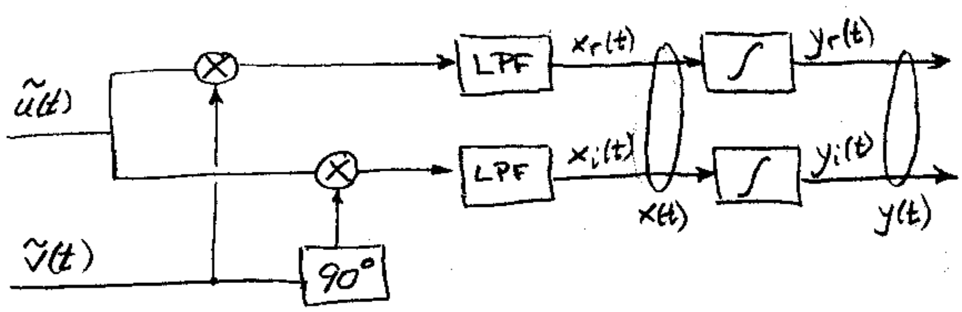
UNIVERSITY OF CANTERBURY
Dept. of Electrical and Computer Engineering

ENEL 673

Assignment 1 **March, 2002**

1. The Bandpass Correlator



(a) I'll use a prime (') to denote real bandpass signals, since Mathcad has no tilde. We have

$$u'(t) = \mathrm{Re}\left(u(t)\cdot e^{j\cdot 2\cdot\pi\cdot f_c\cdot t}\right) \qquad v'(t) = \mathrm{Re}\left(v(t)\cdot e^{j\cdot 2\cdot\pi\cdot f_c\cdot t}\right)$$

The output of the upper multiplier, from the identity, is

$$u'(t)\cdot v'(t) = \frac{1}{2}\cdot\mathrm{Re}\left(u(t)\cdot\overline{v(t)} + u(t)\cdot v(t)\cdot e^{j\cdot 4\cdot\pi\cdot f_c\cdot t}\right)$$

where the overhead bar denotes complex conjugate. Therefore, after the lowpass,

$$x_r(t) = \frac{1}{2}\cdot\mathrm{Re}\left(u(t)\cdot\overline{v(t)}\right)$$

For the other component, denote the output of the 90 degree phase advance as

$$w'(t) = \mathrm{Re}\left(j\cdot v(t)\cdot e^{j\cdot 2\cdot\pi\cdot f_c\cdot t}\right) \quad \text{with complex envelope} \quad w(t) = j\cdot v(t)$$

Following through as above, but with $w(t)$, instead of $v(t)$, we get

$$x_i(t) = \frac{1}{2}\cdot\mathrm{Re}\left(-j\cdot u(t)\cdot\overline{v(t)}\right) = \frac{1}{2}\cdot\mathrm{Im}\left(u(t)\cdot\overline{v(t)}\right)$$

Combining the two lowpass outputs into a "conceptual" complex signal, we have

$$x(t) = x_r(t) + j\cdot x_i(t) = \frac{1}{2}\cdot u(t)\cdot\overline{v(t)}$$

As for the output of the integrators, the conceptual complex signal is

$$y(t) = y_r(t) + j\cdot y_i(t) = \int_{-\infty}^{t} x(t)\,d\alpha = \int_{-\infty}^{t} u(\alpha)\cdot\overline{v(\alpha)}\,d\alpha$$

The structure calculates the correlation (inner product) of the two complex envelopes. In fact, the integrator acts as a lowpass itself, so you can combine it with the lowpass, if you want.

(b) We write the LO output in complex envelope form as

$$v'(t) = \mathrm{Re}\left[e^{j\cdot(2\cdot\pi\cdot\Delta f\cdot t+\phi)}\cdot e^{j\cdot 2\cdot\pi\cdot f_c\cdot t}\right] \qquad \text{so that} \qquad v(t) = e^{j\cdot(2\cdot\pi\cdot\Delta f\cdot t+\phi)}$$

Substitution into the results from (a) gives

$$x(t) = \frac{1}{2}\cdot u(t)\cdot e^{-j\cdot(2\cdot\pi\cdot\Delta f\cdot t+\phi)}$$

which makes the frequency and phase shift explicit.

