6.4 Tree Searching in ML-CDMA

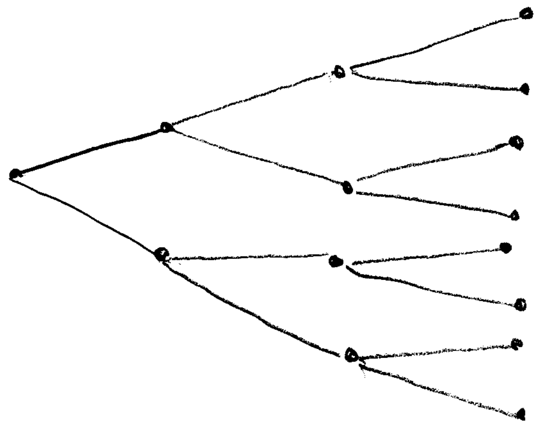
- Maximum likelihood has real attraction, from the performance viewpoint, especially in heavily loaded systems. It also has real drawbacks from the computational viewpoint, especially for heavily loaded systems.

- Can we cut the computation by avoiding full enumeration of all the $\underline{b}$ vectors?

- Since the limited memory (tridiagonal) makes the tree become a trellis, the VA is one such algorithm, and it retains optimality. However, it is still computationally expensive, and it continues to extend many "loser" paths.

- Suboptimal tree search algorithms begin to look attractive. "Breadth first" algorithms [Ande 84], [Simm 90]: retain a limited number of survivor paths; extend only the survivors; harvest the best of the extensions as the new set of survivors. Two candidates:

- M-algorithm: keep the best N_f extensions, regardless of actual metric.
- T algorithm: keep those paths with metrics within a factor T of the best one of the extensions, regardless of number of survivors.



darken branches for $N_f = 2$

- If $N_f = 2^{NK}$ or $T = 0$, then it's optimum, the full tree search.
- They circumvent exponential growth; in fact, they are <u>linear</u> in NK, the number of bits.

- We could apply either algorithm to the causal ML formulations of Section 6.3. However, since they are suboptimal if $N_f < 2^{NK}$ or $T > 0$, a prior transformation by Cholesky can help. (Note that non-sing. linear transformations do not affect <u>full search</u> ML, as shown in Appendix R).

- For perfect CSI, form

$$\underline{z} = (F^{+})^{-1}\underline{y} = FA\underline{b} + \underline{\alpha} \qquad \text{(page 5.3.2)}$$

Then the tree search requires less breadth than in the simple one sided structures of Section 6.3.

- Explored in [Wei97].

Model: Synchronous, but allows multiple paths

First, AWGN, single path.

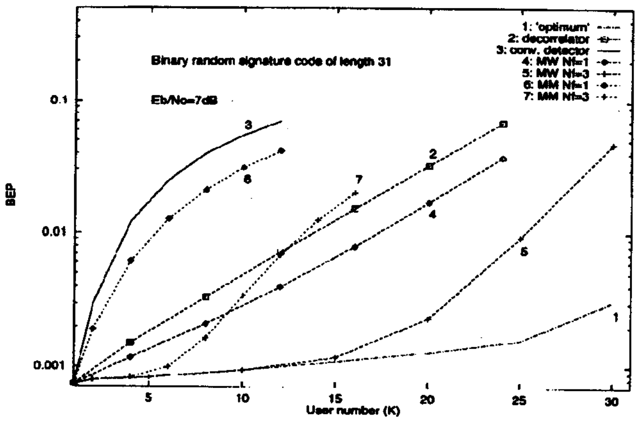


Fig. 4. The BEP's as a function of K for $E_b/N_o = 7$ dB with binary random signature sequences of length 31.

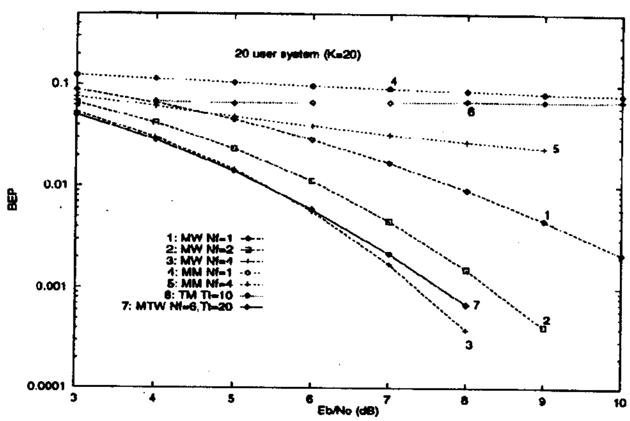


Fig. 5. The BEP's of the MW, MM, TM, and MTW as a function of E_b/N_o with $K = 20$ and binary random signature sequences of length 31.

