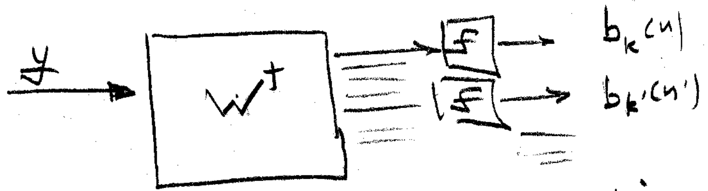
A CDMA MULTIUSER DETECTION WITH LINEAR RECEIVERS

- This section differs from Section 3 in that we use different pulse shapes (signature sequences) to distinguish the users.

- The structures used here are entirely linear for both multiuser separation and diversity combining. No use of the finite alphabet property until the decision point



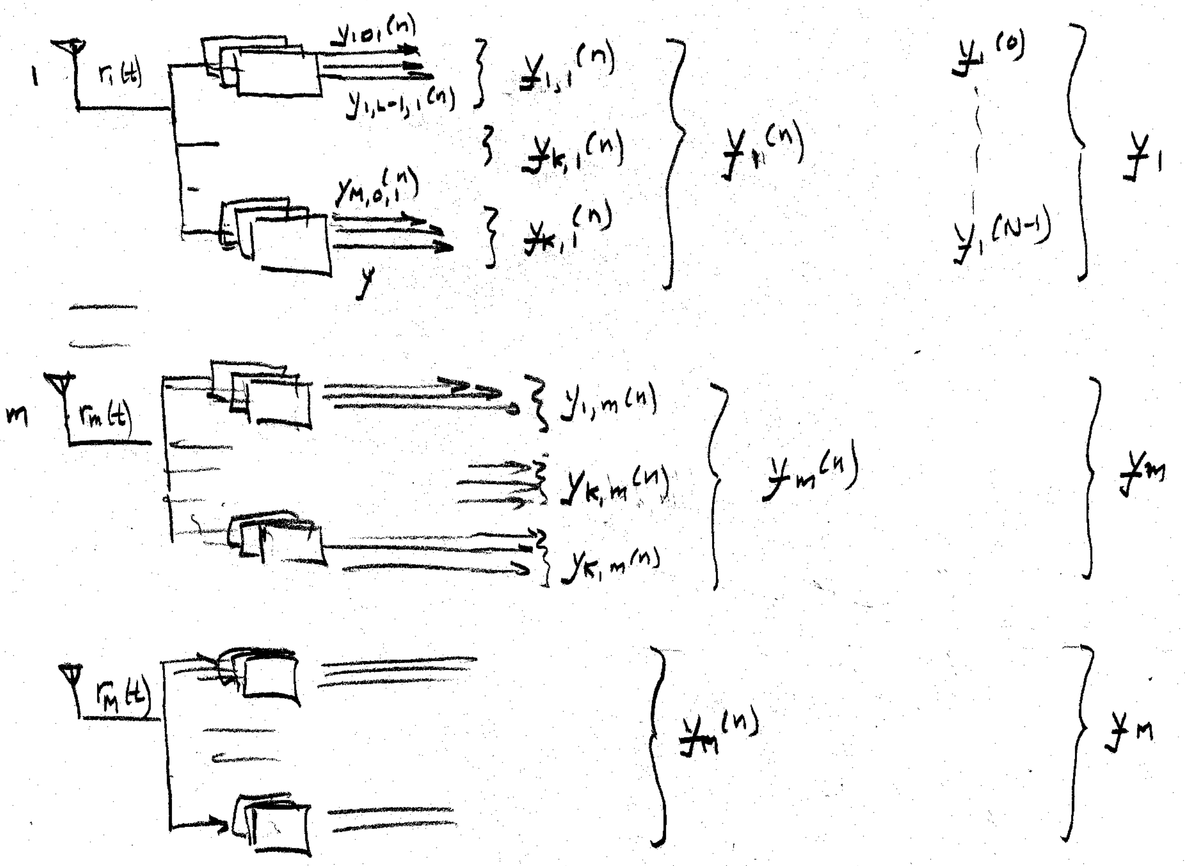
  This class includes the conventional detector, the decorrelator (zero forcing) and mmse detectors.

- A few of the <u>many</u> references that deal, in whole or in part, with linear CDMA detection.

