4.4 Error Performance of Linear Receivers

- Finally — the payoff for working through Sections 4.1 to 4.3. We get to see how well the linear methods work.

- Things to look for:
  - required window size
  - performance in power-controlled and in near-far conditions
  - number of users against processing gain, number of paths, number of antennas
  - effect of channel estimation error.

- How the BER is determined:
  - Conventional: Quadratic form if single user, simulation if several users.
  - Decorrelator: Easy for static channel, since

$$\underline{\hat{b}} = \underline{b} + \underline{\alpha} \qquad R_\alpha = N_0 A^{-1} (C^+ \underline{R} C)^{-1} A^{-1}$$

Just use the Q function, user by user.

Fading channels? No analytical solution available, resort to simulation (semianalytic style, average static channel result over C)

– MMSE : Easy for static channel, since

$$\underline{\hat{b}} = \underbrace{W_\varsigma^{+} C^{+} \underline{R} C A \underline{b}}_{H \text{ (another def'n)}} + \underbrace{W_\varsigma^{+} \eta}_{\beta}$$

Data pattern dependent, so user k BER depends on other data

$$P_{ek} = Q\left(\frac{H_{kk} - \sum_{i \neq k} H_{ki} b_i}{\sqrt{R_{\beta\, kk}}} \right)$$

and average over other-user bit patterns.

Fading channels – another semianalytic simulation, averaging over C.

– Pre combining: quadratic form for PSK, fading

$$\underline{\hat{b}} = C^{+}(\underline{R}^{-1} \underline{y})$$

For bit i, form $\underline{\xi} = \begin{bmatrix} C^{\langle i \rangle} \\ R^{-1} \underline{y} \end{bmatrix}$ col i of C / MUDed MF output

and BER is prob that

$$\frac{1}{2} \underline{\xi}^{+} \left[\begin{array}{c|c} O_{2MNKL} & I_{2MNKL} \\ \hline I_{2MNKL} & O_{2MNKL} \end{array}\right] \underline{\xi} < 0$$

when $b_i > 0$.

- Two specialized measures are widely used in the literature to get rid of clutter.
  - Asymptotic multiuser efficiency (AME), [Verd 86] and many subsequent papers. It's defined for static channels as [Lupa 89] (for user k)

$$\eta_k = \sup_{0 \le \rho \le 1} \left\{ \lim_{\sigma \to 0} P_k(\sigma) \Big/ Q\left(\frac{\sqrt{\rho E_k}}{\sigma}\right) \right\}$$

where $P_k(\sigma)$ is user-k BER with noise std dev σ and E_k is energy.

Reason for $\lim_{\sigma \to 0}$: in a mixture of BERs, typically one dominates at high SNR (the worst case one)

Reason for $\sup_{0 \le \rho \le 1}$: adjusts slopes of log BERs to match.

Interpretation: the SNR penalty compared with single user caused by the other users.

