Typical Correlation Matrices of Signature Sequences

This worksheet generates examples of the correlation matrix R that plays a central role in MUD. For simplicity, it assumes chip alignment and one sample per chip. It is asynchronous by symbol and includes delay spread, but uses only short codes.

Definitions:

$N_c := 15$ chips per symbol $K := 3$ number of users

$N := 4$ number of symbols in the transmission

$L := 2$ number of multipath components

Set the delay (in chips) of each user manually:

$Nd_0 := 0$ $Nd_1 := 3$ $Nd_2 := 6$ $N_{del} := \max(Nd)$

Generate random signature sequences.

$\mathrm{data2}(x) := \mathrm{if}(\mathrm{rnd}(1) > 0.5, 1, -1)$ random +1, -1 generator

$$\mathrm{gensignature}(N_c, K) := \left| \begin{array}{l} \text{for } i \in 0..N_c - 1 \\ \quad \text{for } k \in 0..K - 1 \\ \quad\quad seq_{i,k} \leftarrow \mathrm{data2}(i) \\ seq \end{array} \right.$$

$seq := \mathrm{gensignature}(N_c, K)$ <=== To get a new set, click on this equation and press F9, then scroll down to see the effect.

Have a look at the signatures:

$seq =$

	0	1	2
0	-1	-1	1
1	-1	1	-1
2	1	-1	-1
3	-1	1	-1
4	-1	1	1
5	-1	-1	-1
6	1	1	1
7	1	1	-1
8	1	1	1
9	1	-1	1
10	-1	1	-1
11	-1	1	1
12	-1	1	-1
13	1	1	1
14	1	-1	-1

Next, the procedure for creating **S** for symbol zero. Expand and shift each signature to account for delay spread, then pack the users into successive columns. The number of rows will be

$$
\mathbf{makeS}(K, L, N_c, Nd, seq) := \left|
\begin{array}{l}
N_{rows} \leftarrow N_c + N_{del} + L - 1 \\
N_{del} \leftarrow \max(Nd) \\
\text{for } nm \in 0 .. N_{rows} - 1 \\
\quad \text{for } km \in 0 .. K \cdot L - 1 \\
\quad\quad S_{nm, km} \leftarrow 0 \\
\text{for } k \in 0 .. K - 1 \\
\quad \text{for } l \in 0 .. L - 1 \\
\quad\quad \text{for } i \in 0 .. N_c - 1 \\
\quad\quad\quad \left| \begin{array}{l} row \leftarrow Nd_k + l + i \\ col \leftarrow k \cdot L + l \\ S_{row, col} \leftarrow seq_{i,k} \end{array} \right. \\
S
\end{array}
\right.
$$

The number of rows and number of columns in **S** are

$$N_{rows} := N_c + N_{del} + L - 1 \qquad N_{rows} = 22$$

$$N_{cols} := K \cdot L \qquad N_{cols} = 6$$

Compare the sequences and the **S** matrix

$$\mathbf{S} := \mathbf{makeS}\left(K, L, N_c, Nd, seq\right)$$

seq =

	0	1	2
0	-1	-1	1
1	-1	1	-1
2	1	-1	-1
3	-1	1	-1
4	-1	1	1
5	-1	-1	-1
6	1	1	1
7	1	1	-1
8	1	1	1
9	1	-1	1
10	-1	1	-1
11	-1	1	1
12	-1	1	-1
13	1	1	1
14	1	-1	-1

S =

	0	1	2	3	4	5
0	-1	0	0	0	0	0
1	-1	-1	0	0	0	0
2	1	-1	0	0	0	0
3	-1	1	-1	0	0	0
4	-1	-1	1	-1	0	0
5	-1	-1	-1	1	0	0
6	1	-1	1	-1	1	0
7	1	1	1	1	-1	1
8	1	1	-1	1	-1	-1
9	1	1	1	-1	-1	-1
10	-1	1	1	1	1	-1
11	-1	-1	1	1	-1	1
12	-1	-1	-1	1	1	-1
13	1	-1	1	-1	-1	1
14	1	1	1	1	1	-1
15	0	1	1	1	1	1

This shows only some of the rows. To see the remaining entries, click on the matrix and use the scroll bar.

