UNIVERSITY OF CANTERBURY

Dept. of Electrical and Computer Engineering

ENEL 673

Assignment 2 **Due: 27 May, 2002**

1. Two Seemingly-Different MMSE Solutions

If you are not familiar with the singular value decomposition (SVD) of matrices, you can either skip part (b) of this question or read the ENSC 810 notes on SVD to come up to speed.

There are K synchronous users transmitting over frequency-flat fading channels to a receiver with M antennas, where $M \geq K$. The output of the matched filters can be represented, as in class, by

$$\mathbf{y} = \mathbf{CAb} + \mathrm{n} \tag{1}$$

where n is a length-M vector of i.i.d. complex variates with zero mean and unit variance. If $\mathbf{C}$ and $\mathbf{A}$ are known, we obtain an MMSE estimate $\hat{\mathbf{b}}$ of the user data by

$$\hat{\mathbf{b}} = \mathbf{W}_{\mathrm{y}}^{\dagger}\mathbf{y} \tag{2}$$

where

$$\mathbf{W}_{\mathrm{y}} = \left(\mathbf{FF}^{\dagger} + 2\mathbf{I}_M\right)^{-1}\mathbf{F} \tag{3}$$

and $\mathbf{F} = \mathbf{CA}$. However, we also know that we can form a set of sufficient statistics for the bits by $\mathrm{z} = \mathbf{F}^{\dagger}\mathbf{y}$. From this, we can also estimate the data by

$$\hat{\mathbf{b}} = \mathbf{W}_{\mathrm{z}}^{\dagger}\,\mathrm{z} \tag{4}$$

where

$$\mathbf{W}_{\mathrm{z}} = \left(\mathbf{F}^{\dagger}\mathbf{F} + 2\mathbf{I}_K\right)^{-1} \tag{5}$$

(a) Derive (2)-(4) from linear estimation principles, remembering the factor ½ in variances and correlations. Starting from the normal equations will reduce the length of your derivation, but you could start from the basic minimisation, if you want practice.

(b) The two formulations look different. Using an SVD of $\mathbf{F}$, show that they produce the same result for $\hat{\mathbf{b}}$.

2. ZF MUD With Imperfect CSI

A cryptic title for a relatively straightforward problem. We have the same situation as in (1) above, but this time we separate the users by zero forcing. If we had perfect channel state information, then we would form

$$\hat{\mathbf{b}} = \mathbf{A}^{-1}\mathbf{C}^{\#}\mathbf{y} \tag{6}$$

where $\mathbf{C}^{\#} = \left(\mathbf{C}^{\dagger}\mathbf{C}\right)^{-1}\mathbf{C}^{\dagger}$ is the pseudo-inverse of the channel matrix $\mathbf{C}$. The signals have equal power, so $\mathbf{A} = \sqrt{2\Gamma_s}\,\mathbf{I}_K$, and the channel matrix consists of i.i.d. Gaussian variates with zero mean and variance $s_c^2 = \frac{1}{2}$. The noise has unit variance. From class notes, we know that the error rate is that of single -user maximal ratio combining (MRC) with diversity order M-K+1, and we have expressions for the error rate, as well.

If the channel estimates are not perfect, then nulling of the unwanted users is not perfect, either, and there will be some residual interference to hamper the performance. In this question, you will determine the magnitude of the effect.

Specifically, assume that the quality of channel estimates is summarized by r, the same for all channel gains, and that $\mathbf{C} = \hat{\mathbf{C}} + \mathbf{E}$. The components of $\hat{\mathbf{C}}$ and $\mathbf{E}$ are i.i.d. Gaussians, with variances $r^2 s_c^2$ and $\left(1 - r^2\right) s_c^2$, respectively, and are also independent of each other. Now we form

$$\hat{\mathbf{b}} = \mathbf{A}^{-1}\hat{\mathbf{C}}^{\#}\mathbf{y} = \mathbf{A}^{-1}\hat{\mathbf{C}}^{\#}\left(\mathbf{CAb} + \mathrm{n}\right) \qquad (7)$$

(a) Substitute the expression for $\mathbf{C}$, then relate it to the original system with an altered noise level.

(b) Obtain an expression for the error rate if the transmitted symbols are ± 1. It should be a straightforward modification of a result you already know.

(c) What does this imply about the required accuracy of channel estimation for an error rate of, say, 10^{-3}?

(d) Now consider the case of two users with a power difference of 20 dB. Does this present a problem for the stronger user? How about the weaker user? Use your analysis of (c) here.