

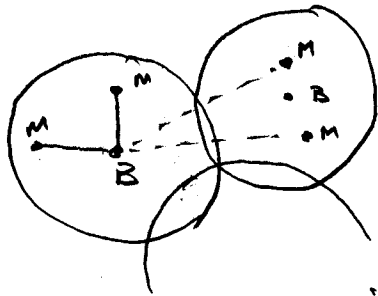
### 3. PROPAGATION, FADING AND DIVERSITY

- This chapter outlines the features of the propagation environment that affect MUD
  - scattering and fading
  - what fading does to BER
  - diversity: why you need it, and how to get it
    - MUD by spatial processing
    - imperfect channel state information.
- It's all classical, so highlights plus references.

## 3.1 Scattering Channels

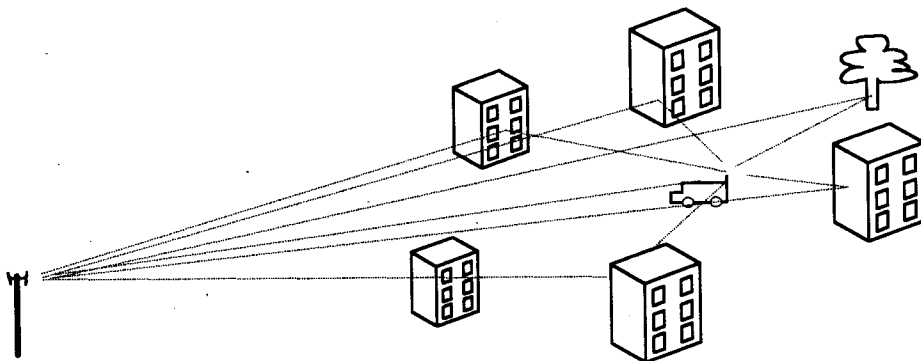
3.1.1

- In the mobile environment, received signal strength depends on distance and obstructions (path loss and shadowing), both with a coarse spatial scale



It also has a spatial microstructure caused by the scattering environment around the mobile.

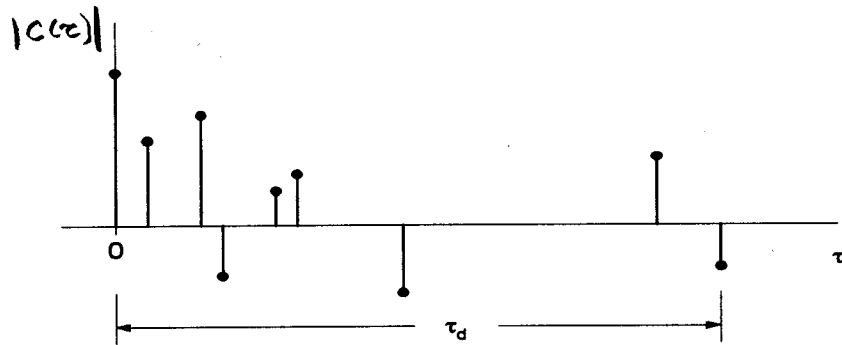
- We can infer most of what we need in this course from the sketch below



[Cave00]

- It's a linear time-dispersive (frequency selective) filter with Channel Impulse Response

