6.3 ML-MUD for CDMA: Problem Setup

- In this section, we address the full-blown CDMA problem, formulating it for ML.
  - no numerical results (yet)
  - demonstrates structure of detector with channel estimation error
  - demonstrates a causal form of the ML detector
  - sets it up for performance analysis by quadratic forms
  - suggests a more realistic solution method.

The work is incomplete, unpublished.

6.3.1 The MLSD Receiver for Perfect CSI

- As a warm-up, we'll derive the MLSD for perfect CSI. Recall that

$$\underline{y} = \underline{R} C A \underline{b} + \underline{\nu}$$

for multiple antennas, paths, users and symbol times.

- Conditional likelihood:

$$p_{y|bc}(\underline{y}\,|\,\underline{b}, C) = \frac{1}{(2\pi)^{KLMN}|R_\nu|} \exp\left[-\frac{1}{2}\left(\underline{y} - \underline{R}CA\underline{b}\right)^{+} R_\nu^{-1}\left(\underline{y} - \underline{R}CA\underline{b}\right)\right]$$

Note $R_\nu = N_0 \underline{R}$, independent of $\underline{b}$

– Log, discard hypothesis independent terms, factors

$$\underline{\hat{b}} = \arg\min_{\underline{b}} \left(\underline{y} - \underline{R}CA\underline{b}\right)^{+} \underline{R}^{-1} \left(\underline{y} - \underline{R}CA\underline{b}\right)$$

– Expand exponent, drop hypothesis independent stuff and we have the MLSD as a metric maximization:

$$\underline{b} = \arg\max_{\underline{b}} \underbrace{\mathrm{Re}\left[\underline{b}^{+} A C^{+} \underline{y}\right]}_{\text{correlation term}} - \underbrace{\frac{1}{2N_0}\, \underline{b}^{+} A C^{+} \underline{R} C A \underline{b}}_{\text{energy term}}$$

- Note incorporation of Rake/MRC through $C^{+}\underline{y}$ in the correlation term, which can be written

$$\mathrm{Re}\left[\underline{b}^{+} A \underline{\zeta}\right]$$

- The energy term is important. If we drop it, then

$$\arg\max_{\underline{b}} \mathrm{Re}\left[\underline{b}^{+} A \underline{\zeta}\right]$$

could be achieved by $\hat{b}_k(n) = \mathrm{sgn}\left[\zeta_k(n)\right]$, that is,
the conventional detector. It's the energy term
that makes it ML<u>SD</u>. Unfortunately, it is
quadratic in $\underline{b}$, so it seems that we must receive
all of the packet to process any of it — no
causal form here ...

. . . or is there?

- A trick, originally formulated by Ungerboeck [Unge 74] gives an optimal causal form, even without whitening the noise. Here's how:
  - Denote $\underline{C}^{\dagger}\underline{R}\,\underline{C} = H$, so energy term is $(A\underline{b})^{\dagger} H A\underline{b}$
  - Decompose $H = \underset{H_\ell}{\cdot} + \underset{H_d}{\cdot} + \underset{H_u = H_\ell^{\dagger}}{\cdot}$
  - For convenience, denote $A\underline{b} = \underline{b}'$. Then energy term is

$$\underline{b}'^{\dagger} H \underline{b}' = \underline{b}'^{\dagger} H_d \underline{b}' + \sum_{i=1}^{NK} b_i'^{*}\left(\sum_{j=1}^{i-1} [H_\ell]_{ij}\, b_j'\right) + \sum_{j=1}^{NK} b_j'\left(\sum_{i=1}^{j-1} [H_u]_{ij}\, b_i'^{*}\right)$$

$$= \sum_{i=1}^{NK}\left(H_{ii}|b_i'|^2 + 2\,\mathrm{Re}\left[b_i' \sum_{j=1}^{i-1} H_{ij}^{*}\, b_j'^{*}\right]\right)$$

and this is causal.