  - Near-far resistance (NFR), [Lupa 90], also for static channels

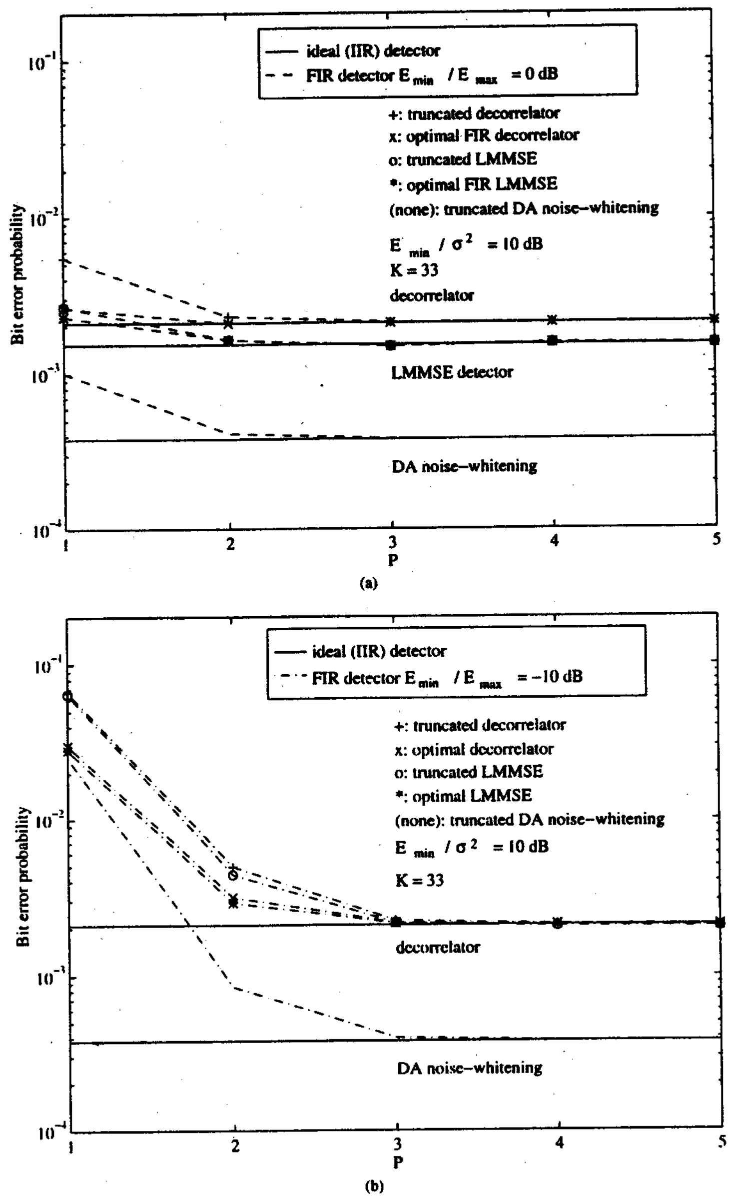
$$\bar{\eta}_k = \inf_{\substack{E_i \ge 0 \\ i \ne k}} \eta_k$$

Interpretation — can large powers of other users drive user-k η_k to zero? If $\bar{\eta}_k \ne 0$, then the method is near-far resistant.

Sometimes restricted to a particular range, like 10 dB, or 20 dB, power disparities.

- We can't present all the available studies, but here are a few.

- Window size
  [Junt97] AWGN, $N_c = 31$, $K = 33$, window size $2P+1$



short codes

Fig. 3. Probabilities of bit error as a function of parameter P with time-invariant waveforms. (a) Equal received energies $E_{min}/E_{max} = 0$ dB. (b) Near-far problem $E_{min}/E_{max} = -10$ dB.

- long codes require slightly smaller windows in near/far conditions

- Basic performance
- [Xie 90] AWGN, asynch, flat, $N_c = 127$, $K = 18$ (lightly loaded)

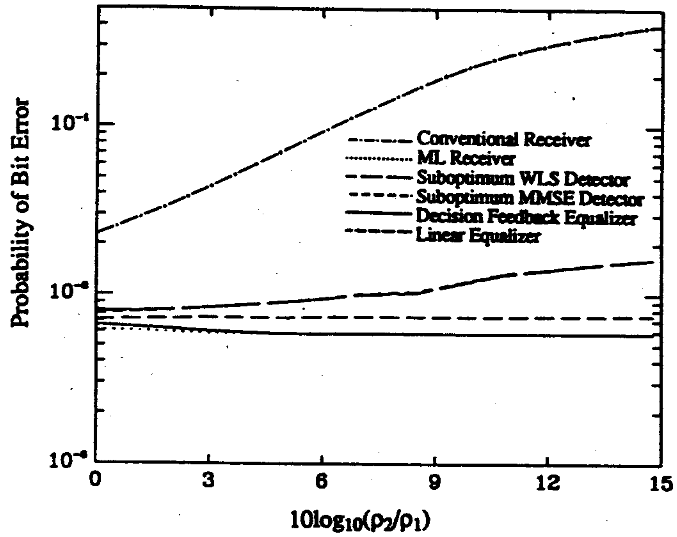


Fig. 7. Bit-error probability versus interfering user's power, for conventional receiver, ML receiver, suboptimum MMSE detector, suboptimum WLS detector, linear equalizer, and decision feedback equalizer with $K = 2$, $L = 5$, and $w_1/N_0 = 5$ dB.

