7.5 Another Nonlinear Method - Parallel IC

- PIC is the Jacobi counterpart of the discrete G-S that we saw as SIC in Section 7.4

- PIC was first proposed clearly in [Vare 90] for asynchronous and [Vare 91] for synchronous.
  - For synchronous or one shot, adaptation of [Vare 91] gives received signal for multiple antennas

$$\underline{r}(t) = \underline{S}(t) C A \underline{b} + \underline{n}(t)$$

  - The ML detector minimizes the exponent of the Gaussian noise pdf as usual, producing

$$\hat{\underline{b}} = \underset{\underline{b} \in Q^{NK}}{\arg\max} \left[2\,\mathrm{Re}\left[\int \underline{r}(t) \left(\underline{S}(t) C A \underline{b} \right)^{*} dt \right] - \int \left| \underline{S}(t) C A \underline{b} \right|^2 dt \right]$$

$$= \underset{\underline{b} \in Q^{NK}}{\arg\max} \left[2\,\mathrm{Re}\left[\underbrace{\underline{b}^{+} \underbrace{A^{+} C^{+} \underline{y}}_{\underline{\xi}}}_{\underline{\xi}'} \right] - \underline{b}^{+} \underbrace{A^{+} C^{+} \underline{R}\, C A}_{G} \underline{b} \right]$$

  - Do the maximization in stages, or iterations. Given previous $\underline{b}^{(j-1)} \in Q^{NK}$, find the user-$k$ value $\hat{b}_k^{(j)} \in Q$ that maximizes the likelihood above. That's what we do next.

Expanding the likelihood and discarding terms that do not depend on $b_k^{(j)}$

$$2\,\mathrm{Re}\left[\zeta_k' \, b_k^{(j)*}\right] - G_{kk}|b_k|^2 - \sum_{i\neq k} G_{ik}\, b_i^{(j-1)*} b_k^{(j)} - \sum_{\ell\neq k} G_{k\ell}\, b_k^{(j)*} b_\ell^{(j-1)}$$

(PSK)

$$\arg\max_{b_k^{(j)}\in Q} \left\{ \mathrm{Re}\left[\left(\zeta_k - \sum_{\ell\neq k} G_{k\ell}\, b_\ell^{(j-1)}\right) b_k^{(j)*}\right]\right.$$

$$= \mathrm{dec}\left[\zeta_k - \sum_{\ell\neq k} G_{k\ell}\, b_\ell^{(j-1)}\right] \qquad \mathrm{dec}(x) = \arg\min_{b\in Q} |x-b|$$

This gives the likelihood-maximizing value of $b_k^{(j)}$, given measurements and previous tentative decisions.

- As an array, it is

$$\underline{b}^{(j)} = \underline{\mathrm{dec}}\left[\underline{\zeta}' - (L+U)\,\underline{b}^{(j-1)}\right]$$

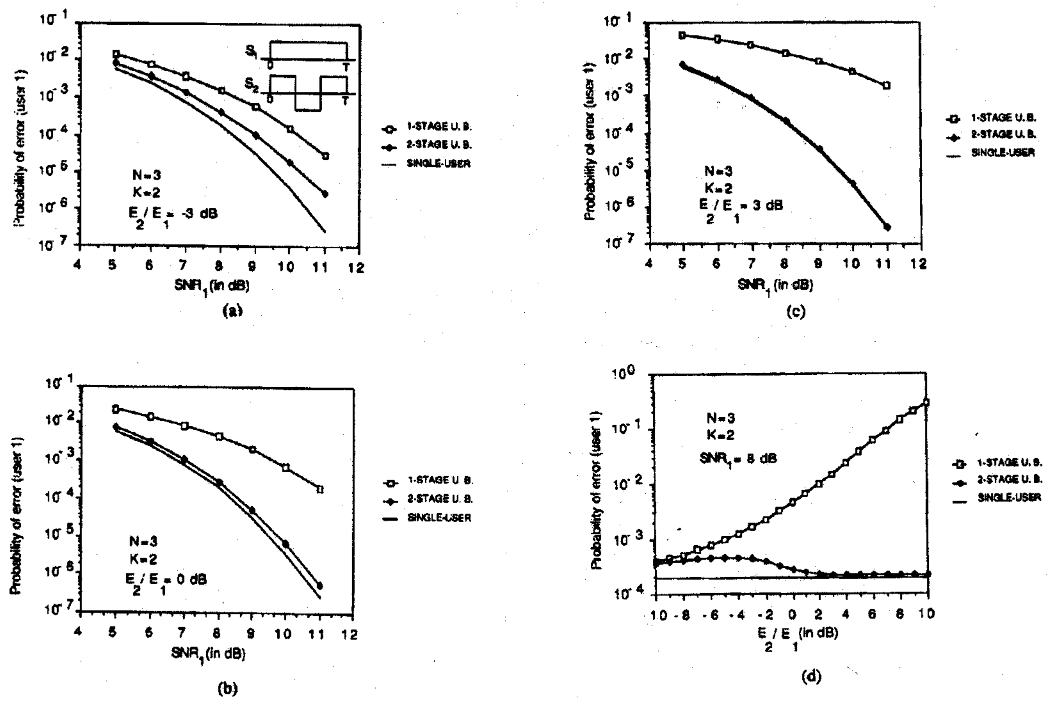
which we recognize as Jacobi iteration with the decision nonlinearity applied.

