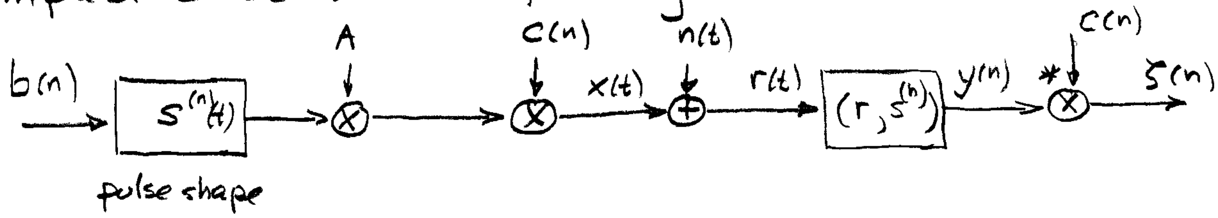
3.2 Effect of Fading on Single-User BER

- Fading has a devastating effect on BER, as we'll see below.

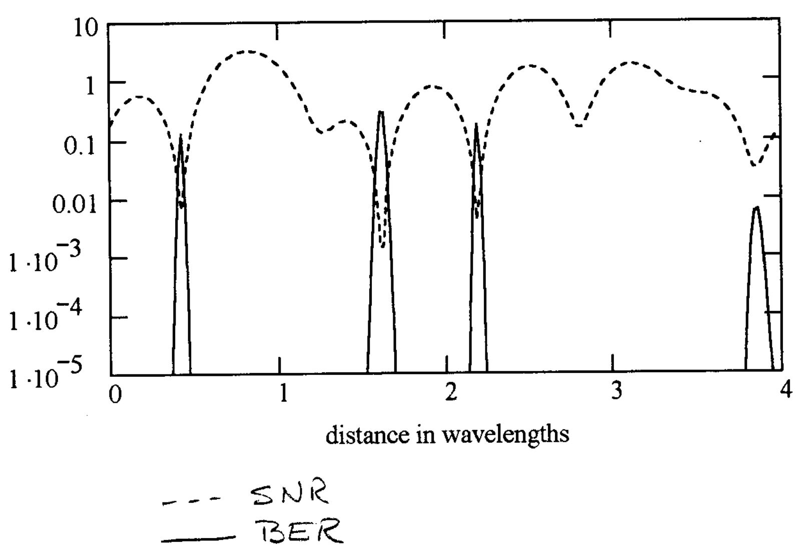
- Simplest case is flat fading.



This model assumes Rx has perfect channel state information (CSI). For given $c(n)$, the signal component of $\zeta(n)$ is $|c(n)|^2 A\, b$ and the variance of the noise component is $N_0 |c|^2$. Hence BER is (BPSK)

$$P_e(c) = Q\left(\frac{|c|^2 A}{\sqrt{N_0 |c|^2}}\right) = Q\left(\sqrt{\frac{|c|^2 2E_b}{N_0}}\right) = Q\left(\sqrt{2\Gamma_b |c|^2}\right)$$

- Typical variation with time:



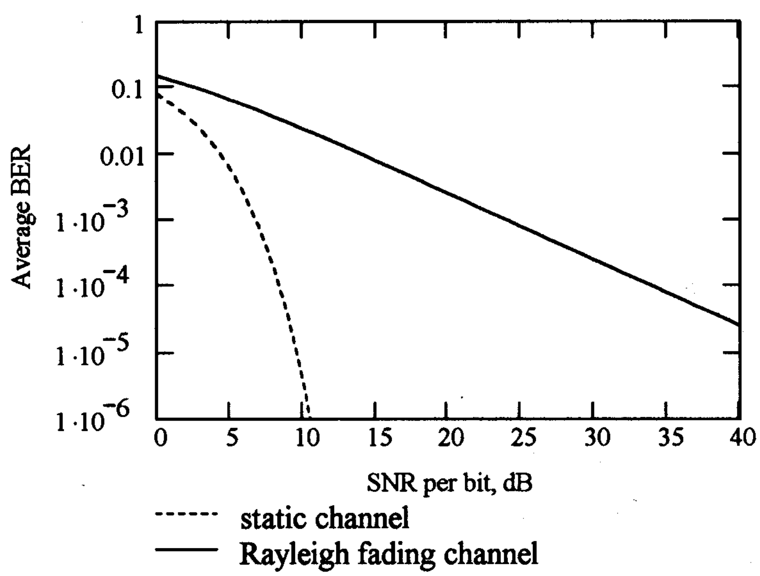
- bursty
- prob of deep fades determines the average BER

- To obtain the BER, average the instantaneous BER with the pdf of channel gain. Assume $\sigma_c^2 = \frac{1}{2}$, so $\overline{|c|^2} = \overline{z} = 1$, which leaves the average received SNR equal to average transmitted SNR.

