

## 6.2 ML-MUD With Signatures - Classic Results

- With signatures, each antenna output expands to an inner product per user - in fact,  $KL$  correlations per symbol time for synchronous, or  $NKL$  correlations per packet if aynch. This is our familiar

$$\underline{y}_m = R C_m \underline{A} \underline{b} + \underline{v}_m$$

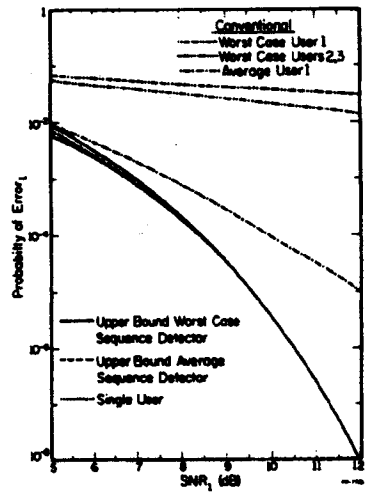
- The problem is still to find which of the candidate sequences has the highest likelihood.
- [Verd 86], a densely-written paper, is the base of much of what we know of ML-MUD with signatures. System: AWGN, asynchronous, flat, single antenna.

Contributions:

- decomposition of the metric into a cyclically causal format, limiting the state set explosion
- upper and lower bounds on error performance
- tightening the union (upper) bound through definition of "decomposable" error sequences and discard of unnecessary terms

- definition of asymptotic multiuser efficiency

Examples

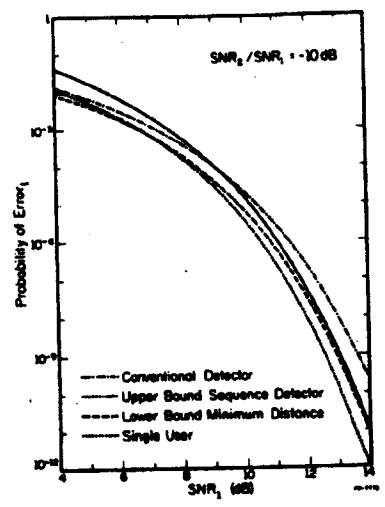


$N_c = 31$   
 $K = 3$   
 BER dep. on value of rel. delays

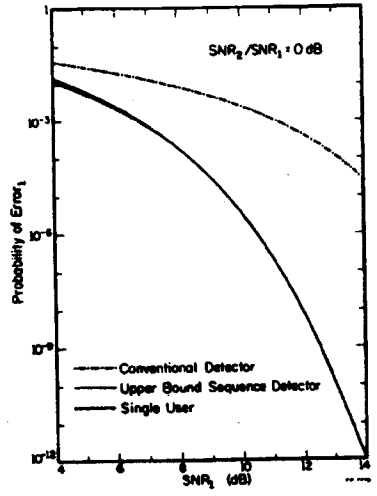
Worst-case relative delay is infrequent, but devastating for the conventional detector. MLSD doesn't even notice it

Fig. 5. Worst-case and average error probabilities achieved by conventional and optimum multiuser detectors with three active users employing m-sequences of length 31.

- Down to  $K=2$ , examine near far. This is  $P_e$  for stronger user when interferer is  $-10$  dB,  $0$  dB



(a)



(c)

Fig. 6. Bounds on minimum error probability of user 1. Worst-case delays and two active users. (a)  $w_2/w_1 = -10$  dB. (b)  $-5$  dB. (c)  $0$  dB.

As interferer becomes stronger, conventional detector has seriously compromised performance, but strong user moves to the single user bound!

- [Lupa 90] focuses power disparities, near-far issues.  
Model: asynch CDMA, AWGN, flat

Contributions:

- introduced definition of near-far resistance
- demonstrated near-far resistance of ML and decorrelator

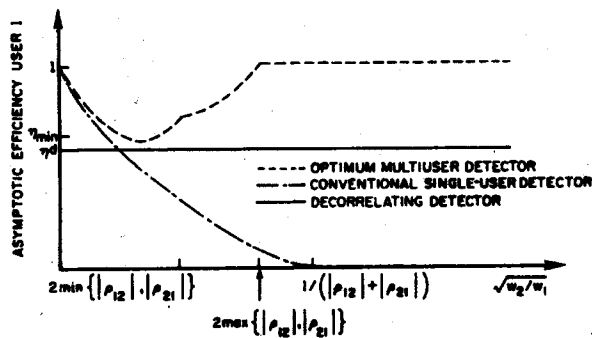


Fig. 7. Asymptotic efficiencies in the two-user case for infinite transmitted sequence length, when the user energies are constant over time (here we chose  $|\rho_{12}|, |\rho_{21}| = 0.3, 0.5$  which yields  $\eta_{\min} = 0.68$ ,  $\eta^d = 0.59$ ).

- noted that imperfect decorrelation (due, e.g., to model error) remove the NFR defined in a strict sense.

- [Zvon 94] took it to fading channels.

Model: asynchronous, flat fading, single antenna, perfect CSI

- New results:

- upper and lower bounds using quadratic forms
- tightened the union bound by extending decomposability to fading channels
- asymptotic multiuser efficiency definition and calculation for fading channels.

- Numerical presentation

$$N_c = 127 \quad K = 2$$

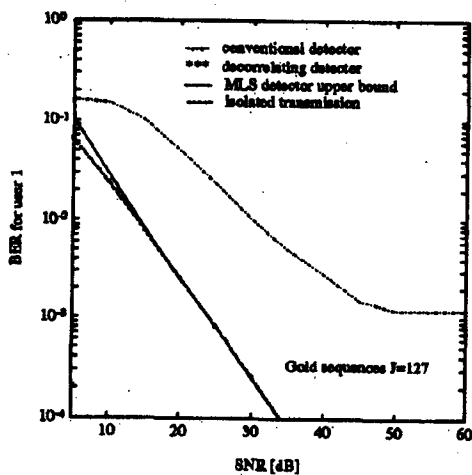


Fig.2. Bit error rate of user 1 for the two-user case with Rayleigh faded paths (same average path strength) and Gold sequences of period  $J=127$ .

- Low correlation between users, but fading produces error floor in conventional (easily derived). ML is indistinguishable from single user bound.
- Lightly loaded

$$N_c = 3 \quad K = 2$$

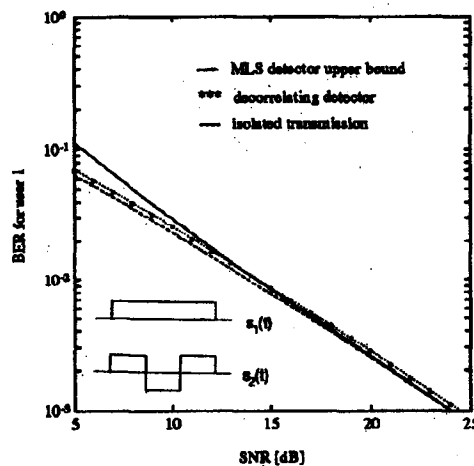


Fig.3. Bit error rate of user 1 for the two-user case with Rayleigh faded paths (same average path strength), spreading ratio  $J=3$ .

- High correlation. MLSD is virtually identical to single user, supporting speculation (Sect. 6.1) that spreading reduces the SNR penalty. Note decorrelator does well - it could handle at most one more user.
- Heavily loaded