APPENDIX J

ZERO FORCING GIVES MAXIMAL RATIO OF LOWER DEGREE.

- We have $\underline{y} = C A \underline{b} + \underline{\nu}$

  where $C_{M \times K}$ contains independent components of variance σ_c^2

  and $\underline{\nu}$ contains iid components (spatially white)

- For user k, we form $\underline{w}_k^\dagger \underline{y}$ with some vector that is orthogonal to $\underline{c}_i$, $i \neq k$. As an easy first step, project $\underline{y}$ onto the subspace orthogonal to the K-1 vectors $\underline{c}_i$, $i = 1..K$, $i \neq k$. Let some orthonormal basis of that space be the $M-(K-1)$ cols of $\Phi_k = [\phi_{k1} | \phi_{k2} | \cdots | \phi_{k,M-K+1}]$

  Components of the projection are of length $M-(K-1)$:

$$\underline{y}' = \Phi_k^\dagger \underline{y} = \Phi_k^\dagger C A \underline{b} + \Phi_k^\dagger \underline{\nu}$$

$$= \Phi_k^\dagger (\underline{c}_1 A_1 b_1 + \underline{c}_2 A_1 b_2 + \cdots + \underline{c}_K A_K b_K) + \underline{\nu}_k'$$

$$= \Phi_k^\dagger \underline{c}_k A_k b_k + \underline{\nu}_k'$$

$$= \underline{c}_k' A_k b_k + \underline{\nu}_k'$$

- What are the statistical properties of $\underline{c}_k'$, $\underline{\nu}_k'$? $\sqrt{2}$
  - Recall $R_c = \frac{1}{2} E[\underline{c}_k \underline{c}_k^\dagger] = \sigma_c^2 I_M$

    Now
    $$R_{c'} = \tfrac{1}{2} E\left[\underline{c}_k' \underline{c}_k'^\dagger\right] = E\left[\Phi_k^\dagger \tfrac{1}{2} \underline{c}_k \underline{c}_k^\dagger \Phi_k\right]$$
    $$= \sigma_c^2 \Phi_k^\dagger \Phi_k = \sigma_c^2 I_{M-(K-1)}$$
  - $R_{\nu'} = \frac{1}{2} E\left[\underline{\nu}_k' \underline{\nu}_k'^\dagger\right] = \Phi_k^\dagger N_0 I_M \Phi_k = N_0 I_{M-(K-1)}$

- These are the same as the stats of a system with only $M-(K-1)$ antennas. Hence we get BER performance exactly equal to single-user MRC with diversity order $M-(K-1)$.

- For completeness, the weight we apply to $\underline{y}'$ is $w_k' = c_k'$, from MRC principles applied to the lower dimensionality space. Going back to the original $\underline{y}$, we have
  $$A\hat{b}_k = c_k'^\dagger y_k' = \underline{c}_k'^\dagger \Phi_k^\dagger \underline{y} = \underline{w}_k^\dagger \underline{y}$$
  so the weight vector applied to the original is
  $$\underline{w}_k = \Phi_k \underline{c}_k' = \underbrace{\Phi_k \Phi_k^\dagger}\, \underline{c}_k$$

  a projection operator — projects $\underline{c}_k$ onto the subspace orthogonal to the other $\underline{c}_i$, $i \neq k$.