conventional receiver is not near far resistant

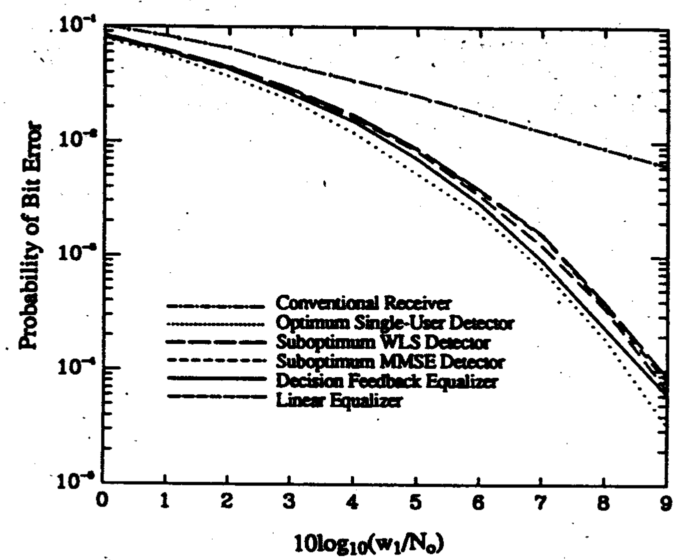


Fig. 6. Bit-error probability for conventional receiver, optimum single-user detector, suboptimum MMSE detector, suboptimum WLS detector, linear equalizer, and decision feedback equalizer with $K = 18$ and $L = 127$.

all MUDs similar for light loading flat channel

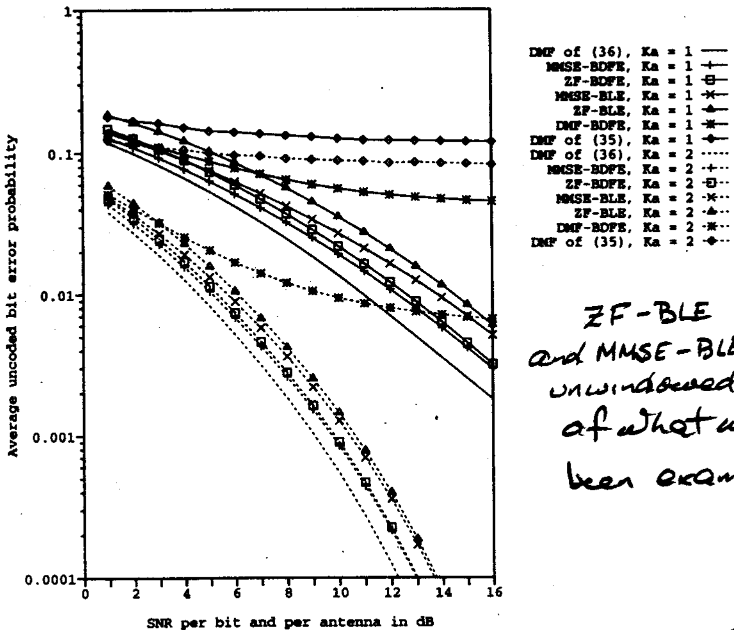
- Linear – fading, multipath
  - [Jung95] (a "must read")
    fading, Ka antennas, different numbers of paths