$$\overline{P_e} = \int_0^\infty e^{-z} Q\left(\sqrt{2\Gamma_b z}\right) dz \qquad [\text{Proa95, Stub96, Schw66}]$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{\Gamma_b}{1+\Gamma_b}}\right) \qquad (\text{integration by parts})$$

$$\sim 1/4\Gamma_b \quad \text{large SNR}$$



- A 3 dB increase in SNR roughly
  - squares BER in static
  - halves BER in fading

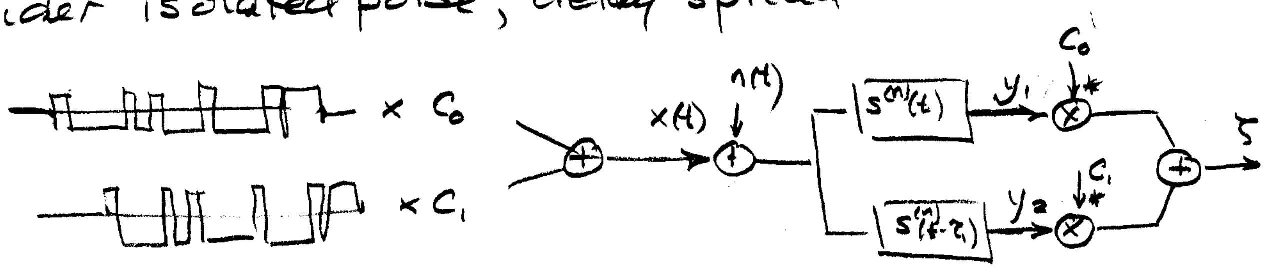
- What if CSI is not perfect?

$$\hat{c}(n) = c(n) + e(n)$$

Very difficult integrals — but there's an easy way using Gaussian quadratic forms, as we'll see.

- Next step: selective fading. We won't do the full solution at this point. Objectives are just:
  - demonstrate that sufficient stat ζ, the Rake receiver, follows from ML principles, too.
  - demonstrate a form of diversity reception.

- Consider isolated pulse, delay spread



- From pp 2.3.7, 2.3.8, sufficient stat is

$$\zeta = \underline{c}^{+} R \underline{c} A b + \underline{c}^{+} \underline{v} = \underline{c}^{+} \underline{y}$$

$$R = \begin{bmatrix} 1 & r^{(n)}(-\tau) \\ r^{(n)}(\tau) & 1 \end{bmatrix}, \quad R_v = \tfrac{1}{2} E\left[\underline{v}\,\underline{v}^{+}\right] = N_0 R$$

R is Hermitian, positive definite, so $\underline{c}^{+} R \underline{c} A$ is pos, real $\Longrightarrow$ slice ζ for decision

- This combining also follows from ML principles.
The conditional prob is

$$P_{y|b}(y|\underline{b}) = \frac{1}{(2\pi)^2 |R|} \exp\left(-\frac{1}{2}(\underline{y} - R\underline{c}Ab)^{+} R^{-1} (\underline{y} - R\underline{c}Ab)\right)$$

and we max wrt b. Log likelihood, keep exponent

$$\hat{b} = \arg\min \left[(y - R\underline{c}Ab)^{+} R^{-1} (\underline{y} - R\underline{c}Ab) \right]$$

$$= \arg\min \left[\underline{y}^{+} R^{-1} \underline{y} - 2bA\,\mathrm{Re}\left[\underline{c}^{+} R R^{-1} \underline{y}\right] + bA\underline{c}^{+} \underbrace{R R^{-1} R}_{R} \underline{c} A b \right]$$

$$= \arg\max \left[bA\,\mathrm{Re}[\underline{c}^{+} \underline{y}] \right]$$

$$= \mathrm{sgn}\left(\mathrm{Re}[\zeta]\right)$$

So we have another justification of this combining.

- Next, the BER:

$$\zeta = \underline{c}^{+} \begin{bmatrix} 1 & r^{(n)}(-\tau) \\ r^{(n)}(\tau) & 1 \end{bmatrix} \underline{c} A b + \eta \qquad \sigma_\eta^2 = N_0 \underline{c}^{+} R \underline{c}$$

The conditional prob is easy. Average over $\underline{c}$?

$$\underline{c}^{+} R \underline{c} = |c_0|^2 + |c_1|^2 + 2\,\mathrm{Re}\left[c_0 c_1^{*} r^{(n)}(\tau)\right]$$

The cross term makes it tough — again, need theory of Gaussian quad forms.

However, if $|r^{(n)}(\tau)| \ll 1$, as in spread spectrum,

$$\zeta \approx \left(|c_0|^2 + |c_1|^2\right) A b + \eta \qquad \sigma_\eta^2 = N_0 \left(|c_0|^2 + |c_1|^2\right)$$

we can <u>see</u> the diversity effect:

the prob that <u>both</u> $|c_0|^2$ and $|c_1|^2$ are very small is much less than the individual probabilities of being small, so the multipath should improve the BER

- Postpone further analysis until we have the mathematical tools.