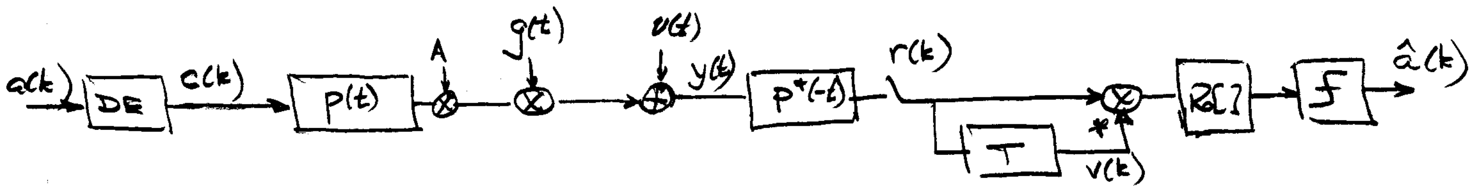
APPENDIX L:

Examples of Detectors Describable by Gaussian Quadratic Forms

- We now have a means for calculating error rate of detectors based on quadratic forms. Let's try it out...

- Example Use this on DPSK in slow, flat Rayleigh fading



$$c(k) = a(k)\,c(k-1) \qquad a(k) = \pm 1$$

from pp 4.4, 4.6: $\quad r(k) = A\,c(k)\,g(kT) + n(kT)$

where A is amplitude, $\quad R_g(t) = \sigma_g^2 J_0(2\pi f_D \tau)$

$$\text{and } \tfrac{1}{2}\,\overline{n(kT)\,n^*(iT)} = N_0\,\delta_{ki}$$

the statistics are (expectations over gain and noise, not data)

$$\bar{r} = 0 \qquad \sigma_r^2 = \tfrac{1}{2}\,\overline{|A\,c(k)\,g(kT) + n(kT)|^2} = A^2\sigma_g^2 + N_0 = E_s + N_0$$

$$\bar{v} = 0 \qquad \sigma_v^2 = \sigma_r^2 = A^2\sigma_g^2 + N_0 = E_s + N_0$$

$$\sigma_{rv}^2 = \tfrac{1}{2}\,\overline{\left(A\,c(k)\,g(kT) + n(kT)\right)\left(A\,c^*(k-1)\,g^*((k-1)T) + n^*((k-1)T)\right)}$$

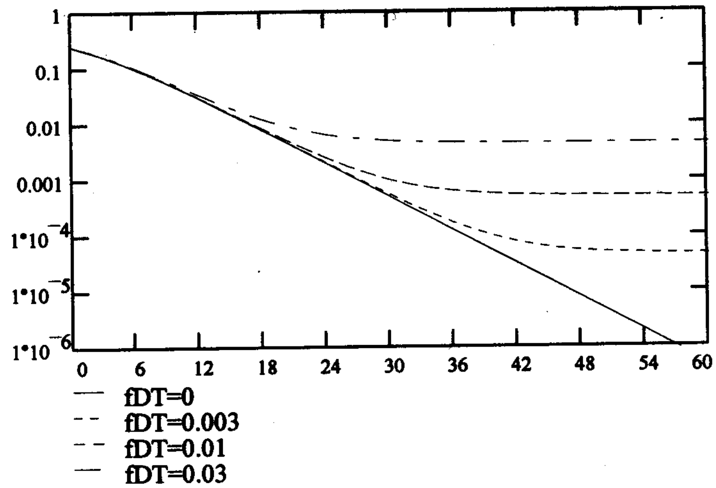
$$= a(k)\,A^2 R_g(T) = a(k)\,E_s\,J_0(2\pi f_D T)$$

$$\rho = \frac{a(k)}{1+\Gamma_s^{-1}}\,J_0(2\pi f_D T) \qquad \Gamma_s = \frac{E_s}{N_0}$$

and for $a(k) = 1$, prob of error is prob $d = \mathrm{Re}[r v^*] < 0$

$$P_e = \tfrac{1}{2}\left(1 - \frac{\rho_r}{\sqrt{1-\rho_i^2}}\right) = \tfrac{1}{2}(1-\rho) = \frac{1}{2}\,\frac{1 + \Gamma_s\left(1 - J_0(2\pi f_D T)\right)}{1+\Gamma_s}$$

