2.4 Random Gaussian Vectors [Proakis]

- Recall multivariate Gaussian pdf for reals:

$\underline{z}$ real vector, N components

$$E[\underline{z}] = \underline{m} \qquad E\left[(\underline{z}-\underline{m})(\underline{z}-\underline{m})^t\right] = S$$

Then
$$p_{\underline{z}}(\underline{z}) = \frac{1}{(2\pi)^{N/2}|S|^{\frac{1}{2}}} \exp\left(-\tfrac{1}{2}(\underline{z}-\underline{m})^t S^{-1}(\underline{z}-\underline{m})\right)$$

Recall S, S^{-1} are real, symmetric, positive definite

- For complex Gaussian:

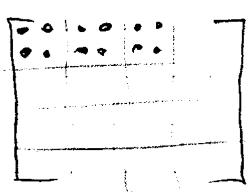
$$E[\underline{v}] = \underline{m}, \qquad S = \tfrac{1}{2} E\left[(\underline{v}-\underline{m})(\underline{v}-\underline{m})^\dagger\right] \qquad (\dagger = *t)$$

Now S is Hermitian $S^\dagger = S$, so it still has real eigenvalues. Since covariance matrix they are all pos, i.e. pos definite matrix.

$$p_{\underline{v}}(\underline{v}) = \frac{1}{(2\pi)^N |S|} \exp\left(-\tfrac{1}{2}(\underline{v}-\underline{m})^\dagger S^{-1}(\underline{v}-\underline{m})\right)$$

- Alternatively, write it as a real vector of length $2N$:

$$\underline{w} = (x_1\, y_1\, x_2\, y_2 \cdots x_N\, y_N)^t$$

$S =$ 

$$p_{\underline{w}}(\underline{w}) = \frac{1}{(2\pi|S|)^N} \exp\left(-\tfrac{1}{2}(\underline{w}-\underline{m})^t S^{-1}(\underline{w}-\underline{m})\right)$$

∴ The characteristic function of the complex form is

$$M_{\underline{z}}(j\underline{\omega}) = E\left[e^{j\underline{\omega}^t\underline{z}}\right]$$

$$= \exp(j\,\underline{m}^t\underline{\omega})\,\exp(-j\tfrac{1}{2}\,\underline{\omega}^t S\,\underline{\omega})$$

Note $\left.\dfrac{\partial M_z}{\partial \omega_k}\right|_{\underline{\omega}=0} = j\,m_k$, $\nabla_{\underline{\omega}} M_z(j\underline{\omega}) = j\,\underline{m}$

etc for other moment generation