  [Junt 97], [Junt98] [Junt99] [Junt 00a], [Junt 00b], [Latv 00], [Lupa 89], [Lupa90], [Tide 99], [Vara 91], [Vasu94], [Xie 90]

4.1 Linear Receivers

- Recall that the receiver front end produces $y_{k,\ell,m}^{(n)}$



$$\underline{y}_m(n) = R(n,n)\, C_m(n)\, A\, \underline{b}(n) + \underline{v}_m(n)$$

$$\underline{y}_m = R\, C_m\, A\, \underline{b} + \underline{v}_m$$

$$\underline{y} = \underline{R}\, C\, A\, \underline{b} + \underline{v}$$

- We'll define the receivers on a whole-block basis, even though the matrices are impractically large. More reasonable implementations are in Section 4.3.

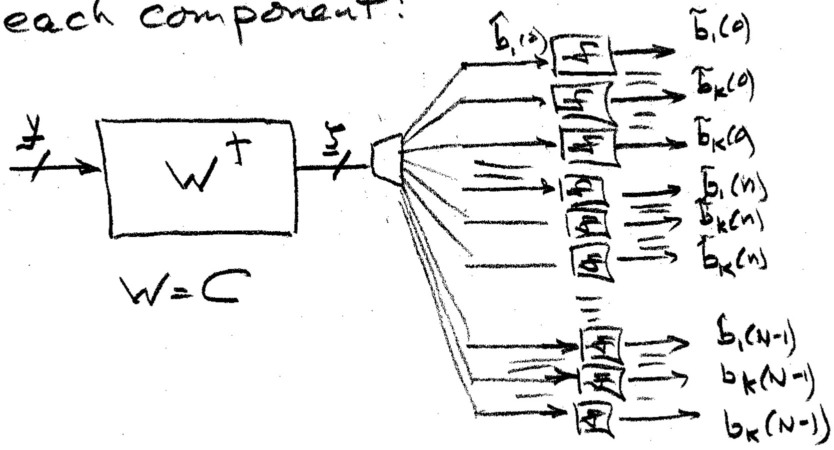
4.1.1 The Conventional Receiver

- The conventional receiver first performs Rake/MRC on the MF outputs to reduce to a set of sufficient stats, one for each transmitted bit

$$\underline{\xi} = C^{+}\underline{y} = C^{+}\underline{R}CA\underline{b} + \eta \quad , \quad R_{\eta} = N_0 C^{+}\underline{R}C$$

No information relevant to decisions has been lost, as shown in Section 2.3

- The conventional receiver ignores the mutual interference among bits in the stat $\underline{\xi}$ and makes separate decisions on each component:



Denote $C^{+}RC = H$. It is block tridiagonal, so each bit $b_k(n)$ experiences interference from other users at symbol n ($b_i(n)$, $i \neq k$) and from itself (ISI) at other times $n-1$, $n+1$ and from other users at $n-1$, $n+1$.

- If all but one A_k are negligible ($A_i \ll A_k,\ i \neq k$) then it's optimum for that lucky user. The weaker users experience intolerable interference — vulnerable to near-far power disparities.

- For any given bit, the multiplication produces a LC of MF outputs

$$\zeta_k(n) = \underline{w}_k^{+}(n)\,\underline{y} = \sum_{m=1}^{M}\sum_{l=0}^{L-1} c_{k,l,m}^{*}(n)\, y_{k,l,m}(n)$$

Only LM terms. Other-user MFs and other-time MFs ignored.

4.1.2 The Decorrelating Receiver

- This is just the zero forcing receiver we have already examined. Start with

$$\underline{\zeta} = C^{+}\underline{y} = \underbrace{C^{+}\underline{R}C}_{H}\, A\underline{b} + \underline{\eta}$$

- Since $C^{+}\underline{R}C$ is a square matrix, remove all interference by

$$\underline{\hat{b}} = A^{-1}(C^{+}\underline{R}C)^{-1}\underline{\zeta} = A^{-1}(C^{+}\underline{R}C)^{-1}C^{+}\underline{R}CA\underline{b} + A^{-1}(C^{+}\underline{R}C)^{-1}\underline{\eta}$$

$$= \underline{b} + \underline{\alpha}$$

where

$$R_{\underline{\alpha}} = A^{-1}(C^{+}\underline{R}C)^{-1}R_{\eta}(C^{+}\underline{R}C)^{-1}A^{-1}$$

$$= N_0\, A^{-1}(C^{+}\underline{R}C)^{-1}A^{-1}$$

- The attractive features:
  - no interference
  - don't need to know the noise level
  - no near-far problems (in principle, though depends on quality of channel estimates)
  - keep that hard-won diversity (unless R becomes singular).

- Drawbacks:
  - noise enhancement if cols of $C^+\underline{R}C$ approach dependency, causing small eigenvalues and large components in $(C^+\underline{R}C)^{-1}$ (as we saw in the toy system of Section 2.5).