same for $a(k) = -1$, since



Note the error floor. Even with no noise, r and v are not perfectly correlated, because complex gain can change over a symbol interval. Slow symbol rate gives error floor problems. Floor value is given by $\Gamma_s \to \infty$

$$P_{e\infty} = \tfrac{1}{2}\left(1 - J_0(2\pi f_D T)\right) \qquad J_0(x) \approx 1 - x^2/4 \quad , |x| \ll 1$$

$$\simeq \frac{\pi^2}{2}(f_D T)^2 \quad \text{typical variation of floor}$$

$f_D = 100\,\text{Hz} \qquad R_s = 2400\ \text{sym/s} \qquad P_{e\infty} \approx 8\times10^{-3}$

- Example Above, with frequency offset.

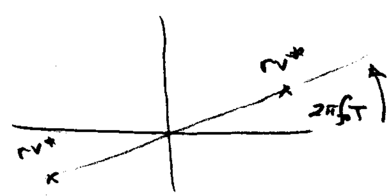
$$r(k) = A\,c(k)\,e^{j2\pi f_0 kT} g(kT) + n(kT)$$

so $\sigma^2_{rv} = a(k)\,E_s\,e^{j2\pi f_0 T} J_0(2\pi f_D T)$

$$\rho = \frac{a(k)}{1+\Gamma_s^{-1}} e^{j2\pi f_0 T} J_0(2\pi f_D T) = a(k)\,e^{j2\pi f_0 T}\rho_0$$

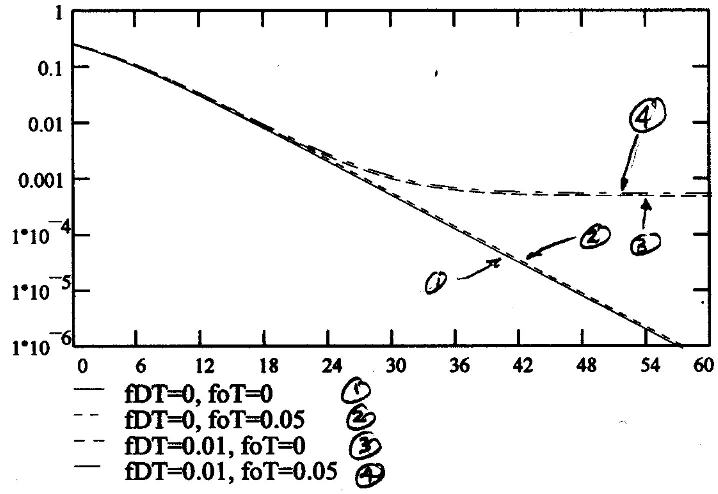
for $a(k)=1$, using $\rho_0 = \frac{J_0(2\pi f_D T)}{1+\Gamma_s^{-1}}$

$$P_e = \tfrac{1}{2}\left(1 - \frac{\rho_0 \cos(2\pi f_0 T)}{\sqrt{1-\rho_0^2 \sin^2(2\pi f_0 T)}}\right)$$



no floor from f_0 if $(f_0 T) \to 0$, just SNR loss

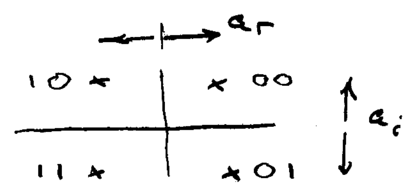
same for $a(k) = -1$



Note frequency offset of 5% makes little difference, but a Doppler spread of 5% is a big problem. The main problem with frequency offset is nonalignment of signal and matched filter, resulting in ISI (not modeled here).