[Wei97]

<u>Code</u>

MW: M-alg, whitened

MM: M-alg, unwhitened

$N_c = 31$, random signatures

- breadth $N_f = 1$, MW, is just DF (curve 4); better than decorr.
- $N_f = 3$, MW (curve 5) allows 25 users at BER $= 10^{-2}$, $\gamma = 7$dB !
- value of whitening? compare curves 5 and 7.
- primary labour is whitening (Cholesky or Gauss elimination), <u>not</u> the ML search.

Next (also from [Wei97]):

Fading, L=2, perfect CSI. Equipower (mean square) users. $N_c = 31$, random signatures.

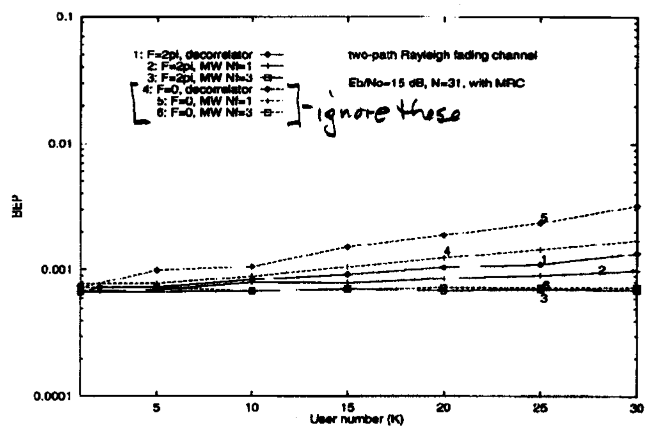


Fig. 8. Average BEP's as a function of K for $E_b/N_o = 15$ dB with binary random signature sequences of length 31 on the two-path Rayleigh-fading channels.

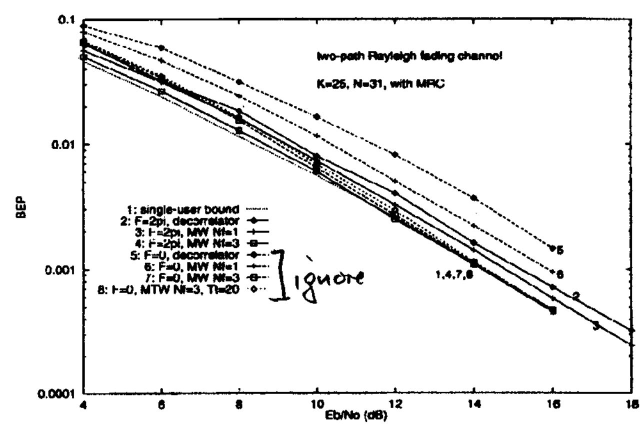


Fig. 9. The BEP's of the MW and MTW as a function of E_b/N_o with $K = 25$ and binary random signature sequences of length 31 on the two-path Rayleigh-fading channels.

(F = 0 means path phases fixed at 0. Not realistic — F = 2π applies to Rayleigh.)

- decorrelator BER rises as K increases and eigenvals become more "profiled"
- the MW receiver with $N_f = 3$ is <u>flat</u> with increasing users. !
- computational effort per bit is determined by whitening; it's $O(K^2)$, and less than decorrelator
- MW with $N_f = 3$ achieves single user bound.
- Very promising, even for heavily loaded systems.
- Caveat: perfect CSI

Here's an exercise related to our most recent topics. Can you resolve this apparent contradiction?

1. In Section 5.3, on whitening receivers, we noted that the first user to be detected suffered from the usual problems with zero forcing, especially for high loading (i.e. high correlation). Only the last of the users enjoyed single-user performance.

2. In Section 6.4, on whitening receivers, we saw that all users get virtually single-user performance.

Why are they different? (It has nothing to do with ranking, by the way).

Here's another thing to look at. In Section 6.3, we represented the ML metric as causal, after examination of the energy term. Yet in Section 6.4, application of the whitening filter, which also makes the interference causal, gave substantially better performance. Why? (Suggestion: study the discussion in [Wei97]).