- Note $W^+ = A^{-1}(C^+\underline{R}C)^{-1}C^+$

is a solution to a weighted LS problem, or to LS after transforming & whiten the noise by premult with

$$R_\eta^{-\frac{1}{2}} = (C^+\underline{R}C)^{-\frac{1}{2}}$$

Solve

$$\min \left\| (C^+\underline{R}C)^{-1/2}\underline{\zeta} - (C^+\underline{R}C)^{-\frac{1}{2}}(C^+\underline{R}C)A\underline{b} \right\|^2$$

$$\text{or } \| \underline{x} - H\underline{b} \|^2$$

H is square, non singular, so $\hat{\underline{b}} = H^{-1}\underline{x}$, and the min is zero.

$$\hat{b} = \underbrace{A^{-1}(C^+RC)^{-1}(C^+RC)^{\frac{1}{2}}}_{H^{-1}}\underbrace{(C^+RC)^{-1/2}\underbrace{C^+\underline{y}}_{\underline{\zeta}}}_{\underline{x}}$$

the LS solution

$$= A^{-1}(C^+\underline{R}C)^{-1}C^+\underline{y}$$

the decorrelator

4.1.3 The MMSE Receiver

- As before, MMSE is more flexible, allowing tradeoff between MAI and noise.

- Uses a stochastic model for $\underline{b}$, and applies the quadratic criterion to the input space $\left(\underline{b} \rightarrow \quad \rightarrow \underline{y} \right)$.

$$\hat{\underline{b}} = W_y^{+} \underline{y} \; ; \quad \text{choose } \underline{W}_y^{+} \text{ to minimize } E\left[\| \hat{\underline{b}} - \underline{b} \|^2 \right]$$

Standard linear estimation as before. Set up normal equations:

$$E[\underline{y}\,\underline{y}^{+}]\, W_y = E[\underline{y}\, \underline{b}^{+}]$$

The covariance matrix

$$R_y = E[\underline{y}\,\underline{y}^{+}] = \underline{R}\, C A^2 C^{+} \underline{R} + N_0 \underline{R}$$

and cross correlation matrix

$$R_{yb} = E[\underline{y}\, \underline{b}^{+}] = \underline{R}\, C A$$

and solution

$$R_y W_y = R_{yb}$$

$$W_y = R_y^{-1} R_{yb}$$

$$= \left(\underline{R}\, C A^2 C^{+} \underline{R} + N_0 \underline{R} \right)^{-1} \underline{R}\, C A$$

$$= \left(C A^2 C^{+} \underline{R} + N_0 I_{NKL} \right)^{-1} C A$$

- It's also possible to work from

$$\underline{\xi} = C^+ \underline{y}$$

and form $\underline{\hat{b}} = W_\xi^+ \underline{\xi}$

We have

$$R_\xi = C^+\underline{R}CA^2C^+\underline{R}C + N_0 C^+RC$$

$$R_{\xi b} = C^+\underline{R}CA$$

$$R_\xi W_\xi = R_{\xi b} \quad \text{or} \quad \left(A^2C^+\underline{R}C + N_0 I_{NK}\right) W_\xi = A$$

so

$$W_\xi = \left(A^2C^+\underline{R}C + N_0 I_{NK}\right)^{-1} A$$

In effect, we apply CW_ξ to $\underline{y}$, as $W_\xi^+ C^+ \underline{y}$.

- The two approaches are equivalent, since

$$CW_\xi = W_y$$

or

$$C\left(A^2C^+\underline{R}C + N_0 I_{NK}\right)^{-1} A = \left(CA^2C^+\underline{R} + N_0 I_{NKL}\right)^{-1} CA$$

Show by matrix inversion lemma.

- Obervations:
  - Trades interference against noise through the sum in the covariance matrix
  - Just as near-far resistant as decorrelation:
    - if weak signal has a problem with interference, its solution is close to ZF
    - if strong signal and interferers are below noise, its solution is close to Rake/MRC
  - Residual interference causes data pattern dependence in BER
  - Need to know noise level.

- Again, for any given bit $b_k(n)$, the estimate is just a linear combination of MF output samples

$$\hat{b}_k(n) = \underline{w}_k^{\dagger}(n)\, \underline{y} \qquad \text{where } \underline{w}_k(n) = W^{\langle nK+k \rangle}$$

much like an equalizer.

