cutting own power as much.

- The optimum $\underline{w}$ depends on the noise σ, so we must know SNRs γ_1, γ_2, unlike ZF.
- As $\gamma_1, \gamma_2 \to \infty$, the only disturbance is MAI, and MMSE solution $\to$ ZF.
- At lower SNR, is much better than ZF, unless $|\rho| \ll 1$.
- Insensitive to near-far disparity.
- Also fails both users if $|\rho| \approx 1$.

- Method 4. Max Likelihood, joint detection. Detect both user data at once by selecting the point closest to $\underline{r}$.



* Note

This method works even with $|\rho|=1$ (same spreading code) provided $A_1 \neq A_2$ (or a mutual phase shift, if complex).

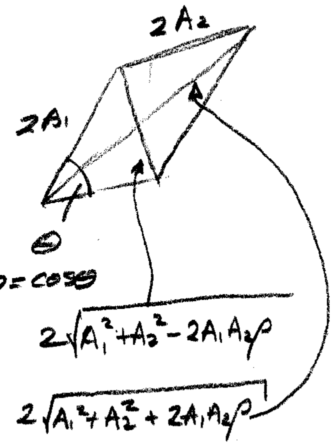
Consider user 1 error rate, with $+v_1$ sent. Then

$$P_{e_1} = \tfrac{1}{2} \Pr[\text{user 1 error} \mid ++ \text{ sent}] + \tfrac{1}{2} \Pr[\text{user 1 error} \mid +- \text{ sent}]$$

$$\leq \tfrac{1}{2}\Big(\Pr[++ \to -+] + \Pr[++ \to --]\Big)$$

$$+ \tfrac{1}{2}\Big(\Pr[+- \to --] + \Pr[+- \to -+]\Big)$$

union bound

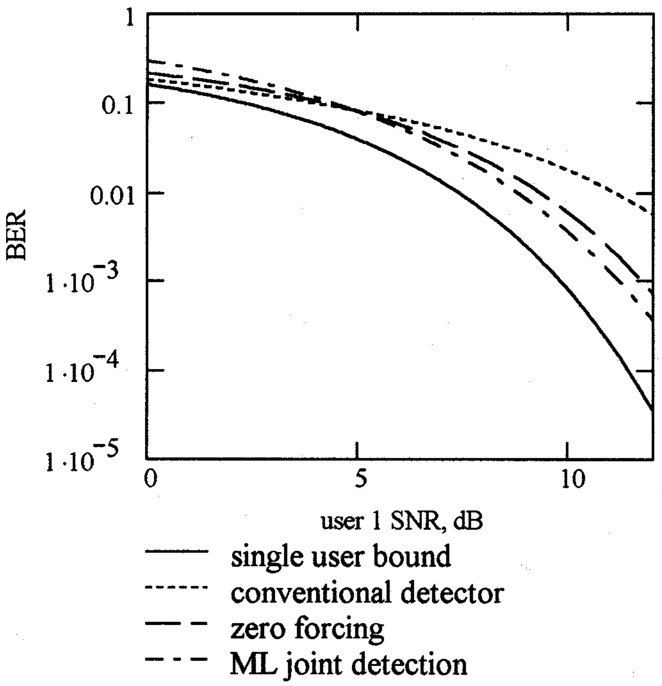


$$= \tfrac{1}{2}\left(Q\left(\frac{A_1}{\sigma}\right) + Q\left(\frac{\sqrt{A_1^2 + A_2^2 + 2A_1A_2\rho}}{\sigma}\right)\right)$$

$$+ \tfrac{1}{2}\left(Q\left(\frac{A_1}{\sigma}\right) + Q\left(\frac{\sqrt{A_1^2 + A_2^2 - 2A_1A_2\rho}}{\sigma}\right)\right)$$

$$\approx P_{e,\text{single}} + \tfrac{1}{2} Q\left(\sqrt{\gamma_1 (1 + \lambda^{-1} - 2\rho\lambda^{-\frac{1}{2}})}\right)$$

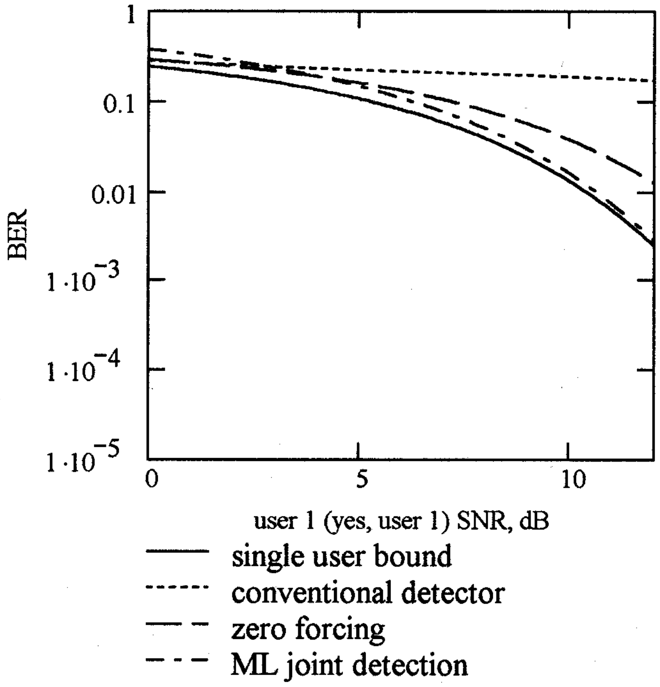
A minor increase from the single user bound.



User 1 performance, various detectors

$\rho = 0.6$ $\lambda = 2$

Notes:
* A large value of ρ.
* ZF worse than conv. at low SNR, because noise enhancement.
* ML union bound diverges at low SNR.
* ML bound is too loose for the strong user - a problem with standard union bound.



User 2 performance, various detectors

Notes:
* Conv. detector crippled by MAI.
* ML benefits this weaker user, effectively eliminating MAI.

- The multiuser detectors are readily understood from simple sketches. They do offer significant improvement over conventional detection (for the cases examined).

Here are a few more questions I'd like you to think about if you ever have a moment.

1. We determined that Rake processing produces sufficient statistics? Does this mean that the conventional Rake receiver is optimum? If so, in what sense and for what environment?

2. In our toy system of Section 2.5, the signal space had dimensionality 2 for convenience in sketching. If it were 3 (e.g., because of 3 chips/symbol or because of 3 antennas), then:

(a) How many users can you support with zero forcing? Is this a hard limit?

(b) How many users can you support with MMSE? Is this a hart limit or a practical limit?

(c) Is there a limit, soft or hard, on the number of users you can support with ML detection?

(d) If the signals are linearly dependent, e.g., v3 is expressible as a linear combination of the other two, can ZF separate them? Can MMSE? Can ML?

(e) In a special case of linearly dependent signals, where two of them are colinear and the third is linearly independent of the other two, what can zero forcing salvage?