K = 8 users

$N_c = 14$ chips

N = 20 bits in the burst

perfect CSI

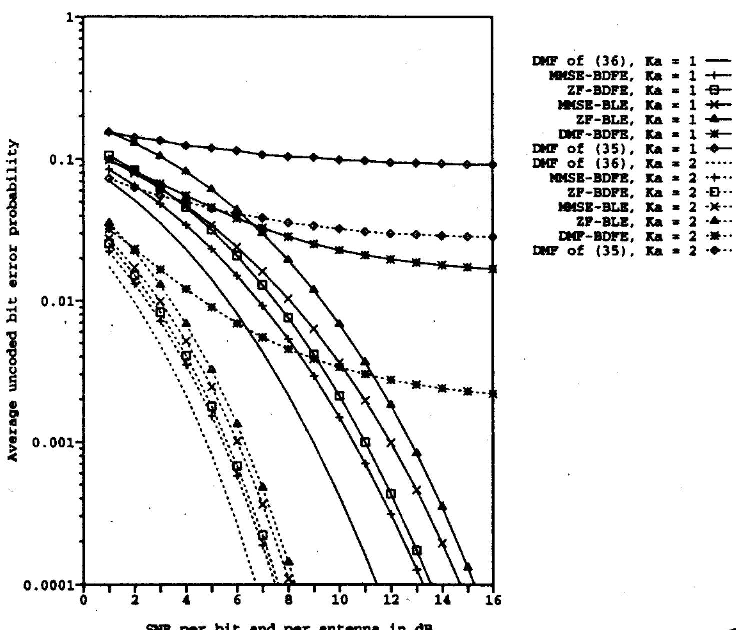


ZF-BLE and MMSE-BLE are unwindowed versions of what we have been examining

L = 2

Fig. 4. Average uncoded bit error probabilities P_e versus the average SNR per bit and per antenna $\bar{\gamma}_b/K_a$ for transmission over rural area radio channels when DMF, DMF–BDFE, ZF–BLE, MMSE–BLE, ZF–BDFE, and MMSE–BDFE are used for JD in the case of $K_a = 1, 2$.

K_a is our M (antennas)



successfully exploits additional diversity

L = 21

Fig. 6. Average uncoded bit error probabilities P_e versus the average SNR per bit and per antenna $\bar{\gamma}_b/K_a$ for transmission over bad urban radio channels when DMF, DMF–BDFE,ZF–BLE, MMSE–BLE, ZF–BDFE, and MMSE–BDFE are used for JD in the case of $K_a = 1, 2$.

4.47

– [Junt99] $L = 2$, $N_c = 31$, $K = 10, 20, 30$

LMMSE-RAKE is precombining

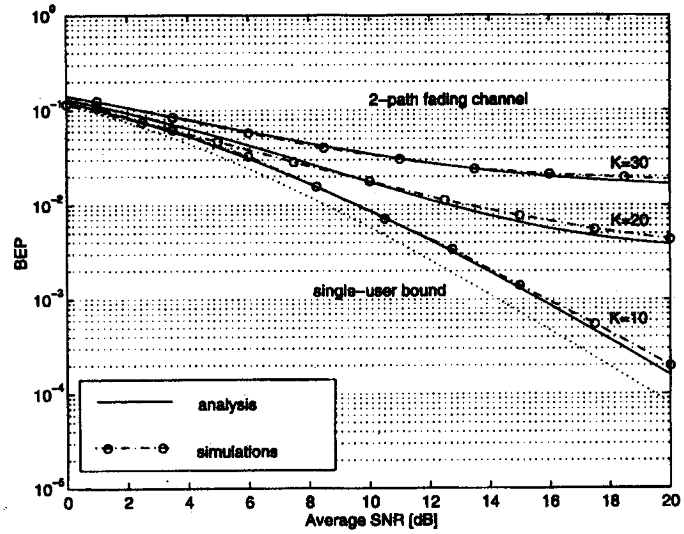


Fig. 2. BEP's computed by semianalytic BEP evaluation and bit-error rates obtained by Monte-Carlo computer simulations of LMMSE-RAKE receivers versus signal-to-noise ratio per symbol for different numbers of users ($N_c = 31$).

