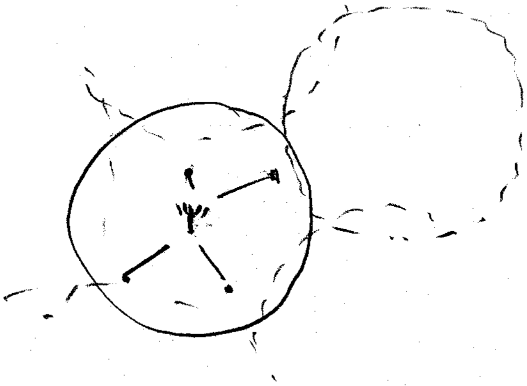
3.4 Diversity Reception - Multiple Users

3.4.1 The Model

- At last, we tackle mutual interference. In the cellular context we interpret as intra cell interferers. Again, other cell interferers either do not exist or are treated as white noise.



- We'll deal only with spatial filtering to remove MAI in this section. Classic. Other MUD methods later.
- Model:
  - perfect diversity (zero correlation)
  - flat channels
  - all users have same pulse shape

Note diversity implies no directionality. Just as no "beamforming" in Section 3.3, there will be no nulls tied to azimuthal directions here.

- To focus on spatial filtering, give the signal as little temporal structure as possible:
  - all same pulse shape (spreading sequence)
  - synchronous users
  - flat channel

- Our standard model

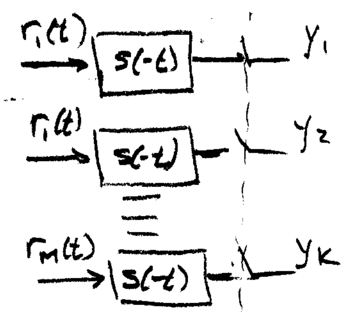
$$\underline{r} = \underline{\underline{S}}\ \underline{\underline{C}}\ A\ \underline{b} + \underline{n}$$

is shown in Appendix I to reduce after correlate onto

$$\underline{y}(n) = C(n)\, A\, \underline{b}(n) + \underline{\nu}(n)$$

or

$$\underline{y} = C A \underline{b} + \underline{\nu}$$



$$\underline{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix} \qquad C = \begin{bmatrix} \underline{c}_1 \,|\, \underline{c}_2 \,|\, \cdots \,|\, \underline{c}_K \end{bmatrix} \begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & \ddots & \\ & & & A_K \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix}$$

$M \times 1$ $\qquad$ $M \times K$ $\qquad$ $K \times K$ $\qquad$ $K \times 1$

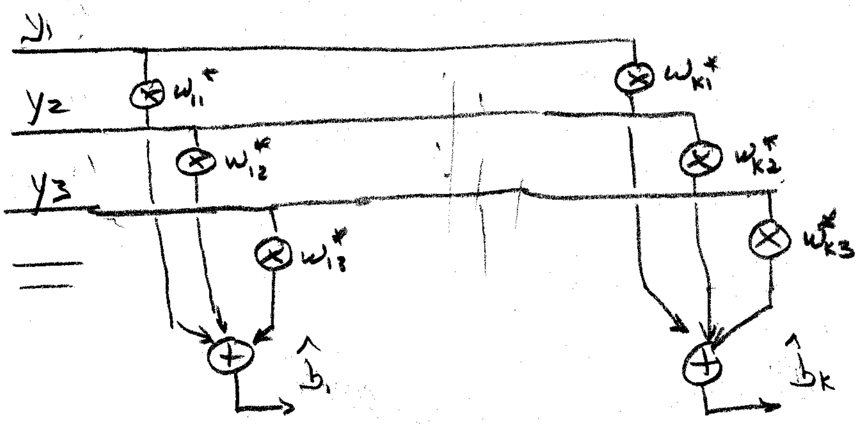
and $R_\nu = N_0 I_K$

We'll assume perfect CSI here, so must track $C(n)$.

Signals are coupled to all outputs through

$$\underline{y} = \underline{c}_1 A_1 b_1 + \underline{c}_2 A_2 b_2 + \cdots + \underline{c}_K A_K b_K + \underline{\nu}$$

- We can decouple the signals with a linear transformation.



3.4.2 Zero Forcing Approach

- Zero forcing is one way. Eliminate all interference in $\hat{b}_k$, regardless of effect on SNR.