2. CDMA in Delay Spread

(a) The easiest way to get sufficient statistics is to observe that the signal space is spanned by the $2MK$ waveforms

$$\begin{matrix} p_1(t) & p_1(t-\tau_1) & p_1(t-T) & p_1(t-T-\tau_1) & \cdots & p_1[t-(M-1)\cdot T-\tau_1] \\ p_2(t) & p_2(t-\tau_1) & p_2(t-T) & p_2(t-T-\tau_1) & \cdots & p_2[t-(M-1)\cdot T-\tau_1] \\ \cdots & \cdots & \cdots & \cdots & & \cdots \\ p_K(t) & p_K(t-\tau_1) & p_K(t-T) & p_K(t-T-\tau_1) & \cdots & p_K[t-(M-1)\cdot T-\tau_1] \end{matrix}$$

or any scalar multiple of them (for example, some of you used the set $\{h_{0,k}p_k(t-mT), h_{1,k}p_k(t-mT-\tau_k); k=1..K, m=0..M-1\}$). Therefore, correlation against this set of waveforms will give you sufficient statistics; that is, you do not need any more information from the received signal in order to make your decisions on the MK bits. Of course, what you do with the correlation values after obtaining them is an interesting question, and one that we will spend much of the course addressing.

One very important point: the data symbols $b_k(n)$ are *not* part of the basis. They are in the coefficients that multiply the basis functions, because they are variable.

A more compact set can be obtained by noting that each user's received waveform consists of translates of

$$h_{0,k} \cdot p_k(t) + h_{1,k} \cdot p_k\left(t - \tau_k\right)$$

The received signal therefore occupies a subspace of the one just described, and is spanned by the *MK* waveforms

$$h_{0,1} \cdot p_1(t) + h_{1,1} \cdot p_1\left(t - \tau_1\right) \quad .....$$

$$........ \qquad h_{0,1} \cdot p_1[t - (M-1) \cdot T] + h_{1,1} \cdot p_1\left[t - (M-1) \cdot T - \tau_1\right]$$

....

$$h_{0,K} \cdot p_K(t) + h_{1,K} \cdot p_K\left(t - \tau_K\right) \quad .....$$

$$........ \qquad h_{0,K} \cdot p_K[t - (M-1) \cdot T] + h_{1,K} \cdot p_K\left[t - (M-1) \cdot T - \tau_K\right]$$

Again, the data symbols act as the coefficients, so that the received waveform, *in the absence of noise*, is

$$\sum_{m=0}^{M-1} \sum_{k=1}^{K} b_k(m) \cdot \left(h_{0,k} \cdot p_k(t - m \cdot T) + h_{1,k} \cdot p_k\left(t - m \cdot T - \tau_k\right)\right)$$

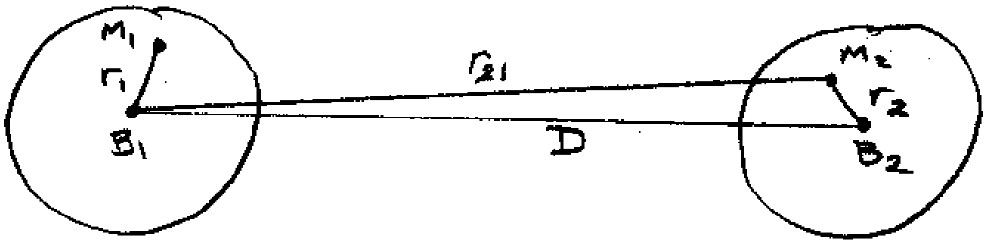
Therefore, if the receiver has the channel coefficients available at this stage, it can correlate against the basis waveform

$$h_{0,k} \cdot p_k(t - m \cdot T) + h_{1,k} \cdot p_k\left(t - m \cdot T - \tau_k\right)$$

for symbol m of user k. Again, what you do with the correlation values is the next question.

(b) If you were interested only in user 1, you would certainly need all the correlations against that user's pulse. You might also be tempted to discard the correlations against other pulses (after all, that is what the conventional CDMA detector does). Unless the waveforms are orthogonal, however, you would lose information; what you pick up from the user-*k* correlation also contains components from the other-user pulses even if those contributions are small. Thus the vector of all correlations contains information about all symbols and, in principle, you should use the vector to make a collective decision on all symbols. Untangling the interactions is one of the central problems of multiuser detection.