- The extension to asynch is obvious [Varo 91]

- The discrete PIC (and SIC) receivers can therefore be viewed as multistage attempts to reach an ML solution.

- The pipeline structure on p 7.3.3 still works with discrete Jacobi. PIC and SIC have the same delay for a given number of iterations

- Toy system performance [vara90].



user 1 error rate

Fig. 9. A comparison between the worst case and upper bound of the average error probability of a two-user direct-sequence spread-spectrum system with $N = 3$ for the conventional receiver and the two-stage receiver and the single-user bit-error probability; (a) $E_2/E_1 = -3$ dB, (b) $E_2/E_1 = 0$ dB, (c) $E_2/E_1 = 3$ dB.

- We have an interesting question of what we use for the initial estimate, if this is viewed as a process of refinement. After all, this is a nonlinear iteration that may have local minima, in addition to variations in convergence speed. Obvious choices for first stage: conventional detector, zero (equiv to conventional), decorrelator, MMSE.

Results from [Vara91] show that the decorrelator is preferable as a first stage.

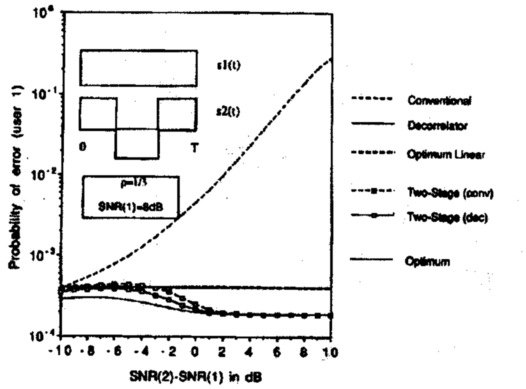


Fig. 1. Error probability comparison of the linear, two-stage, and optimum detectors for a two-user channel with $r_{12} = 1/3$ and signal-to-noise ratio of user 1 fixed at 8 dB.

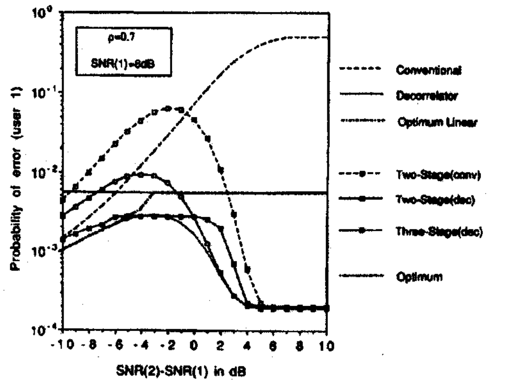


Fig. 2. Error probability comparison of the linear, two-stage, three-stage and optimum detectors for a two-user channel with $r_{12} = 0.7$ and signal-to-noise ratio of user 1 fixed at 8 dB.