$$c(\tau) = \sum_{\ell=0}^{L-1} c_{\ell} \delta(\tau - \tau_{\ell})$$

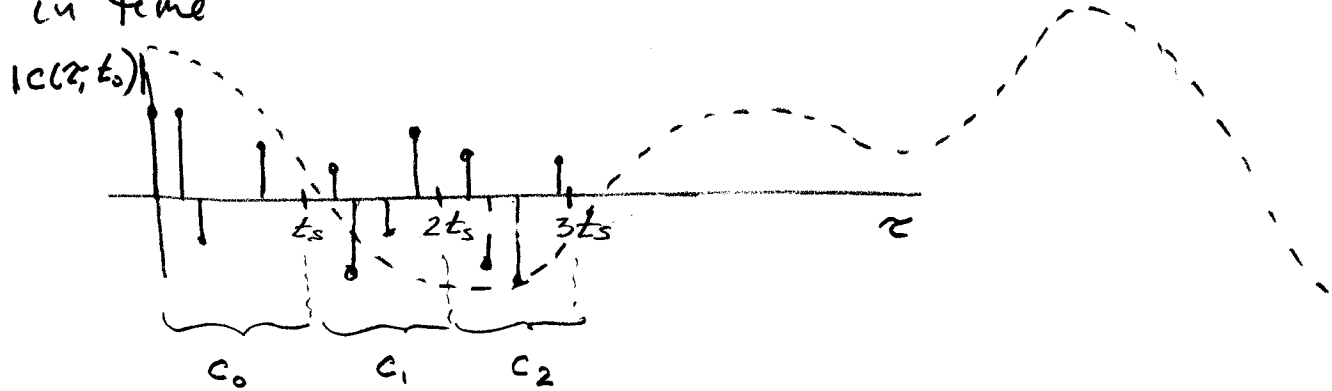


- If mobile moves, phases of components vary in amounts depending on angle of ray w.r.t. direction of travel; it's a linear time variant filter

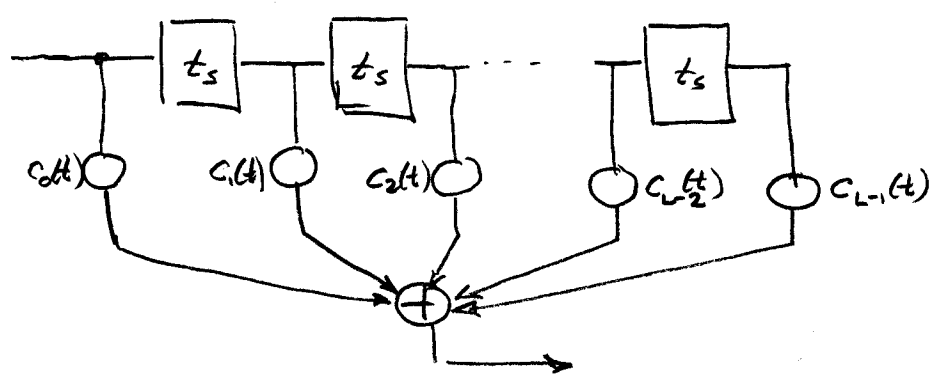
$$c(\tau, t) = \sum_{\ell=0}^{L-1} c_{\ell}(t) \delta(\tau - \tau_{\ell})$$

- CIRs of different mobiles are uncorrelated, even if mobiles are only a few wavelengths apart.

- Signal bandwidth determines its temporal resolution. Usually aggregate all components into resolution bins for a model equispaced in time



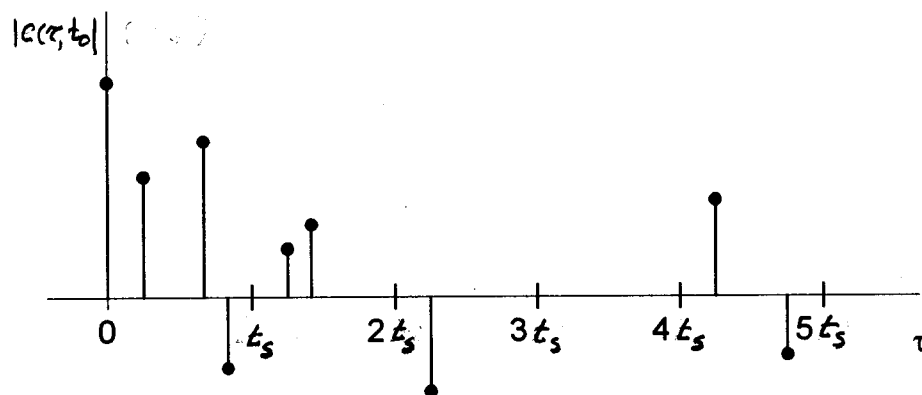
$$c(r, t) = \sum_{l=0}^{L-1} c_l(t) \delta(r - lt_s)$$



