Eigenvalue Distribution of Typical
Signal and Channel Correlation Matrices

This worksheet generates examples of the correlation matrix **R** that plays a central role in MUD and displays its eigenvalue distribution. For simplicity, the calculations assume short codes, chip alignment and one sample per chip. Also, a single antenna. However, transmission is asynchronous by symbol and includes delay spread. For contrast, examples of the Rake/MRC combined matrix $\mathbf{C}^H\mathbf{R}\mathbf{C}$ and its eigenvalue distribution are also shown.

You are invited to change any parameter value highlighted in yellow. For generation of random matrices, click once on the equation and press F9 (every F9 gives a new realization).

Definitions (for display here, set values further below):

$N_c = 15$ chips per symbol $K = 6$ number of users

$N = 3$ number of symbols in the transmission

$L = 3$ number of multipath components

Nd delay (in chips) of each user relative

Procedures to generate random signature sequences.

$\text{data2}(x) := \text{if}(\text{rnd}(1) > 0.5, 1, -1)$ random +1, -1 generator

$$\text{gensignature}(N_c, K) := \left| \begin{array}{l} \text{for } i \in 0..N_c - 1 \\ \quad \text{for } k \in 0..K - 1 \\ \quad\quad \text{seq}_{i,k} \leftarrow \text{data2}(i) \\ \text{seq} \end{array} \right.$$

We'll need zero fill arrays further below. Here's a routine to create one:

$$\text{zeromatrix}(\text{Nrows}, \text{Ncols}) := \left| \begin{array}{l} \text{for } ir \in 0..\text{Nrows} - 1 \\ \quad \text{for } ic \in 0..\text{Ncols} - 1 \\ \quad\quad F_{ir,ic} \leftarrow 0 \\ F \end{array} \right.$$

Next, the procedure for creating the signal matrix **S** for symbol zero using the signatures in the columns of *seq*. Expand and shift each signature to account for delay spread, then pack the users into successive columns.

```
makeS(K, L, N_c, Nd, seq) := | N_del ← max(Nd)
                             | N_rows ← N_c + N_del + L − 1
                             | for nm ∈ 0 .. N_rows − 1
                             |   for km ∈ 0 .. K·L − 1
                             |     S_nm,km ← 0
                             | for k ∈ 0 .. K − 1
                             |   for l ∈ 0 .. L − 1
                             |     for i ∈ 0 .. N_c − 1
                             |       | row ← Nd_k + l + i
                             |       | col ← k·L + l
                             |       | S_row,col ← seq_i,k
                             | S
```

At last, a routine to create the signal matrix for the whole received sequence, by replicating the time-zero matrix *N* times: ***S***=diag(**S**,**S**,**S** ,,**S**)

```
makeS(N, K, L, N_c, S) := | shortfill ← zeromatrix(N_c, K·L)
                          | S ← S
                          | S ← stack[S, zeromatrix[(N − 1)·N_c, K·L]]  if  N > 1
                          | temp ← S
                          | for n ∈ 1 .. N − 1
                          |   | newlast ← rows(S) − N_c − 1
                          |   | temp ← stack(shortfill, submatrix(temp, 0, newlast, 0, K·L
                          |   | S ← augment(S, temp)
                          | S
```

To generate the channel gains matrix **C** (*NKL* x *NK*) as independent Gaussian variates, we use

$\text{cgauss}(x) := \sqrt{-\text{rnd}(2)}\cdot\exp(j\cdot\text{rnd}(2\cdot\pi))$ unit variance complex Gaussian

$$\textbf{\textit{makeC}}(N,K,L) := \left|\begin{array}{l} C \leftarrow \text{zeromatrix}(K\cdot L\cdot N, K\cdot N) \\ \text{for } i \in 0..K\cdot N-1 \\ \quad \text{for } j \in 0..L-1 \\ \quad\quad \left|\begin{array}{l} jj \leftarrow i\cdot L + j \\ C_{jj,i} \leftarrow \text{cgauss}(i) \end{array}\right. \\ C \end{array}\right.$$

The eigenvalue ratio of a matrix is a good measure of its conditioning.

$$\text{eigenvalratio}(R) := \left|\begin{array}{l} \lambda \leftarrow \text{eigenvals}(R) \\ \dfrac{\max(\lambda)}{\min(\lambda)} \end{array}\right.$$

Now that we have all the required procedures, try different values and see how the eigenvalue distributions change.