More users, more signatures give a better picture

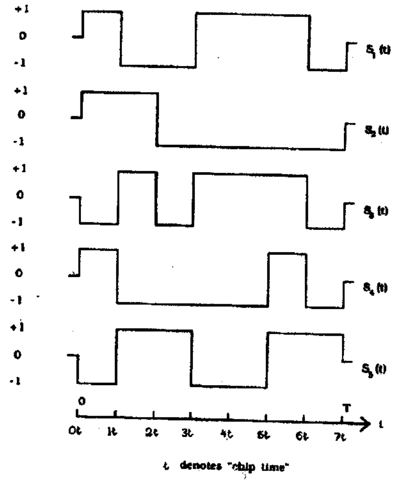
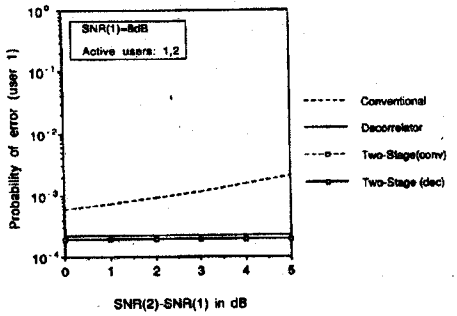
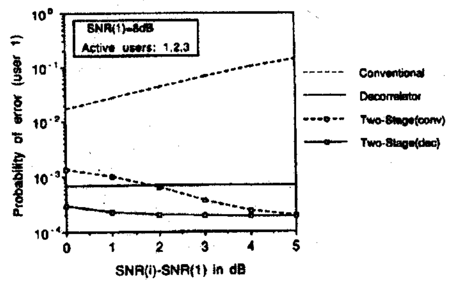


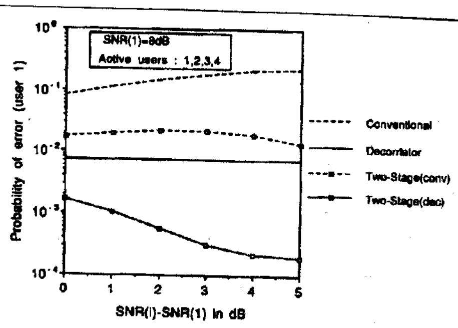
Fig. 4. Baseband signature signals assigned to a five-user CDMA system derived from Gold sequences of length 7.



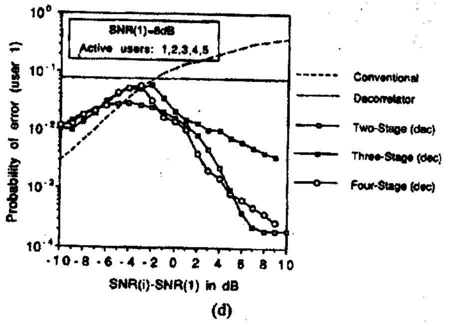
(a)



(b)



(c)



(d)

Fig. 5. Error probability comparison of the linear and multistage detectors for the system assigned the signature signals of Fig. 4. The signal-to-noise ratio of user 1 fixed at 8 dB. The number of active users considered in each figure are (a) two users (1–2), (b) three users (1–3), (c) four users (1–4), (d) five users (1–5).

- No evidence of different solutions with different 1st stages; even (c) looks as though conventional 1st stage will converge properly.
- Note – better than decorr alone

- [Junt 00a] examined decorrelating and PIC receivers in fading. The numerical results also included the effects of imperfect channel estimation, so they are (a) more realistic, but (b) harder to interpret in terms of the inherent qualities.