We'll need zero fill arrays further below. Here's a routine to create one:

```
zeromatrix(Nrows, Ncols) := | for ir ∈ 0 .. Nrows − 1
                            |   for ic ∈ 0 .. Ncols − 1
                            |     F[ir, ic] ← 0
                            | F
```

Finally, a routine to create the signal matrix for the whole received sequence, by replicating the time-zero matrix N times: ***S***=diag(**S**,**S**,**S** ,,**S**)

```
makeS(N, K, L, N_c, S) := | longfill ← zeromatrix(rows(S), K·L)
                          | shortfill ← zeromatrix(N_c, K·L)
                          | S ← S
                          | for n ∈ 1 .. N − 1
                          |   S ← stack(S, longfill)
                          | temp ← S
                          | for n ∈ 1 .. N − 1
                          |   | newlast ← rows(S) − N_c − 1
                          |   | temp ← stack(shortfill, submatrix(temp, 0, newlast, 0, K·L
                          |   | S ← augment(S, temp)
                          | S
```

Create the matrix and compare with predecessors above

$$\boldsymbol{S} := \textbf{makeS}(N, K, L, N_c, S)$$

$\boldsymbol{S} =$

	0	1	2	3	4	5	6	7	8	9
0	-1	0	0	0	0	0	0	0	0	0
1	-1	-1	0	0	0	0	0	0	0	0
2	1	-1	0	0	0	0	0	0	0	0
3	-1	1	-1	0	0	0	0	0	0	0
4	-1	-1	1	-1	0	0	0	0	0	0
5	-1	-1	-1	1	0	0	0	0	0	0
6	1	-1	1	-1	1	0	0	0	0	0
7	1	1	1	1	-1	1	0	0	0	0
8	1	1	-1	1	-1	-1	0	0	0	0
9	1	1	1	-1	-1	-1	0	0	0	0
10	-1	1	1	1	1	-1	0	0	0	0
11	-1	-1	1	1	-1	1	0	0	0	0
12	-1	-1	-1	1	1	-1	0	0	0	0
13	1	-1	1	-1	-1	1	0	0	0	0
14	1	1	1	1	1	-1	0	0	0	0
15	0	1	1	1	1	1	-1	0	0	0

This does not show the full matrix. To see more entries, click on it and use the scroll bars.

This is a big matrix: $\quad \mathrm{rows}(\boldsymbol{S}) = 88 \qquad \mathrm{cols}(\boldsymbol{S}) = 24$

Now form the signal correlation matrix relating the correlator outputs

$$\boldsymbol{R} := \boldsymbol{S}^{\mathrm{T}} \cdot \boldsymbol{S}$$

It's smaller: $\quad \mathrm{rows}(\boldsymbol{R}) = 24 \qquad \mathrm{cols}(\boldsymbol{R}) = 24$

It is also Hermitian and diagonal dominant, as the displays on the next page demonstrate.

The eigenvalue ratio of a matrix is a good measure of its conditioning.

$$\mathrm{eivalratio}(R) := \left| \begin{array}{l} \lambda \leftarrow \mathrm{eigenvals}(R) \\ \dfrac{\max(\lambda)}{\min(\lambda)} \end{array} \right.$$

