UNIVERSITY OF CANTERBURY

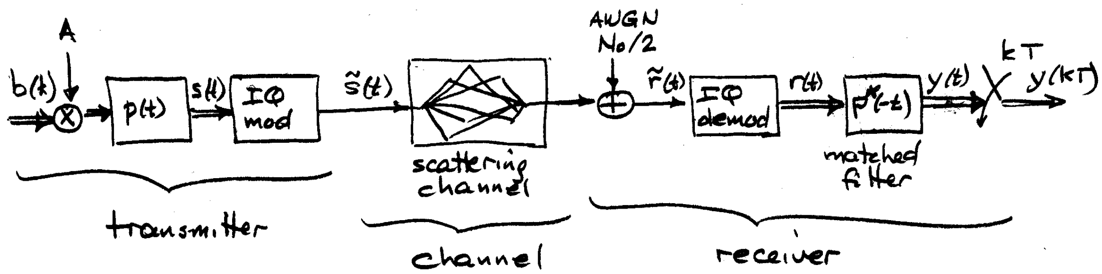
Dept. of Electrical and Computer Engineering

ENEL 673

Assignment 2 **Due: 22 April, 2002**

1. SNR Calculations

Many analyses end up 3 dB out simply through incorrect calculation of signal and noise powers. This question is intended to give you practice in these calculations. It may appear easy, but it should make you think carefully in places.



The sketch above shows a model of a simple link. For emphasis, complex signals are shown with double arrows, real signals with a single arrow. The complex envelope of the transmitted signal is

$$s(t) = A\sum_k b(k)p(t-kT)$$

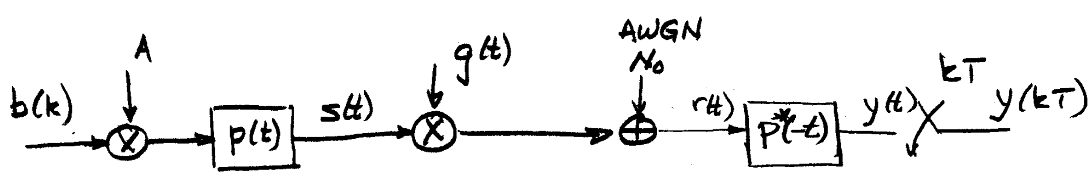
where

$$E\left[\left|b(k)\right|^2\right] = 1 \quad \text{and} \quad \int_{-\infty}^{\infty} \left|p(t)\right|^2 dt = 1$$

and T is the symbol spacing.

(a) The average energy per transmitted symbol in the RF signal $\tilde{s}(t)$ is $E_b = P_s T$, where P_s is the transmitted RF power. In this model, what is the value of the amplitude factor A?

Next, consider the sketch below, which represents the above link entirely from a complex envelope viewpoint, but allows flat fading only. For simplicity, the double/single arrow convention is dropped, because virtually all of the signals are complex.



(b) What is the variance of the complex noise in the matched filter output $y(t)$? (Remember the factor ½, since it's complex.) What is the discrete autocorrelation function of the noise components of the samples $y(kT)$?

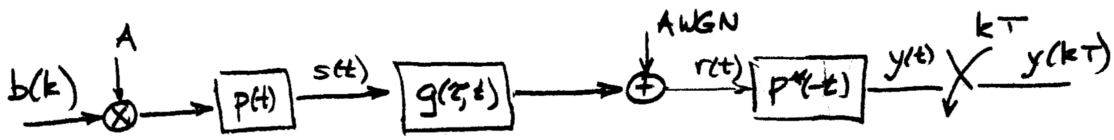
(c) If the complex gain $g(t)$ is constant at 1, what is the signal component in the matched filter output samples $y(kT)$? What is its power (i.e. variance)? What is the ratio of signal power to noise power in terms of A and in terms of E_s?

(d) Now suppose the complex gain $g(t)$ varies with time, but slowly, compared with the time scale T of signal variation. Give an approximation for the signal component in the matched filter output samples $y(kT)$ in terms of gain samples $g(kT)$.