the TDL model

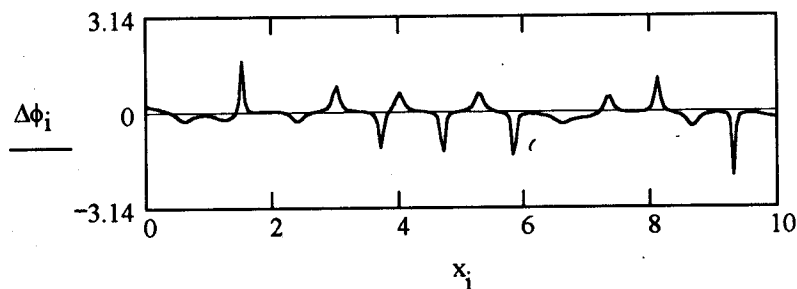
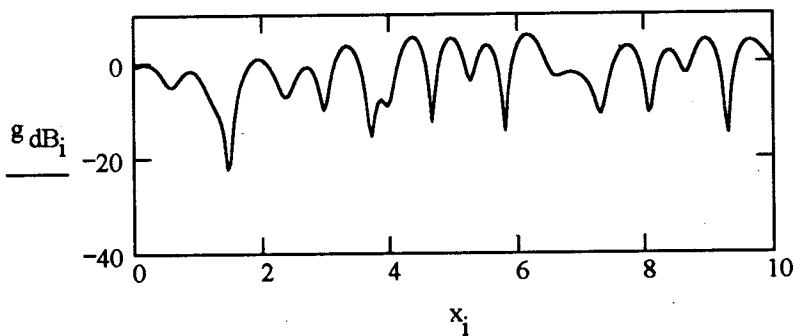
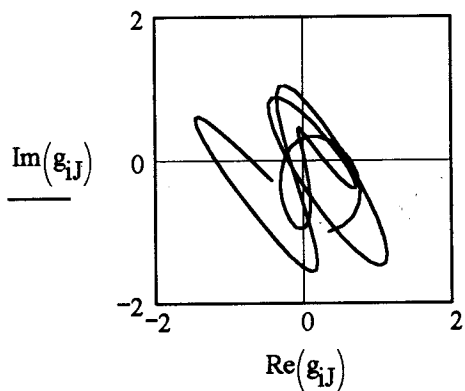
- The behaviour of the aggregated coefficients depends on number of components. Extremes:
  - lots of components; invoke central limit theorem, claim  $c_p(t)$  is complex Gaussian
  - or
  - one component; it just rotates.

- Commonly, the situation is ambiguous,  
especially with W-CDMA (4 Mcchip/s,  $f_s = 8$  MHz)



We'll be conservative and assume Gaussian

- The trajectory of a coefficient is instructive:



- If coefficients are Gaussian, then

$$p_c(c) = \frac{1}{2\pi\sigma_c^2} \exp\left(-\frac{1}{2} \frac{|c-\bar{c}|^2}{\sigma_c^2}\right), \quad \sigma_c^2 = \frac{1}{2} E[|c-\bar{c}|^2]$$

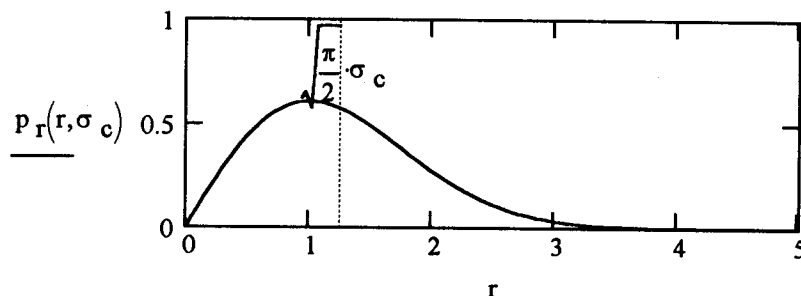
- In polars,  $c = r e^{j\theta}$ . If mean  $\bar{c} = 0$ , then

$$p_{r,\theta}(r,\theta) = \frac{r}{2\pi\sigma_c^2} e^{-r^2/2\sigma_c^2}; \quad r \geq 0, \quad -\pi \leq \theta \leq \pi$$

so  $r, \theta$  are independent,  $\theta$  uniform, and

$$p_r(r) = \frac{r}{\sigma_c^2} e^{-r^2/2\sigma_c^2}, \quad r \geq 0 \quad \text{Rayleigh}$$

Received amplitude of the component has a Rayleigh pdf.