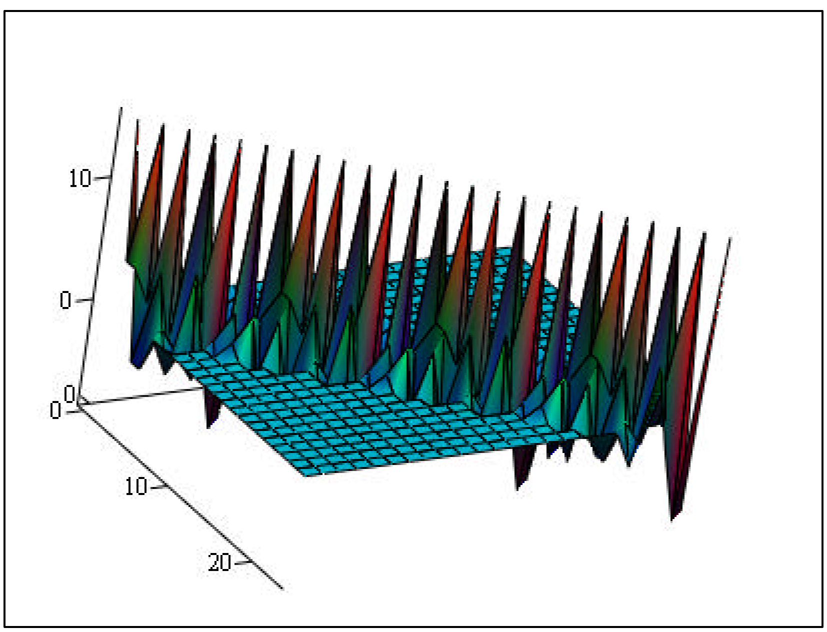
R =

	0	1	2	3	4	5	6	7	8	9
0	15	4	4	-3	-3	0	0	0	0	0
1	4	15	1	4	0	-3	-1	0	0	0
2	4	1	15	-2	-2	5	-3	0	0	0
3	-3	4	-2	15	3	-2	0	-3	1	0
4	-3	0	-2	3	15	-8	2	-1	3	-2
5	0	-3	5	-2	-8	15	-5	2	-4	3
6	0	-1	-3	0	2	-5	15	4	4	-3
7	0	0	0	-3	-1	2	4	15	1	4
8	0	0	0	1	3	-4	4	1	15	-2
9	0	0	0	0	-2	3	-3	4	-2	15
10	0	0	0	0	0	-1	-3	0	-2	3
11	0	0	0	0	0	0	0	-3	5	-2
12	0	0	0	0	0	0	0	-1	-3	0
13	0	0	0	0	0	0	0	0	0	-3
14	0	0	0	0	0	0	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0

This does not show the full matrix, so click on it and use the scroll bars if you want to see more entries.

eivalratio(***R***) = 10.708

The surface plot shows the full matrix. Use your mouse to drag it to different orientations.



R

Now that we have all the required procedures, try different values and see how the surface plot changes.

$N_c := 15$ chips per symbol $K := 4$ number of users

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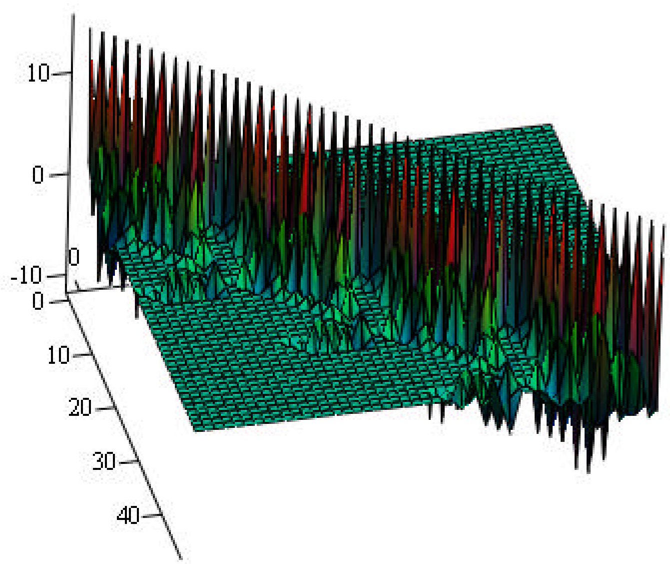
Set the delay (in chips) of each user manually: $k := 0..K-1$ $Nd_k := 0$

$Nd_0 := 0$ $Nd_1 := 3$ $Nd_2 := 6$ $N_{del} := \max(Nd)$

$seq := \mathrm{gensignature}(N_c, K)$ <== put cursor on this equation, press F9

$\mathbf{S} := \mathbf{makeS}(K, L, N_c, Nd, seq)$ $\boldsymbol{S} := \boldsymbol{makeS}(N, K, L, N_c, S)$ $\boldsymbol{R} := \boldsymbol{S}^T \cdot \boldsymbol{S}$



R

$\mathrm{eivalratio}(\boldsymbol{R}) = 223.841$