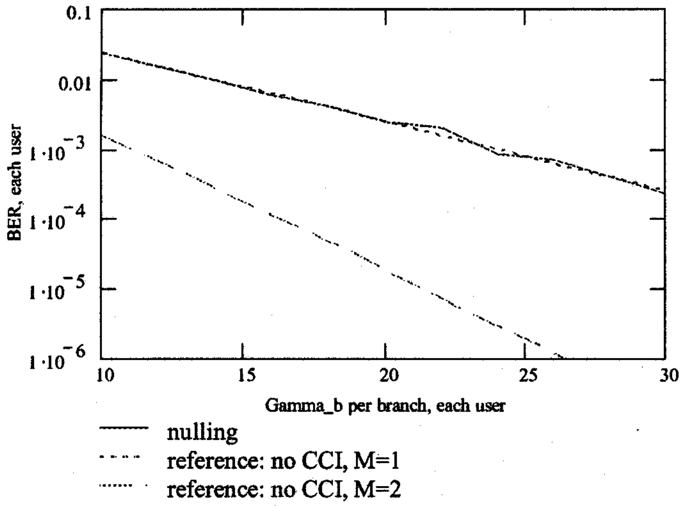
  $\hat{b}_k = \underline{w}_k^\dagger \underline{y}$ is to be unaffected by b_i, $i \neq k$

- Choose $\underline{w}_k$ to be orthogonal to the K-1 gain vectors $\underline{c}_i$, $i \neq k$, of other users. If there were no other users, we would align $\underline{w}$ with $\underline{c}_k$ to maximize SNR – but each orthogonality condition costs a degree of freedom in choosing $\underline{w}$. Consequently,

  $\Rightarrow$ lose a degree of diversity for each other user: net diversity order $= M-(K-1)$.

  $\Rightarrow$ can support only $K \leq M$ users, since we can't find a non-zero $\underline{w}$ that is orthogonal to M or more vectors.

– Appendix J shows that the resulting BER in zero forcing is <u>exactly</u> that of max ratio combining with $M-(K-1)$ antennas



K=2 , M=2, null steered, equipower users

These results are for Λ=0 dB! Equipower users.

The performance is identical to that of a single user with only one antenna (see reference curve) because zero forcing costs an order of diversity to null the other user.

Each interferer is invisible to the other.

- Despite the loss of diversity, ZF has some virtues:
  - It is completely insensitive to power disparities, since each user has nulled out the others. No near-far problem.
  - You don't need to know the SNR to use ZF (although you do need the vectors of complex gains).
  - You don't even need the signal structure here, because it's entirely spatial (but further improvement if we do use signal structure, as we'll see).

- So what <u>are</u> the weight vectors in ZF?

Rather than complicated analysis, use basic principles again.

$$\underline{y} = C A \underline{b} + \underline{\nu}$$

The signal space is spanned by the cols of C and the noise is spatially white. Project onto the column basis and we have lost nothing of value in estimating $\underline{b}$.

The projection is accomplished by

$$\underset{K\times 1}{\underline{\zeta}} = C^{\dagger}\underline{y} = \underbrace{C^{\dagger}C}_{K\times K}\,\underbrace{A\,\underline{b}}_{K\times 1} + C^{\dagger}\underline{\nu}$$

- To decouple, simply premultiply by the inverse

$$(C^{\dagger}C)^{-1}\underline{\zeta} = (C^{\dagger}C)^{-1}C^{\dagger}\underline{y}$$

$$= A\underline{b} + (C^{\dagger}C)^{-1}C^{\dagger}\underline{\nu}$$

or

$$C^{\#}\underline{y} = A\underline{b} + C^{\#}\underline{\nu} \qquad \text{the pseudo inverse}$$

- The weight vectors are the cols of $W = C^{\#\dagger}$

Check: They are orthogonal to the unwanted columns of C because

$$W^{\dagger}C = C^{\#}C = I_K$$

- Note that use of pseudo inverse means we have also solved the least squares problem