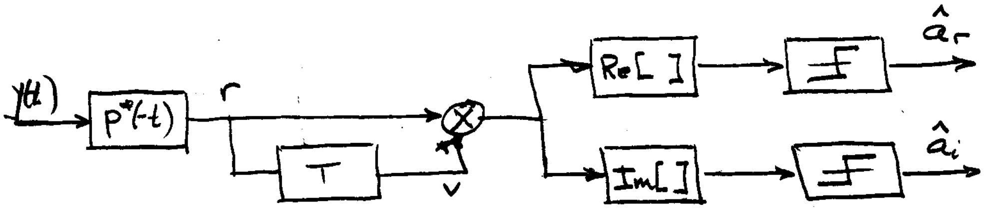
- Example Same, but with $\pi/4$ DQPSK

$$a(k) = \frac{1}{\sqrt{2}}(\pm 1 \pm j) = a_r(k) + j\,a_i(k)$$



$c(k) = c(k-1)a(k)$ always rotated.

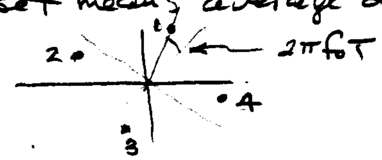
Here we slice the real and imaginary parts separately



Equivalently, we have two quad forms $\mathrm{Re}[rv^*]$, $\mathrm{Re}[-jrv^*]$

$$\rho = a(k)e^{j2\pi f_o T}\rho_o \quad \text{and} \quad \rho = -ja(k)e^{j2\pi f_o T}\rho_o$$

The rotation induced by freq offset means average over data symbols, since BER increased for 1, decreased for 4



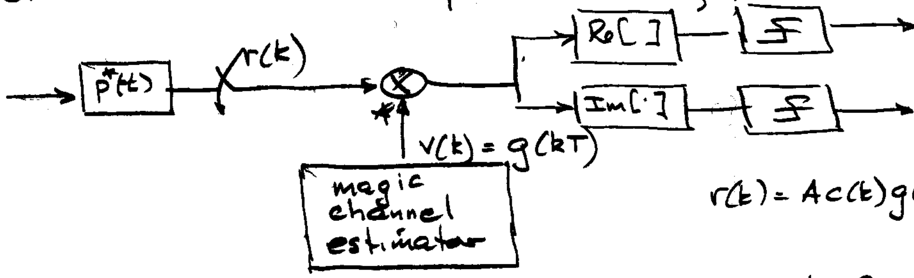
for point 1, $\rho_1 = e^{j\pi/4}e^{j2\pi f_o T}\rho_o \qquad P_{e_1} = \frac{1}{2}\left(1 - \frac{\rho_{1r}}{\sqrt{1-\rho_{1i}^2}}\right)$

for point 4, $\rho_2 = e^{-j\pi/4}e^{j2\pi f_o T}\rho_o \qquad P_{e_4} = \frac{1}{2}\left(1 - \frac{\rho_{2r}}{\sqrt{1-\rho_{2i}^2}}\right)$

$P_e = \frac{1}{2}(P_{e_1} + P_{e_4})$ note this is bit error rate

$$= \frac{1}{2}\left(1 - \frac{\Gamma_s}{\sqrt{2+4\Gamma_s+\Gamma_s^2}}\right) \quad \text{for } f_oT = f_DT = 0$$

- coherent detection is simple to derive, too



$r(k) = A\,c(k)\,g(kT) + n(kT)$

$$\sigma_r^2 = A^2\sigma_g^2 + N_0 = E_s + N_0 \quad \text{as before}$$

$$\sigma_v^2 = \sigma_g^2$$

$$\sigma_{rv}^2 = A\,c(k)\,\sigma_g^2$$

$$\rho = \frac{c(k)\,A\sigma_g^2}{\sigma_g\sqrt{A^2\sigma_g^2 + N_0}} = \frac{c(k)}{\sqrt{1+\Gamma_s^{-1}}}$$

for BPSK, use $c(k) = 1$ (same for $c(k) = -1$)

$$P_{eB} = \tfrac{1}{2}\left(1 - \frac{1}{\sqrt{1+\Gamma_s^{-1}}}\right) \doteq \tfrac{1}{2}\left(1 - \left(1 - \tfrac{1}{2}\Gamma_s^{-1}\right)\right) = \frac{1}{4\Gamma_s}, \quad \Gamma_s >> 1$$

for QPSK, use $c(k) = e^{j\pi/4}$ (same for others)

$$\text{define } \rho_0 = \frac{1}{\sqrt{1+\Gamma_s^{-1}}} \qquad \rho = e^{j\pi/4}\rho_0$$

