3.7 Rank and Dimensionality in Diversity Reception

- So far, Section 3 has taken a stochastic view of diversity reception. However, deterministic models also give us insight into the role of multiple measurements of the arriving signal.

- The deterministic view is tied to classical linear algebra, where considerations of dimensionality, rank and degeneracy are central. We'll look at multiuser narrowband systems for
  - flat fading
  - delay spread.

3.7.1 Flat Fading

- Consider the narrowband multiuser system of Section 3.4: all users have same pulse shape and are synchronous; flat fading; array reception.

$$\underline{y} = C A \underline{b} + \underline{v}$$

and use of zero forcing for simplicity

$$\hat{\underline{b}} = A^{-1} C^{\#} \underline{y} .$$

- The columns of C must be LI in order for the b_k's to be distinguishable by linear processing

$$\underline{y} = \underline{c}_1 A_1 b_1 + \underline{c}_2 A_2 b_2 + \cdots + \underline{c}_K A_K b_K + \underline{v}$$

If $\underline{c}_1$ is proportional to $\underline{c}_2$, there is a nullspace at the input involving b_1, b_2 — you can specify their values only to with a LC; e.g. the value of $A_1 b_1 + A_2 b_2$, if $\underline{c}_1 = \underline{c}_2$, and the individual bits are lost.

More generally, if a linear dependence involves several $\underline{c}_i$, $i \in \{1, \cdots K\}$, then all the associated bits are caught in the null space and are lost.

More generally, there can be several such dependences and the null space can have dimensionality greater than one (the degeneracy). Only bits associated with $\underline{c}_i$ that are LI of all others can be salvaged.

- The issue shows up in the formation of

$$C^{\#} = (C^{+}C)^{-1}C^{+}$$

since $C^{+}C$ is singular if any cols of C are LD.

- Statistical view: this is a probability zero event (but near misses at the fade rate).

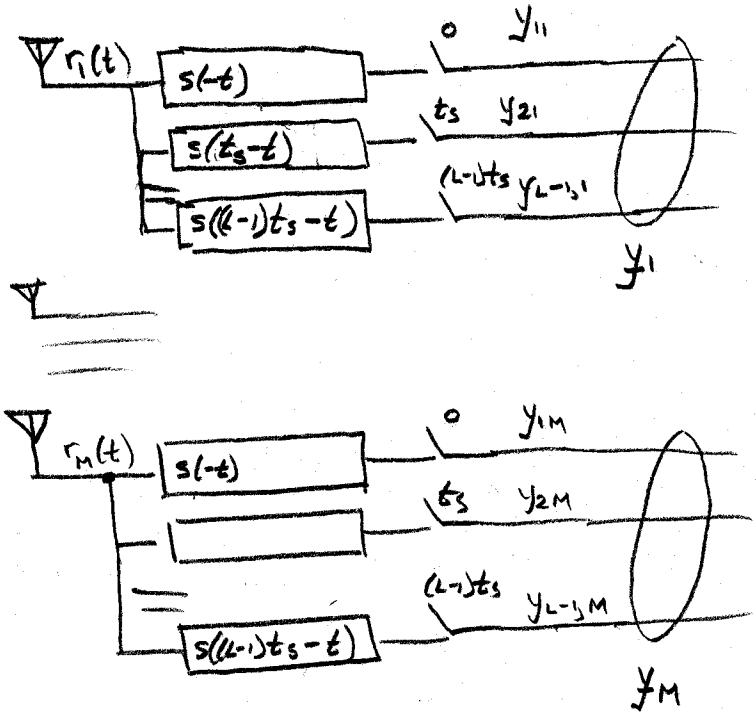
Deterministic view: It doesn't take much shift in the resultants (due to geometry) to cause the vectors $\underline{c}_k$ to be LI (at least in principle).

- Value of the array? Increasing number of antennas M, increases the dimensionality of the received space, since $\underline{y}$ has length M. This allows more LI propagation vectors $\underline{c}_k$, hence more users, with $K \leq M$. A proportional capacity increase.

- Why use $K < M$, instead of $K = M$? Statistical answer: preserve some diversity. Deterministic answer: it reserves some unused dimensions to ensure LI vectors. Also, the antenna gain effect: noise averaging give an SNR boost of $10\log(M-K)$ if $C_{m,k}$ values are of comparable magnitude.

3.7.2 Delay Spread

- Same narrowband multiuser system, but with delay spread. All users, antennas, have same number of multipaths, same delays. Consider one-shot (isolated pulse) transmission, for simplicity).
- Correlators for each delay on each antenna.



$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix}$$

- On antenna m

$$r_m(t) = \underbrace{[s(t), s(t-t_s) \cdots, s(t-(L-1)t_s]}_{\underline{s}(t),\ 1\times L} \underbrace{\begin{bmatrix} c_{1om} & \cdots & c_{Kom} \\ c_{11m} & & c_{K1m} \\ & & \\ c_{1,L-1,m} & & c_{K,L-1,m} \end{bmatrix}}_{C_m,\ L\times K} \underbrace{\begin{bmatrix} A_1 & & \\ & A_2 & \\ & & \ddots \\ & & A_K \end{bmatrix}}_{A,\ K\times K} \underbrace{\underline{b}}_{K\times 1} + n_m(t)$$

So

$$\underline{y}_m = R C_m A \underline{b} + \underline{v}$$

where

$$R = \int \underline{s}^+(t)\,\underline{s}(t)\,dt = \begin{bmatrix} R_s(0) & R_s(-t_s) & \cdots & R_s(-(L-1)t_s) \\ R_s(t_s) & R_s(0) & & \\ & & \ddots & \\ R_s((L-1)t_s) & & & R_s(0) \end{bmatrix}$$

Toeplitz
Hermitian

Rewriting,

$$\underline{y}_m = H_m A \underline{b} + \underline{v} = \underline{h}_1 A_1 b_1 + \underline{h}_2 A_2 b_2 + \cdots + \underline{h}_K A_K b_K + \underline{v}$$

where $H_m = R C_m$, $L \times K$

Now H_m has full column rank iff C_m does, too – provided that R is non-singular. Depends on pulse shape, but easy if R strongly diagonal (good spreading code).

- Thus delay spread also increases the dimensionality of the received signals. Even with a single antenna, we can have $K \leq L$ users, in principle. In practice, poor pulse shape can bring it down, with high eigenvalue ratio in R.

With M antennas, we can support

$$K \leq ML \text{ users.}$$

Delay spread and antenna array combine to significantly expand dimensionality, hence capacity.