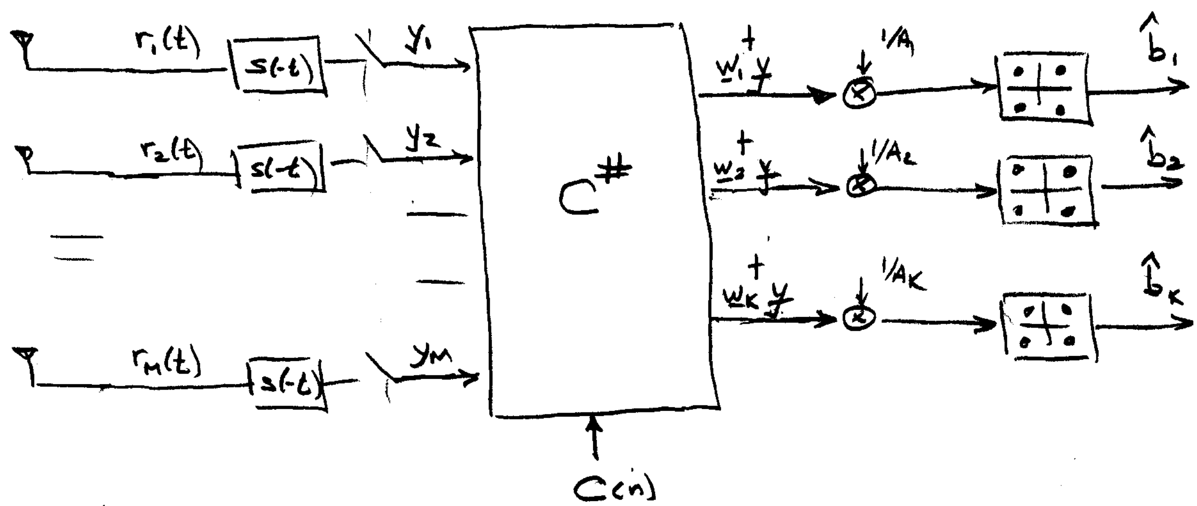
$$\underline{y} = C A \underline{b} + \underline{v}$$

for $A\underline{b}$. That is, given C and $\underline{y}$, we have found the $A\underline{b}$ that gives the best match in the output space $\underline{y}$ in the sense

$$\underline{y} - C A \underline{b} = \underline{v}, \quad \text{minimize} \sum_{m=1}^{M} |v_m|^2$$

a deterministic criterion.

- In operation,



- Spatial filtering separates the signals and delivers to a set of K single-user detectors
- No use of finite alphabet or signature sequence — the signals could equally well have been analog
- Don't need SNR to separate them, but do need it for decision if multiamplitude constellation.

- The probability of error is easily calculated. We have

$$C^{\#}\underline{y} = A\underline{b} + \underline{\alpha} \qquad \underline{\alpha} = C^{\#}\underline{\nu}$$

where the noise has cov matrix

$$C_\alpha = C^{\#} R_\nu C^{\#+} = N_0 (C^+C)^{-1} C^+C (C^+C)^{-1} = N_0 (C^+C)^{-1}$$

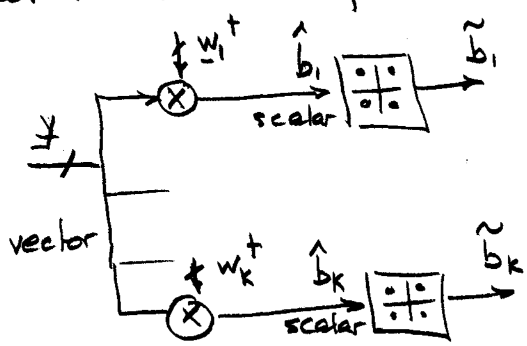
For user k, the BER is then

$$P_{ek} = Q\left(\frac{A_k}{\sigma_{\alpha k}}\right) = Q\left(\sqrt{\frac{2E_{bk}}{N_0 (C^+C)^{-1}_{k,k}}}\right) = Q\left(\sqrt{\frac{2\Gamma_{bk}}{(C^+C)^{-1}_{k,k}}}\right)$$

Note noise enhancement — if any $\underline{c}_i$ is close to being linearly dependent on other gain vectors, then C^+C is close to singular, and many components of $(C^+C)^{-1}$ are large, causing loss of effective SNR in the Q function

3.4.2 User Separation by MMSE

- Nulling the interferers is satisfying - but not smart
  - the desired signal gain may be weak after nulling the others; equivalently, bringing its gain up to 1 also increases the noise.
  - doesn't depend on C/I - if interferers are no stronger than the noise, why insist on nulling them?
- A compromise solution:
  - Allow some increase in CCI in exchange for a big decrease in noise variance
  - Minimise the sum of interference and noise powers at the detection point. MMSE.



$$\hat{b}_k = \underline{w}_k^{+}\underline{y} = \underline{w}_k^{+}(CA\underline{b} + \underline{\nu})$$

$$e_k = \hat{b}_k - b_k$$

Collectively, $\underline{e} = \hat{\underline{b}} - \underline{b}$. Treat it as statistical, with two ensembles, $\underline{b}$ and $\underline{\nu}$ (solution is conditioned on $\underline{C}$