lost the diversity

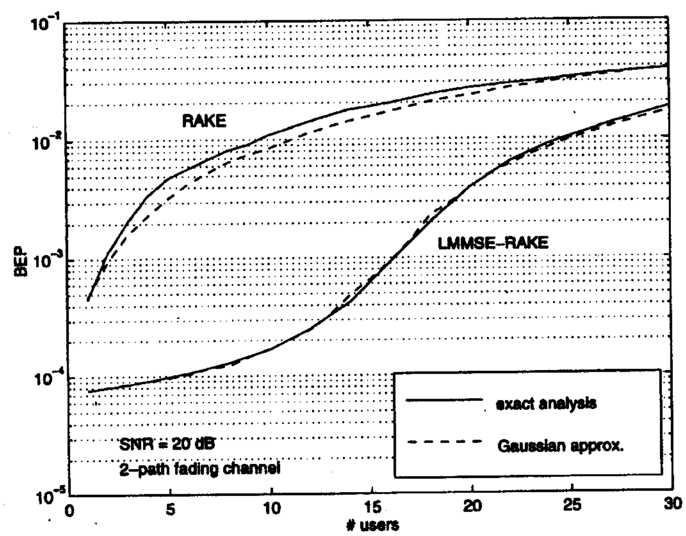


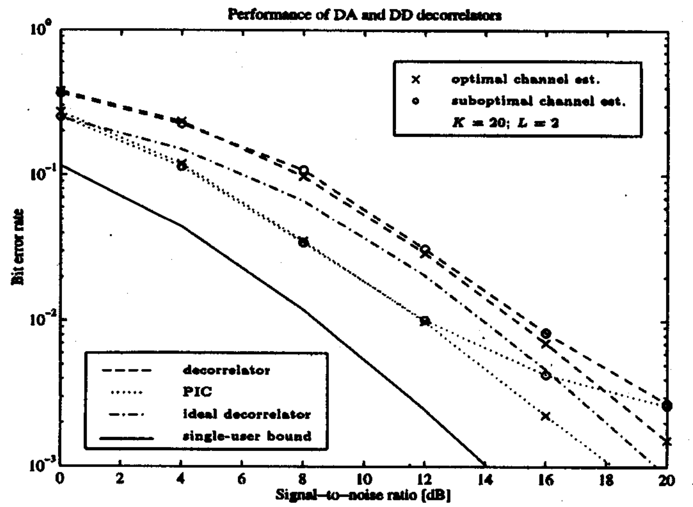
Fig. 3. BEP's of conventional RAKE and LMMSE-RAKE receivers versus number of users computed by Gaussian approximation and semianalytic BEP evaluation (SNR = 20 dB, $N_c = 31$).

LMMSE-RAKE BER rises when $K \approx N_c/L$

but still way better than conventional RAKE

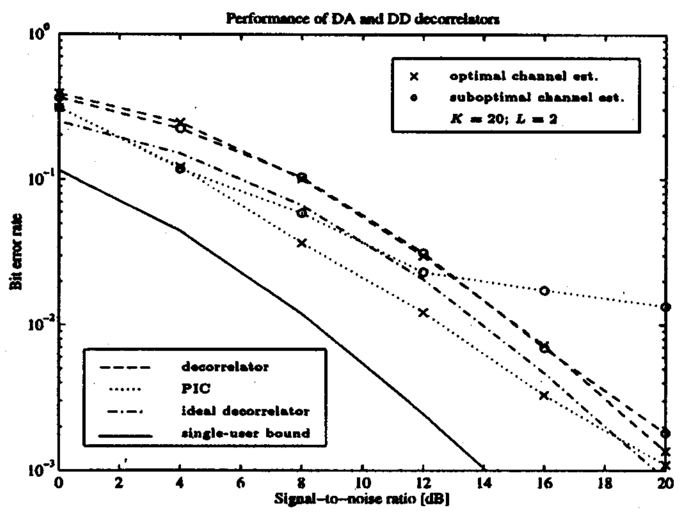
- Channel estimation error

[JuntOO] $L = 2$, $N_c = 16$, fading, precombining $K = 20$



(a)

Precombining facilitates DD channel estimation



(b)

Decorrelator manages estimation error well. PIC (to be discussed) has a problem with it.

Fig. 11. Bit error rates with DD channel estimation in a two-path channel ($L = 2$) for different channel estimation filters: (a) equal received energies and (b) near–far problem. Ideal decorrelator refers to the decorrelating receiver in a known channel.