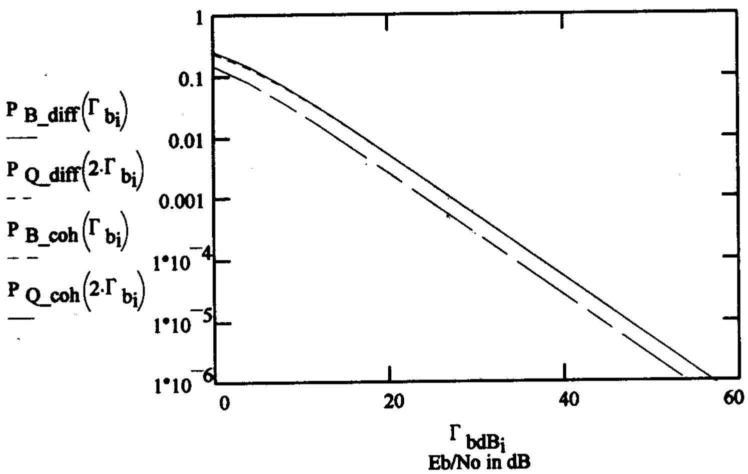
$$P_{eQ} = \tfrac{1}{2}\left(1 - \frac{\rho_r}{\sqrt{1-\rho_i^2}}\right) = \tfrac{1}{2}\left(1 - \frac{\rho_0}{\sqrt{2-\rho_0^2}}\right) \quad (\underline{\text{bit}}\ \text{error rate})$$

$$= \frac{1}{2}\,\frac{\sqrt{\Gamma_s+2} - \sqrt{\Gamma_s}}{\sqrt{\Gamma_s+2}} \approx \frac{1}{2\Gamma_s}, \quad \Gamma_s >> 1$$

- we now have BER for BPSK, QPSK both differentially and coherently detected. Lets compare them, using $\Gamma_b = E_b/N_0$. For BPSK, $\Gamma_s = \Gamma_b$; for QPSK, $\Gamma_s = 2\Gamma_b$. Ignore frequency offset and Doppler spread, and just look at noise-limited region

$$P_{B\_diff}(\Gamma_s) := \frac{1}{2\cdot(1+\Gamma_s)} \qquad P_{Q\_diff}(\Gamma_s) := \frac{1}{2}\cdot\left(1 - \frac{\Gamma_s}{\sqrt{2+4\cdot\Gamma_s+\Gamma_s^2}}\right)$$

$$P_{B\_coh}(\Gamma_s) := \frac{1}{2}\cdot\left(1 - \sqrt{\frac{\Gamma_s}{\Gamma_s+1}}\right) \qquad P_{Q\_coh}(\Gamma_s) := \frac{1}{2}\cdot\left(1 - \sqrt{\frac{\Gamma_s}{\Gamma_s+2}}\right)$$

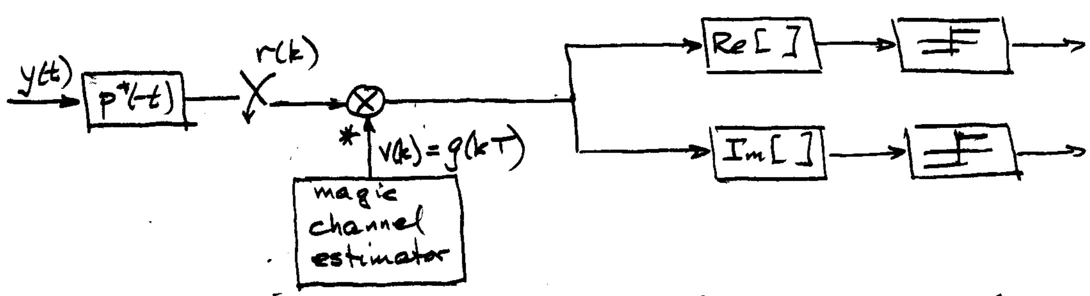


Note binary and quaternary have almost identical performance, if we use *bit* error rate and *bit* SNR. Coherent is 3 dB better than differential.