- Criterion is applied to the input space, unlike the LS solution in ZF.

$$J = E_{\underline{b},\underline{\nu}}\left[|\underline{e}|^2\right]$$ minimum sum of estimation error variances.

where $\underline{e} = \hat{\underline{b}} - \underline{b}$, $\hat{\underline{b}} = W^{\dagger}\underline{y}$ $\quad W = [\underline{w}_1 | \cdots | \underline{w}_K]$

- Solution
  - Choose W to minimise J. It's a sum of variances, and $\underline{w}_k$ does not affect $\hat{b}_i$, $i \neq k$, so we have separate minimisations.
  - We have $e_k = \underline{w}_k^{\dagger}(CA\underline{b} + \underline{\nu}) - b_k$

$$J_k = E\left[|e_k|^2\right] = E\left[e_k e_k^*\right]$$

$$= E\left[\left(\underline{w}^{\dagger}(CA\underline{b}+\underline{\nu}) - b_k\right)\left(\underline{b}^{\dagger}A^{\dagger}C^{\dagger} + \nu^{\dagger}\right)\underline{w}_k - b_k^*\right)\right]$$

$$= \underline{w}_k^{\dagger}\left(CA^2C^{\dagger} + N_0 I_M\right)\underline{w}_k - \underline{w}_k^{\dagger} A_k \underline{c}_k - A_k \underline{c}_k^{\dagger}\underline{w}_k + 1$$

$$\nabla_{\underline{w}} J_k = \left(CA^2C^{\dagger} + N_0 I_M\right)\underline{w}_k - A_k\underline{c}_k = \underline{0}_M$$

so user k weight is

$$\underline{w}_k = \left(CA^2C^{\dagger} + N_0 I_M\right)^{-1} A_k \underline{c}_k$$

- Collectively,

$$W = \left(CA^2C^{\dagger} + N_0 I_M\right)^{-1} CA$$

Solution depends on $C(n)$. Tracking?

- Solution special cases
  - All users but one have $A_k = 0$ (i.e., single user)

    Then solution degenerates to MRC

    $$\underline{w}_1 = \underbrace{(A_1^2 \underline{c}_1 \underline{c}_1^{\dagger} + N_0)^{-1} A_1}_{\text{a scale factor}} \underline{c}_1$$

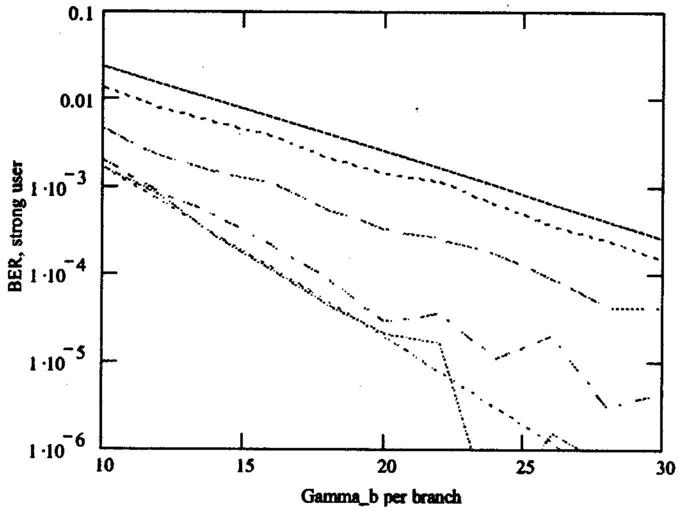
  - Noise is zero. Then it's the zero forcing solution with pseudo inverse (easy to see if $K = M$; if $K < M$, use Moore-Penrose generalised pseudo inv).

- Observations
  - Need the noise level (i.e. SNRs) to calculate it.
  - Good near-far resistance (weak user $\to$ ZF)
  - Users de facto $K \leq M$, as with ZF, and lose degree of diversity with each interferer (shows up asymptotically)
  - BER is data pattern dependent:

    $$\hat{\underline{b}} = W^{\dagger} \underline{y} = \underbrace{A C^{\dagger} (C A^2 C + N_0 I_M)^{-1} C A}_{\text{not diagonal}} \underline{b} + W^{\dagger} \underline{v}$$

    but for a given C, $\underline{b}$, it's just the Q function

And here are some results from the interactive demo:



——— reference: no CCI, M=1
- - - - C/I Lambda = 0 dB (equipower)
······· C/I Lambda = 10 dB
··· ·· C/I Lambda = 20 dB
········· C/I Lambda = 30 dB
- · - · reference: no CCI, M=2