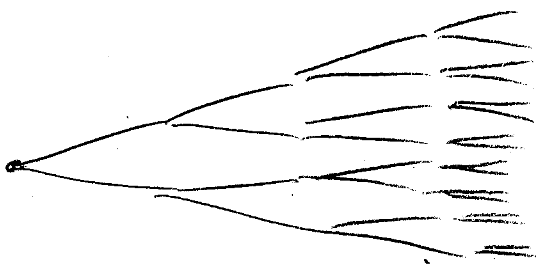
- Combine with the correlation term, and we have the metric up to bit I

$$\Lambda(I) = \sum_{i=1}^{I} \underbrace{\left(\mathrm{Re}\left[b_i'^{*} \xi_i \right] + H_{ii}|b_i'|^2 + 2\,\mathrm{Re}\left[b_i' \sum_{j=1}^{i-1} H_{ij}^{*}\, b_j'^{*}\right]\right)}_{\lambda_i}$$

a branch metric

only the third term has history and it's limited because H is tridiagonal

- Now it's all set up for MLSD by
  - Viterbi. Hmmm — the state set is finite (because $H = C^{+}\underline{R}C$ is block tridiagonal) but it's large.
  - Viterbi with per survivor — well, it's better than DF, but by how much? No results.
  - tree search? We'll see shortly.



eventually folds back on itself to make a messy trellis.

  - or even simple DF (not really ML, but if you're desperate...?)

Exercise:

- Cast the whitened receiver of Section 5.3 into the format of sequence metric and branch metric — show how a tree search algorithm will produce MLSD.

6.3.2 The MLSD Receiver for Imperfect CSI

- Model: the true channel gains and the estimates are related by

$$C = \hat{C} + E$$

where the KLMN × KN matrix E consists of mutually uncorrelated Gaussian estimation errors that are also uncorrelated with $\hat{C}$.

- Then we write the measurements as

$$\underline{y} = \underline{R}\, C A \underline{b} + \underline{v}$$
$$= \underline{R}\, \hat{C} A \underline{b} + \underline{R} \underline{E} A \underline{b} + \underline{v}.$$

The conditional statistics are

$$\mu_{y|b} = \underline{R}\, \hat{C} A \underline{b}$$

$$R_{y|b} = \frac{1}{2} E_{E,\underline{v}}\left[(\underline{R} \underline{E} A \underline{b} + \underline{v})(\underline{R} \underline{E} A \underline{b} + \underline{v})^{+} \right]$$

$$= \frac{1}{2} E\left[\underline{R} \underline{E} \underbrace{A \underline{b}\, \underline{b}^{+} A^{+}}_{H} E^{+} \underline{R}^{+} \right] + N_0 \underline{R}$$

$$= \sigma_e^2 \underline{R}\ tr[H]\, I\, \underline{R}^{+} + N_0 \underline{R}$$

$$= \sigma_e^2\, LMN \sum_{k=1}^{K} |A_k|^2\ \underline{R}^2 + N_0 \underline{R} \qquad \text{independent of } \underline{b}\ !$$

$$= R_y$$

- The MLSD and its metric:

$$p_{y|b}(y|\underline{b},\hat{c}) = \frac{1}{2\pi^{KLMN}|R_y|}\exp\left(-\frac{1}{2N_0}(y-\mu_{y|b})^{+}R_y^{-1}(y-\mu_{y|b})\right)$$

Take log:

$$\arg\min_{\underline{b}} \; (y-\mu_{y|b})^{+} R_y^{-1}(y-\mu_{y|b})$$

or

$$\arg\max_{\underline{b}} \; \underbrace{2\,\mathrm{Re}\left[\mu_{y|b}^{+} R_y^{-1} y\right] - \mu_{y|b}^{+} R_y^{-1}\mu_{y|b}}_{\Lambda(\underline{b},\hat{C})}$$

- Substitute for conditional mean:

$$\Lambda(\underline{b},\hat{C}) = \mathrm{Re}\left[\underline{b}^{+}A^{+}\hat{C}^{+}\underline{R}\,R_y^{-1}y\right] - \frac{1}{2}\,\underline{b}^{+}A^{+}\hat{C}^{+}\underline{R}\,R_y^{-1}\underline{R}\,\hat{C}A\underline{b}$$

Same form as before — another recursive formulation. An interesting receiver.

We can take it further through application of matrix inversion lemma to R_y — but not here!