Easy to see this from asymptotes for large SNR.

Note M-PSK is in Proakis.

- Example Coherent detection, flat Rice fading. What is the effect of the LOS component?



Rice statistics are on page 15 of notes. Use BER formulation in Proakis or on page 4.23. Note variances are central moments.

$$r(k) = c(k)A\,g(kT) + n(kT)$$

$$\bar{r} = c(k)A\sqrt{2K}\,\sigma_g \qquad \sigma_r^2 = A^2\sigma_g^2 + N_0 \qquad \frac{E_s}{N_0} = \frac{A^2\sigma_g^2(1+K)}{N_0}$$

$$\bar{v} = \sqrt{2K}\,\sigma_g \qquad \sigma_v^2 = \sigma_g^2$$

$$\sigma_{rv}^2 = c(k)A\sigma_g^2 \qquad \rho = \frac{c(k)A\sigma_g}{\sqrt{A^2\sigma_g^2+N_0}} = \frac{c(k)}{\sqrt{1+(1+K)\Gamma_s^{-1}}}$$

Keep it simple and do BPSK : $c(k) = \pm 1$. Then from page 4.23 and a little rearranging

$$\left\{ \begin{matrix} a^2 \\ b^2 \end{matrix} \right\} = \frac{K}{2(1+(1+K)\Gamma_s^{-1})} + \frac{K}{2} \mp \frac{c(k)\,K}{\sqrt{1+(1+K)\Gamma_s^{-1}}}$$

But how to calculate $Q(a,b)$?

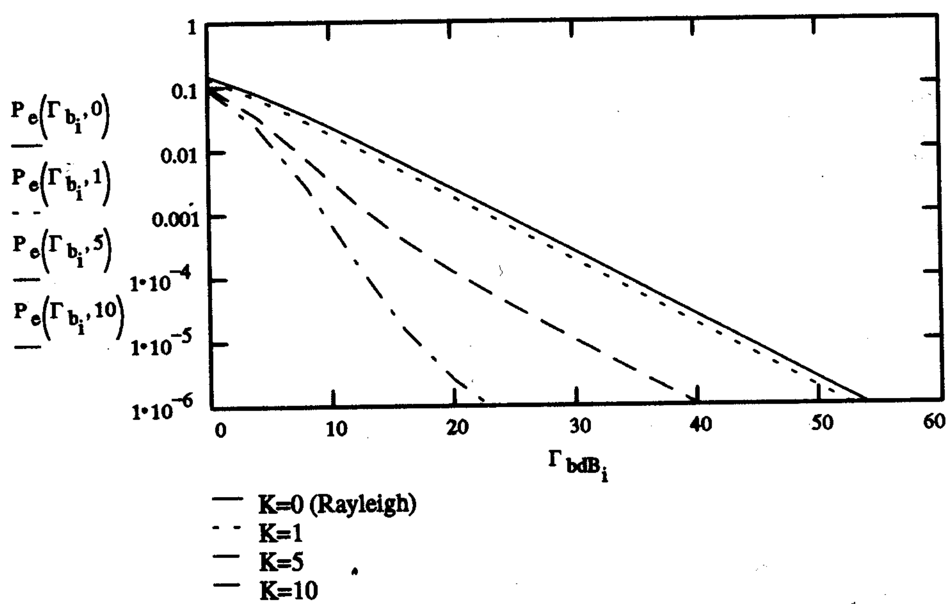
- numerical integration
- series in Bessel functions [Proakis App 4B, S,B & S App A]
- Bird & George, "The Use of Fourier-Bessel Series, ..." IEEE Trans Commun, Sept '81.

$$\rho(\Gamma_s, K) := \frac{1}{\sqrt{1+(1+K)\cdot\Gamma_s^{-1}}}$$

$$a(\Gamma_s, K) := \sqrt{\frac{K}{2}\cdot\left[\frac{1}{1+(1+K)\cdot\Gamma_s^{-1}} + 1\right] - \frac{K}{\sqrt{1+(1+K)\cdot\Gamma_s^{-1}}}}$$