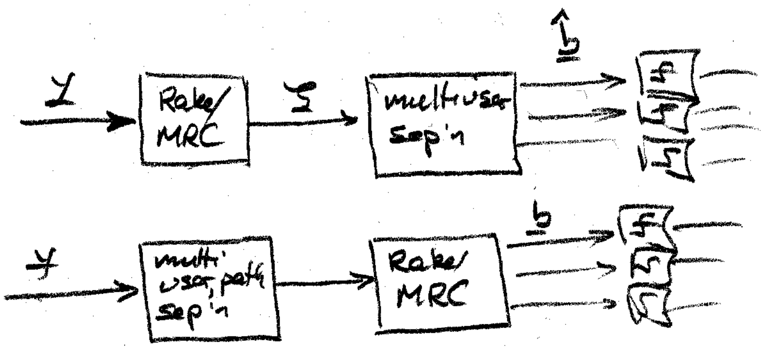
4.1.4 Pre-Combining Multiuser Suppression

- All our analysis to this point has shown the value of Rake/MRC prior to multiuser separation. However, it is possible to reverse the order; it's not optimal, but it has some advantages.

- Terminology:

postcombining

precombining



This follows [Junt00a], [Junt00b], [Latv00], but it may vary in the literature.

- We have at any antenna

$$\underset{KLN\times 1}{\underline{y}_m} = R\,C_m A \underset{KN\times 1}{\underline{b}} + \underline{\nu}_m$$

and R $(KLN \times KLN)$ is the signal correlation matrix.

- Form
$$R^{-1}\underline{y} = \underbrace{C_m A \underline{b}}_{KLN\times 1} + R^{-1}\underline{\nu}_m \qquad \text{(assuming } R \text{ is nonsingular)}$$

to expose each of the path gains at each time in a separate component. For example $K=3$, $L=2$, $N=2$

$$C_m A \underline{b} = [\,,\,,\,,\, c_{k\ell m}(n) A_k b_k(n), \,\cdots\,]^T$$

$$= [c_{10m}(0)A_1 b_1(0),\ c_{11m}(0)A_1 b_1(0),\ c_{20m}(0)A_2 b_2(0),\ c_{21m}(0),\ A_2 b_2(0),$$
$$c_{30m}(0)A_3 b_3(0),\ c_{31m}(0)A_3 b_3(0),\ c_{10m}(1)A_1 b_1(1),\ \cdots$$
$$\cdots\ c_{31m}(1)A_3 b_3(1)]^T$$

- Decimations allow DD channel estimation

e.g. $c_{10m}(0)A_1 b_1(0),\ c_{10m}(1)A_1 b_1(1), \cdots, c_{10m}(n)A_1 b_1(n), \cdots$

as an alternative to using pilot sequences for channel estimation.

- The path combining is ~~performed~~ in the next step.

We have $C_m A \underline{b} + R^{-1} \underline{\nu}_m$ broken out by path and bit

Form

$$C_m^+ (R^{-1} y) = C_m^+ (C_m A \underline{b} + R^{-1} \underline{\nu}_m)$$

$$= C_m^+ C_m A \underline{b} + C_m^+ R^{-1} \underline{\nu}_m$$

where $C_m^+ C_m$ is a diagonal matrix with entries $\sum_{l=0}^{L-1} |c_{k,l,m}(n)|^2$

$$C_m^+ C_m = \operatorname{diag}\left(C_m^+(0) C_m(0), \cdots C_m^+(n) C_m(n), \cdots C_m^+(N-1) C_m(N-1) \right)$$

and

$$C_m^+(n) C_m(n) = \operatorname{diag}\left(\underline{c}_{1m}^+(n) \underline{c}_{1m}(n), \cdots, \underline{c}_{km}^+(n) \underline{c}_{km}(n), \cdots, \underline{c}_{Km}^+(n) \underline{c}_{Km}(n) \right)$$

- Net effect:

$$\hat{b} = A^{-1} (C_m^+ C_m)^{-1} C_m^+ R^{-1} y = A^{-1} C_m^{\#} R^{-1} y$$

$$W = R^{-1} C^{\#+} A^{-1}$$

or if modulation is PSK and amplitude doesn't matter

$$\widehat{C_m^+ C_m A \underline{b}} = C_m^+ R^{-1} y$$

- It's easy to analyze by the methods of Section 3

$$\left[C_m^+ \; (R^{-1} y)^+ \right] \begin{bmatrix} O_{KLN} & I_{KLN} \\ I_{KLN} & O_{KLN} \end{bmatrix} \begin{bmatrix} C_m \\ R^{-1} y \end{bmatrix}$$

or reduce to individual bits

An advantage only if it works well enough to be interesting.