## UNIVERSITY OF CANTERBURY

## Dept. of Electrical and Computer Engineering

ENEL 673

## Assignment 1

Due: $\mathbf{2 5}$ March, 2002

## 1. The Bandpass Correlator



The sketch shows a structure that is often termed a bandpass correlator. The inputs are bandpass signals $\tilde{u}(t)$ and $\tilde{v}(t)$, having complex envelopes $u(t)$ and $v(t)$ with respect to some carrier reference at $f_{c}$. Consider the lowpass signal pairs $x_{r}(t), x_{i}(t)$ and $y_{r}(t), y_{i}(t)$ to be the real and imaginary components of complex signals $x(t)=x_{r}(t)+j x_{i}(t)$ and $y(t)=y_{r}(t)+j y_{i}(t)$.
(a) Start with the equations relating $\tilde{u}(t)$ and $\tilde{v}(t)$ to their complex envelopes. Use the identity $\operatorname{Re}[\alpha] \operatorname{Re}[\beta]=\frac{1}{2} \operatorname{Re}\left[\alpha \beta^{*}+\alpha \beta\right]$ to determine what relationship $x(t)$ and $y(t)$ have to the input complex envelopes $u(t)$ and $v(t)$.
(b) As one application, consider $\tilde{v}(t)$ to be a local oscillator output with some frequency and phase error, so that $\tilde{v}(t)=\cos \left(2 \pi\left(f_{c}+\Delta f\right) t+\phi\right)$. In this case, the structure up to $x(t)$ looks like a quadrature demodulator. Express the complex signal $x(t)$ in terms of the input complex envelope $u(t)$ and the reference errors. Try to use complex notation as much as possible.

## 2. CDMA in Delay Spread

In a particular multiuser scenario, each of $K$ users has its own signature pulse shape $p_{k}(t), k=1 \ldots K$ (not necessarily a real pulse), and uses it to transmit $M$ successive symbols. Each user experiences its own channel with single echo, so that the impulse response for user $k$ is $h_{k}(t)=h_{0 k} \delta(t)+h_{1 k} \delta\left(t-\tau_{k}\right)$. They are all received at a single antenna in white noise.
(a) Identify a set of sufficient statistics for detection of the $K M$ symbols. You should find an easy solution, with $2 K M$ such statistics, and a more condensed solution, with one statistic for each symbol.
(b) If you were interested only in detecting user 1 , are all the sufficient statistics still relevant to the decisions?

## 3. Other-Cell Interference



The sketch shows two cells, each with a mobile transmitting to its base station. Distances are as shown. The path loss follows an inverse fourth power with distance and the shadowing has a standard deviation parameter of $\sigma_{s} \mathrm{~dB}$.

Suppose that each base station exerts power control on its mobile. That is, it sends commands that cause the mobile to adjust its transmit power so that the power received by the base station remains constant at some nominal $P_{0}$.
(a) Write an expression for the power transmitted by each mobile (it will be a random variable because of the shadowing). You will need a constant of proportionality, so make it the same for both cells.
(b) Write an expression for the signalto-interference ratio (SIR) seen at base station 1 in terms of the distance and shadowing variables. Given the distances, what is the probability density function of the SIR?
(c) If there were two or more interferers, instead of just one, would the SIR still have such a simple pdf?