3. Other-Cell Interference



(a) The power received by base k and the power transmitted by its mobile are related by

$$P_{r,k} = K \cdot \frac{P_{t,k} \cdot 10^{\frac{s_k}{10}}}{(r_k)^4} \qquad k = 1..2$$

where $exp_{10}(s_k/10)$ is the log-normally distributed shadowing gain, in which s_k is normal (Gaussian) with mean 0 and standard deviation σ_s dB, and K is the proportionality coefficient.

With power control, the base makes its received power constant at P_0. Inverting the above equation, we then have the transmitted power as

$$P_{t,k} = K^{-1} \cdot P_0 \cdot (r_k)^4 \cdot 10^{-\frac{s_k}{10}} \qquad k = 1..2$$

(b) For the SIR at base station 1, we first calculate the interference power. It is the power transmitted by mobile 2, affected by path loss and shadowing on the way to base station 1. Write it as

$$I_{2,1} = K \cdot P_{t,2} \cdot \frac{10^{\frac{s_{2,1}}{10}}}{(r_{2,1})^4} = P_0 \cdot \left(\frac{r_2}{r_{2,1}}\right)^4 \cdot 10^{\frac{s_{2,1}-s_2}{10}}$$

where $s_{2,1}$ is the random dB change due to shadowing en route to base 1. Since power control by base 1 keeps it receiving its own mobile with power P_0, the SIR is

$$\Lambda_1 = \frac{P_0}{I_{2,1}} = \left(\frac{r_{2,1}}{r_2}\right)^4 \cdot 10^{\frac{s_2-s_{2,1}}{10}}$$

The SIR doesn't depend on K or P_0. It improves quickly with increasing distance ratio $r_{2,1}/r_2$ (which is the point of separating cochannel cells). The dB change due to shadowing $\Delta s = s_2 - s_{2,1}$ is the difference of independent normal random variables defined above, so it also normal, with mean 0 dB and standard deviation $\sigma_{\Delta s}$=sqrt(2)σ_s (the variances add). The pdf of Λ_1 is therefore log-normal. To get an expression for it, we start with the pdf of Δs, given by

$$p_{\Delta s}(\Delta s) = \frac{1}{2\cdot\sqrt{\pi}\cdot\sigma_s}\cdot\exp\left[-\frac{1}{4}\cdot\left(\frac{\Delta s}{\sigma_s}\right)^2\right]$$

Now for the change of variables. We have

$$\Lambda_1 = \left(\frac{r_{2,1}}{r_2}\right)^4\cdot\exp\left(\frac{\ln(10)}{10}\cdot\Delta s\right) \qquad \Delta s = \frac{10}{\ln(10)}\cdot\ln\left[\left(\frac{r_2}{r_{2,1}}\right)^4\cdot\Lambda_1\right]$$

Logarithm is monotonic, which simplifies things, but don't forget the Jacobian. We have

$$d\Delta s = \frac{10}{\ln(10)}\cdot\frac{d\Lambda_1}{\Lambda_1}$$

Therefore

$$p_{\Lambda 1}(\Lambda_1) = \frac{1}{\sqrt{2\cdot\pi}\cdot 0.1\cdot\ln(10)\cdot\sqrt{2}\cdot\sigma_s\cdot\Lambda_1}\cdot\exp\left[\frac{-1}{2}\cdot\left(\frac{\ln(\Lambda_1) - \ln\left(\frac{r_{2,1}}{r_2}\right)}{0.1\cdot\ln(10)\cdot\sqrt{2}\cdot\sigma_s}\right)^2\right]$$

Messy, but at least it's clearly lognormal.

(c) If there were two or more interferers, we would have the total interference at antenna 1 as

$$I_1 = I_{2,1} + I_{3,1} .. I_{N,1}$$

Each of the interferers has a log-normal pdf, as derived above. They are independent, so the pdf of I_1 is the convolution of the N log-normal pdfs. Ugh! No closed form. Often, though, we can approximate the sum as another log-normal pdf, if we pick the two parameters (the moments) carefully. One well-known approximation, Wilkinson's method, equates the first two moments of the left and right sides.