7.8 Constrained ML Detection

- Another example of more formal investigation of nonlinearities for tentative decisions is [Tan 01]. The problem investigated is ML

$$\hat{\underline{b}} = \underset{\underline{b} \in \mathbb{C}}{\arg\min} \; \underline{b}^{+} G \, \underline{b} - 2\,\mathrm{Re}[\underline{y}^{+} \underline{b}]$$

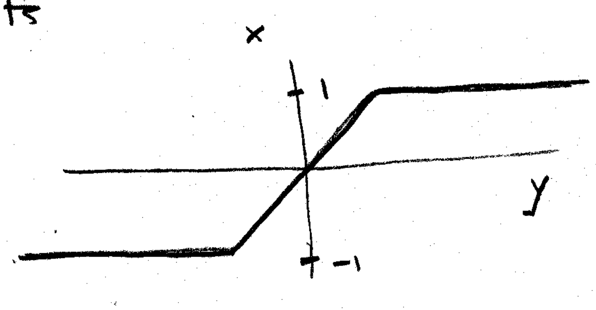
where $\mathbb{C}$ is a closed convex set. Examples include $\mathcal{E}^{NK}$, a sphere $\|b\| \leq NK$ and a box $|b_k(n)| \leq 1, \forall k, n$.

This is one way to approximate the true constraint in the ML problem, $\underline{b} \in \{-1,+1\}^{NK}$.

- Central to the method is the notion of projection of a vector $\underline{y}$ in $\mathcal{E}^{NK}$ onto $\mathbb{C}$. It's the point $\underline{x}$ of $\mathbb{C}$ that's closest to the vector $\underline{y}$ in Euclidean norm.

$$\underline{x} = P_{\mathbb{C}}(\underline{y}) = \underset{\underline{z} \in \mathbb{C}}{\arg\min} \; \|\underline{y} - \underline{z}\|$$

For the sphere constraint, the projection works on the whole vector ($\underline{x} = NK\, \underline{y}/\|\underline{y}\|$). For the box constraint, we can apply it element by element. If real it's



a clipper

• [Tan 01] present an iteration for solution of this constrained ML problem. In its general form:

$$\underline{b}^{(j+1)} = \mu P_c \left[\underline{b}^{(j)} - \omega E_j \left(G\underline{b}^{(j)} - \underline{\xi} + H\left(\underline{b}^{(j+1)} - \underline{b}^{(j)}\right)\right)\right] + (1-\mu)\,\underline{b}^{(j)}$$

where $\left\{ \begin{matrix} 0 < \mu \leq 1 \\ \omega > 0 \end{matrix} \right\}$ are parameters

and for element-separable projections (like box)

H is strictly lower triangular, strictly upper or zero

E_j is positive diagonal

but if not element separable

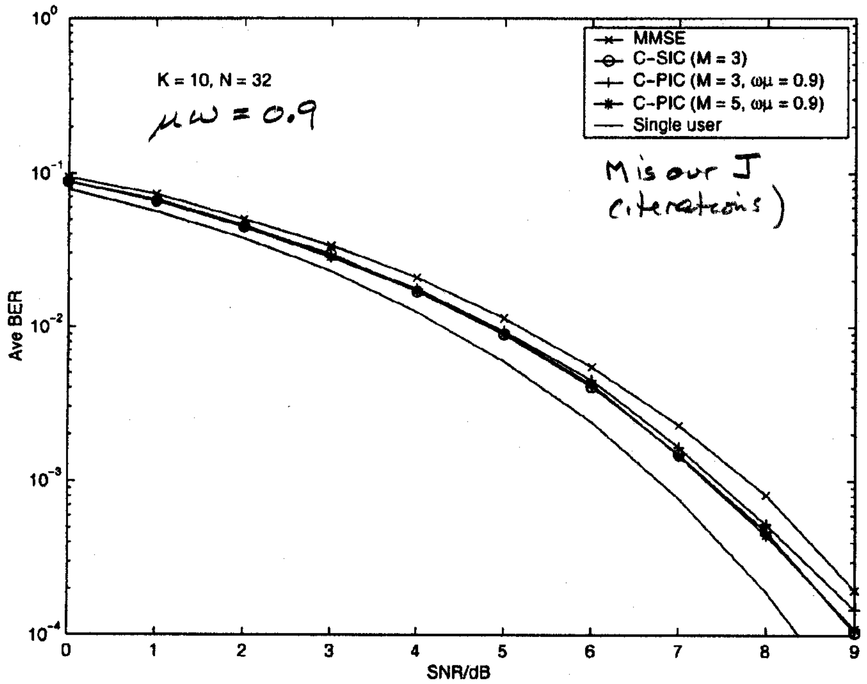
H is zero

$E_j = I$

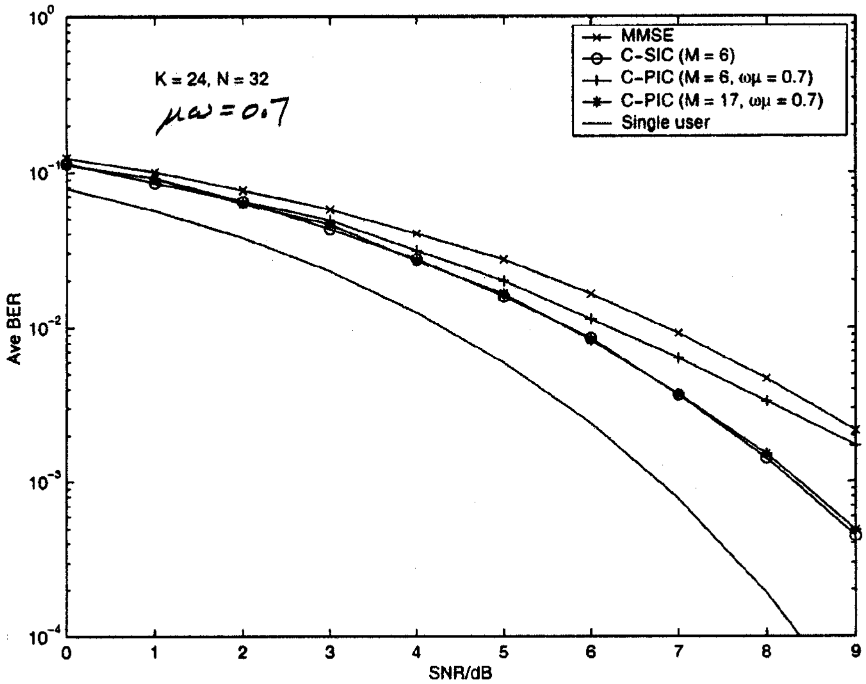
- If $H = 0$, then $\underline{b}^{(j+1)}$ depends only on $\underline{b}^{(j)}$, reminiscent of Jacobi. This is a PIC

- If $H = U$ or $H = L$ then its a SIC, since $\underline{b}^{(j+1)}$ depends on newly determined elements of $\underline{b}^{(j+1)}$ in addition to past values: use $H = L$, $E_j = D^{-1}$, $\mu = \omega = 1$.

- The performance of box constrained (clipped) ML in AWGN, equipower. $N_c = 32$ chips.



- Lightly loaded
- SIC, PIC similar performance
- PIC requires a few more iterations



- Heavy loading
- PIC needs 17 iterations for the same performance as SIC with 6
- significantly better than MMSE

- Adaptable to pipeline in asynch conditions.