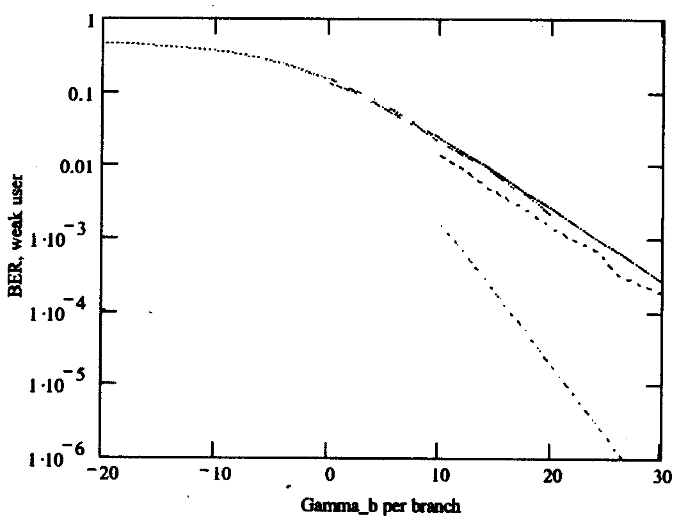
K=2 , M=2, MMSE, Stronger User

Notes:

* operates like max ratio when CCI is much less than noise, hence roughly dual diversity

* when CCI dominates noise, operates more like null steering, hence roughly single diversity (no diversity), as seen in the slope - but there's a still a SNR benefit compared to pure nulling

* when users are equipower, then MMSE is a little better than zero forcing (the reference curve) in the SNR range shown.



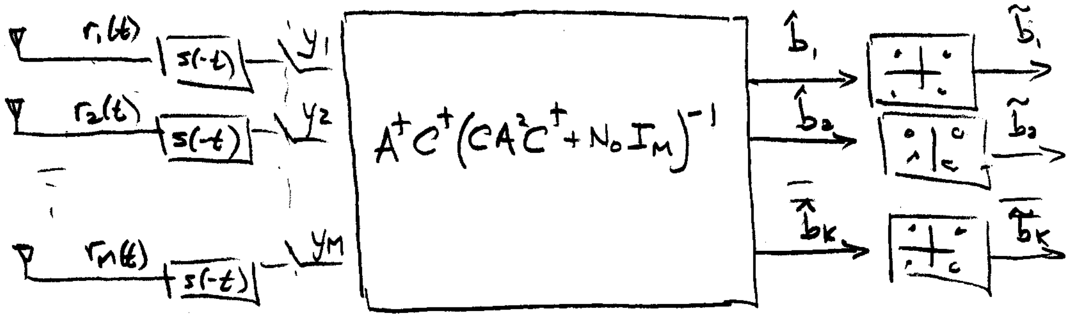
········· reference: no CCI, M=1
- - - - C/I Lambda = 0 dB (equipower)
······· C/I Lambda = 10 dB
··· · · C/I Lambda = 20 dB
········· C/I Lambda = 30 dB
- - - - reference: no CCI, M=2

K=2 , M=2, MMSE, Weaker User

Notes:

* All of the MMSE curves for the weaker user lie on top of the single antenna (M=1) reference curve. This shows that the solution is always very close to the zero forcing (nulling) solution, and it lost an order of diversity.

- Like ZF, MMSE separates to a bank of single-user detectors.



- Read [Wint84] spacefilter, flat fading, CCI
  [Clar92] space filter, selective fading, CCI

- Summary for K mobiles, M antennas

max ratio $\xrightarrow{\text{use all } \{\underline{c}_k\}}$ zero forcing $\xrightarrow{\text{different criterion}}$ mmse

max ratio	zero forcing	mmse
$\underline{w}_k = \underline{c}_k$ (know own $\underline{c}_k$)	$\underline{w}_k = \left[C^{\#+}\right]_{\text{col } k}$	$\underline{w}_k = A_k \underline{c}_k^+ (CA^2C^+ + N_0 I_M)^{-1}$
– need large C/I for satisfactory operation; optimum if single user, white noise	– performance is indep of C/I, so no near-far effect, other mobiles invisible	– minimise sum of all distortions, CCI, noise (even ISI)
– the dominant mobile has diversity M	– equivalent to single user performance with MRC of diversity $M-(K-1)$	– when CCI $\ll$ noise then behaves like max ratio, diversity M
– performance of the others is useless		– when CCI $\gg$ noise then like zero forcing, diversity $M-(K-1)$ but with SNR improvement
		– robust in face of near-far