(e) Now suppose that $g(t)$ is random. What condition does its second moment $\frac{1}{2}E\left[|g|^2\right]$ have to satisfy in order for the power of the signal component in $r(t)$ to be unchanged compared to question 1(c)? (This is a modelling issue – in real channels, the transmitted and received power are normally quite different.) If g has zero mean, what is its variance (second central moment) if the condition is satisfied?

(f) If $g(t)$ is random and slowly varying, satisfying the conditions of parts (d) and (e), calculate the ratio of signal power to noise power in the samples $y(kT)$ in terms of E_s.

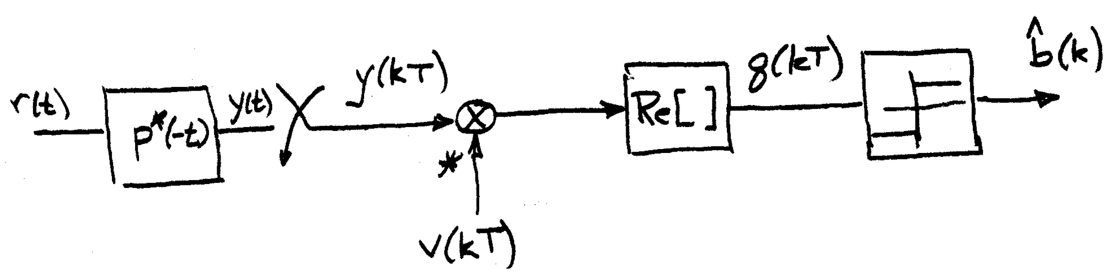
(g) Finally, consider the frequency-selective fading channel below.



The channel impulse response is $g(\tau,t)=\sum_{i=0}^{L-1} g_i(t)\delta(\tau-\tau_i)$ so that the received signal component in $r(t)$ is $\sum_{i=0}^{L-1} g_i(t)s(t-\tau_i)$, where L is the number of arrivals, and $g_i(t)$ is the complex gain experienced at time t by the i^{th} arrival. Assume that those complex gains are uncorrelated. What is the power of the received signal component in $r(t)$? What condition on the second moments of the gains makes the transmitted and received powers equal?

2. Coherent and Incoherent Detection

This question gives you practice at using the Gaussian quadratic form results on the basic system illustrated below.



The matched filter output samples can be written

$$y(kT) = \sqrt{2E_b}\, g(kT) b(k) + n(kT)$$

where the data $b(k) = \pm 1$ and has energy per bit E_b, the noise has variance N_o and the channel gain $g(kT)$ is complex Gaussian with variance ½. The reference is an estimate of the channel gain, given by $v(kT) = g(kT) + e(kT)$. Its error $e(kT)$ is complex Gaussian and independent of $g(kT)$. For purposes of this problem, take its variance to be constant at s_e^2, irrespective of the signal SNR (this isn't usually the case, but I want the problem to be simple). Note that $v(kT)$ is conjugated in the multiplication.

(a) What are the correlation coefficients r, between $g(kT)$ and $v(kT)$, and a, between $y(kT)$ and $v(kT)$?

(b) Write the decision variable $q(kT)$ as a Hermitian quadratic form.

(c) Using the results from our classnotes on Hermitian quadratic forms, give an expression for the BER of this system in terms of the transmitted SNR $\Gamma_b = E_b / N_o$ of the data signal and the reference signal SNR $\Gamma_r = s_g^2 / s_e^2$.

(d) Evaluate your result from (c) with $\Gamma_r \to \infty$ (for coherent detection) and with $\Gamma_r = \Gamma_b$ (the equivalent of differential detection). These are well-known expressions.

(e) Now derive the coherent detection result in a different way. Set $s_e^2 = 0$, so that $v(kT) = g(kT)$, and rewrite $q(t)$ in terms of $x(kT) = |g(kT)|^2$. The probability of error, conditioned on $x(t)$, is now of the type you studied in your first course in communications. Give an expression for that probability of error using either $erfc()$ or $Q(\ )$, whichever you are more comfortable with. Now average it with respect to x, which has an exponential distribution, or with respect to $w = \sqrt{x} = |g|$, which has a Rayleigh distribution. You will probably find it easier to work with w. Use integration by parts.

3. An Easy One

Read and understand the examples of Appendix L. Don't hand anything in on this question.