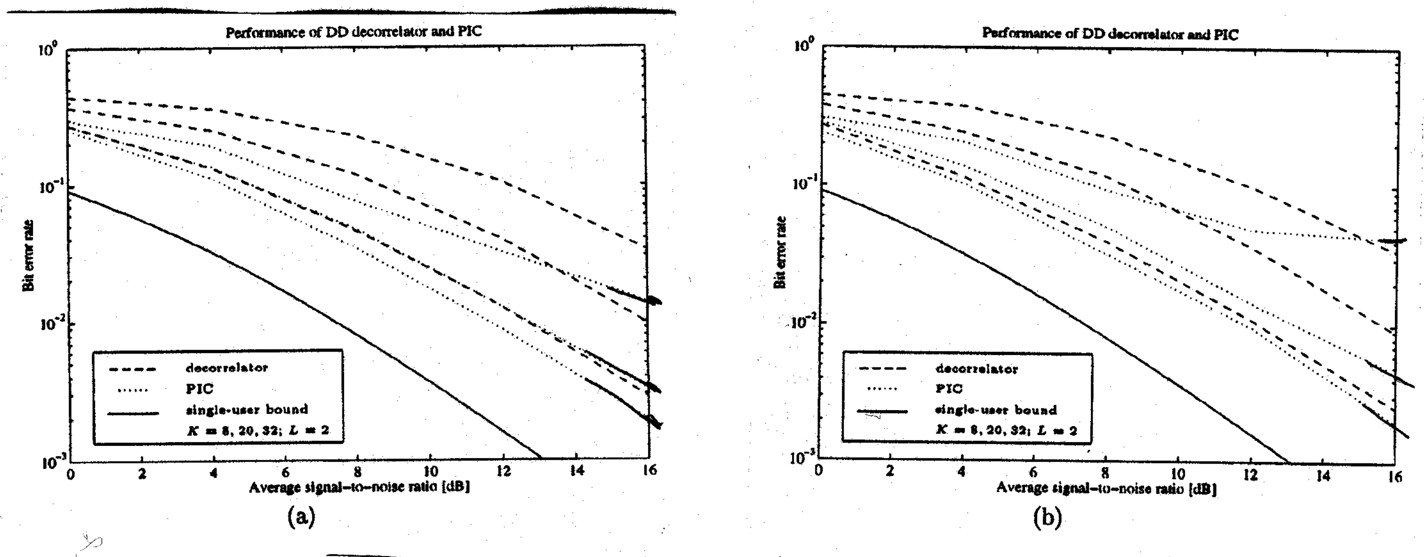


Fig. 10. Bit error rates with DD channel estimation and optimal channel estimation filters in a two-path channel ($L = 2$) for different numbers of users ($K = 8, 20, 32$ from down to upwards): (a) equal received energies and (b) near–far problem. ($\approx \frac{1}{3}$ of users 10 dB stronger than rest).

$N_c = 31$ chips

$J = 2$ stages

- PIC outperforms decorrelator for equipower users, especially for heavier loading and lower SNR, where decorrelator enhances noise.
- For disparate powers and heavy loading, PIC has an error floor due to error propagation. Decorr (as we know) is insensitive to power disparities.

- And now for a suboptimal channel estimator [Junt00a]

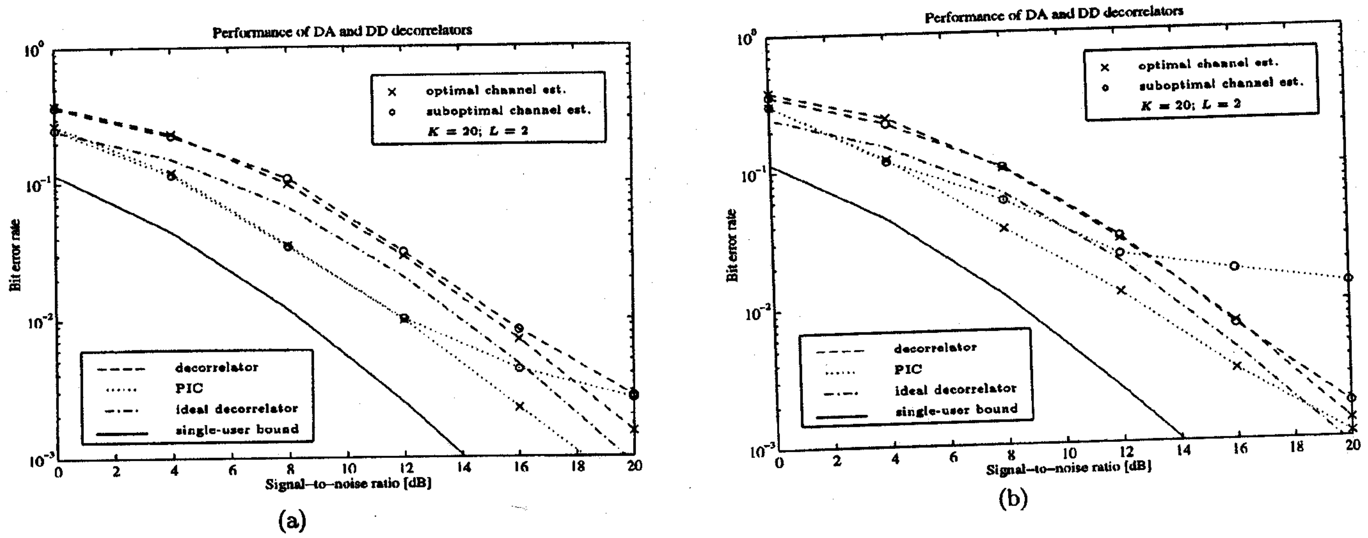


Fig. 11. Bit error rates with DD channel estimation in a two-path channel ($L = 2$) for different channel estimation filters: (a) equal received energies and (b) near–far problem. Ideal decorrelator refers to the decorrelating receiver in a known channel.

- Plots are for moderate load ($N_c = 31$, $K = 20$, $L = 2$) with MUI suppression ~~before~~ combining).
- PIC is better than decorr at lower SNR, but picks up significant floor for poor channel estimation (again error propagation), esp. for near-far conditions
- decorrelator suffers less from imperfect channel estimates

- They are all much worse than single-user, especially for moderate to high load.