$N_c \equiv 15$ chips per symbol $K \equiv 6$ number of users

$N \equiv 3$ number of symbols in the transmission

$M := 1$

$L \equiv 3$ number of multipath components

Set the delay (in chips) of each user to zero or to a random value: $k := 0..K-1$

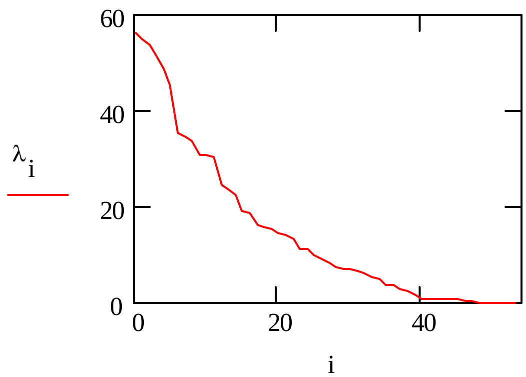
$Nd_k := 0$ $Nd_k := \text{floor}(\text{rnd}(N_c))$ to enable random selection, right-click on this equation and select Enable Evaluation

$seq := \text{gensignature}(N_c, K)$ <== put cursor on this equation, press F9, for new signatures

First the eigenvalues of the $\boldsymbol{R}$ matrix:

$\mathbf{S} := \mathbf{makeS}(K, L, N_c, Nd, seq)$ $\quad \boldsymbol{S} := \boldsymbol{makeS}(N, K, L, N_c, S)$ $\quad \boldsymbol{R} := \boldsymbol{S}^T \cdot \boldsymbol{S}$

$\lambda := \text{eigenvals}(\boldsymbol{R})$ $\quad \lambda := \text{reverse}(\text{sort}(\lambda))$ $\quad i := 0 .. \text{rows}(\lambda) - 1$



$N_c = 15$ $\quad \text{rows}(\boldsymbol{R}) = 54$

$K = 6$ $\quad \text{cols}(\boldsymbol{R}) = 54$

$L = 3$ $\quad \text{rank}(\boldsymbol{R}) = 47$

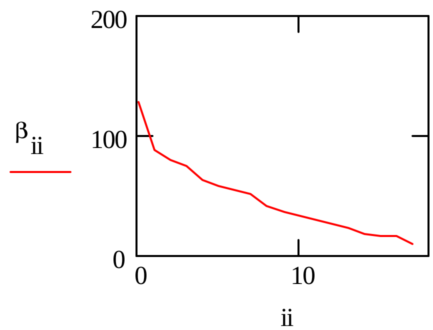
$N = 3$

$\text{eigenvalratio}(\boldsymbol{R}) = -1.255 \cdot 10^{16}$ $\quad \max(\lambda) = 55.987$ $\quad \min(\lambda) = -4.462 \cdot 10^{-15}$

Next, the eigenvalues of the $\boldsymbol{C}^H \boldsymbol{RC}$ matrix:

$\boldsymbol{C} := \boldsymbol{makeC}(N, K, L)$ <== put cursor on this equation, press F9, for new channel gains

$\beta := \text{eigenvals}\left(\overline{\left(\boldsymbol{C}^T\right)} \cdot \boldsymbol{R} \cdot \boldsymbol{C}\right)$ $\quad \beta := \text{reverse}(\text{sort}(\beta))$ $\quad ii := 0 .. N \cdot K - 1$



$N_c = 15$ $\quad \text{rows}\left(\overline{\left(\boldsymbol{C}^T\right)} \cdot \boldsymbol{R} \cdot \boldsymbol{C}\right) = 18$

$K = 6$

$L = 3$ $\quad \text{cols}\left(\overline{\left(\boldsymbol{C}^T\right)} \cdot \boldsymbol{R} \cdot \boldsymbol{C}\right) = 18$

$N = 3$

$M = 1$

$\text{eigenvalratio}\left(\overline{\left(\boldsymbol{C}^T\right)} \cdot \boldsymbol{R} \cdot \boldsymbol{C}\right) = 14.46$