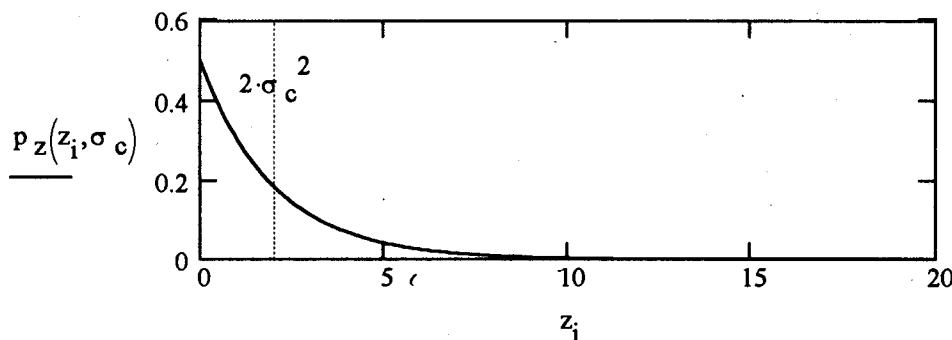


$$\text{mean } \sqrt{\frac{\pi}{2}} \sigma_c$$

$$(\sigma_c = 1, \text{ here})$$

- Often more convenient to work with  $z = r^2 = |c|^2$ . Change of variables gives  $z$  an exponential pdf:

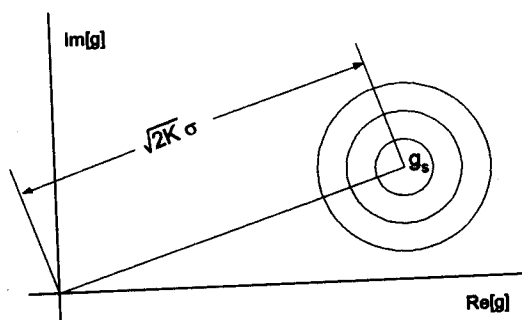
$$p_z(z) = \frac{1}{2\sigma_c^2} e^{-z/2\sigma_c^2} \quad (\text{or note } \chi^2, 2 \text{ d.f.})$$



$$\text{mean } 2\sigma_c^2$$

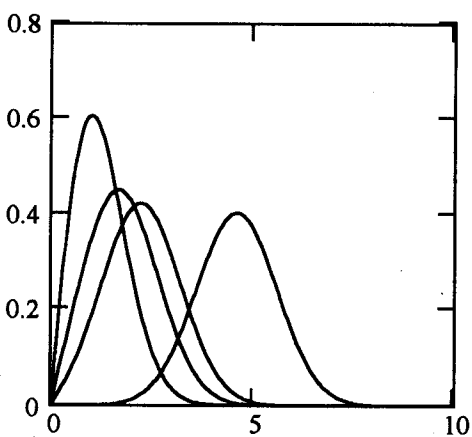
$$(\sigma_c^2 = 1 \text{ here})$$

- Note high probability of deep fades.
- If there's a LOS component (common in microcells) or a dominant component, then Rice model



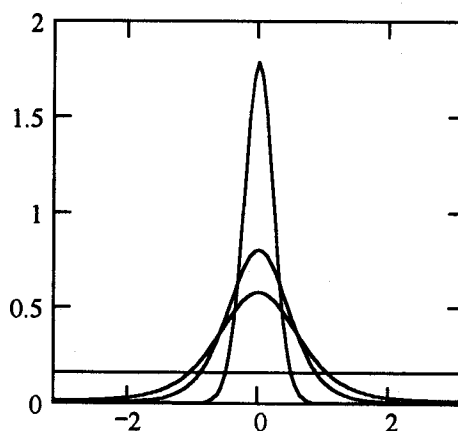
$$p_r(r) = \frac{r}{\sigma_c^2} \exp\left(-\frac{r^2}{2\sigma_c^2} - K\right) I_0\left(\frac{r\sqrt{2K}}{\sigma_c}\right)$$

"K factor": ratio of direct power to scattered power



— K=0 (Rayleigh)  
 — K=1  
 — K=2  
 — K=10

Rice amplitude pdf



— K=0 (Rayleigh)  
 — K=1  
 — K=2  
 — K=10

Rice phase pdf (for zero mean)

Note decreased probability of deep fades

- That's all the channel modeling we need for now.