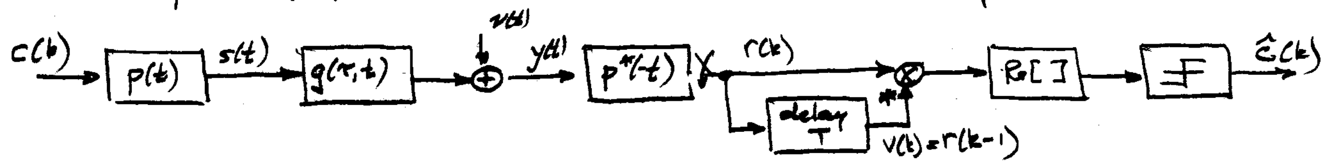
$$b(\Gamma_s, K) := \sqrt{\frac{K}{2}\cdot\left[\frac{1}{1+(1+K)\cdot\Gamma_s^{-1}} + 1\right] + \frac{K}{\sqrt{1+(1+K)\cdot\Gamma_s^{-1}}}}$$

$$P_e(\Gamma_s, K) := Q\left(a(\Gamma_s,K), b(\Gamma_s,K)\right) - \frac{1+\rho(\Gamma_s,K)}{2}\cdot I0\left(a(\Gamma_s,K)\cdot b(\Gamma_s,K)\right)\cdot \exp\left(-\frac{a(\Gamma_s,K)^2 + b(\Gamma_s,K)^2}{2}\right)$$



For higher K, the low SNR region looks much like static channel behaviour. At high SNR, performance is still limited by the $1/\Gamma_b$ asymptote imposed by the scattered component. A major improvement (about 14 dB) even with K=5!

__Example__ Differential detection, selective Rayleigh fading. What is the effect of delay spread? Assume perfect clock recovery.



From page 4.5, slow selective model:

$$r(t) \approx \sum_k c(k) \int g(\tau, t)\, g(t-\tau-kT)\, d\tau + n(t)$$

Consider detection of bit at $t=0$. Then

$$r(0) = \sum_k c(k) \int g(\tau, 0)\, g(-\tau-kT)\, d\tau + n(0)$$

$$v(0) = r(-1) = \sum_k c(k) \int g(\tau, -1)\, g(-1-\tau-kT)\, d\tau + n(-1)$$

All we need is the correlation coefficient of r and v.

$$\sigma_r^2 = \sum_k \sum_i c(k) c^*(i) \tfrac{1}{2} \iint \overline{g(\tau,0) g^*(\eta,0)}\, g(-\tau-kT)\, g^*(-\eta-iT)\, d\tau\, d\eta + N_0$$

Note from page 3.53 WSSUS and separable

$$\tfrac{1}{2} E\left[g(\tau,t) g^*(\eta,s) \right] = \sigma_g^2 G(\tau) R_g(t-s)\, \delta(\tau-\eta)$$

where normalised so $G(\tau)$ has unit area and $R_g(0)=1$

so

$$\sigma_r^2(\underline{c}) = \sum_k \sum_i c(k) c^*(i)\ \sigma_g^2 \int G(\tau)\, g(-\tau-kT)\, g^*(-\tau-iT)\, d\tau + N_0$$

$$\sigma_v^2(\underline{c}) = \sigma_r^2(\underline{c})$$

$$\sigma_{rv}^2(\underline{c}) = \sum_k \sum_i c(k) c^*(i) \tfrac{1}{2} \iint \overline{g(\tau,0) g^*(\eta,-1)}\ g(-\tau-kT)\, g(-T-\eta-iT)\, d\tau\, d\eta$$

$$= \sum_k \sum_i c(k) c^*(i)\ \sigma_g^2 R_g(T) \int G(\tau)\, g(-\tau-kT)\, g^*(-\tau-(i+1)T)\, d\tau$$

all three are sequence-dependent.

the correlation coefficient $\rho(\underline{c}) = \dfrac{\sigma_{rv}^2(\underline{c})}{\sigma_r(\underline{c})\,\sigma_v(\underline{c})}$

assume differentially encoded $c(k) = a(k)\,c(k-1)$, consider $a(0) = +1$

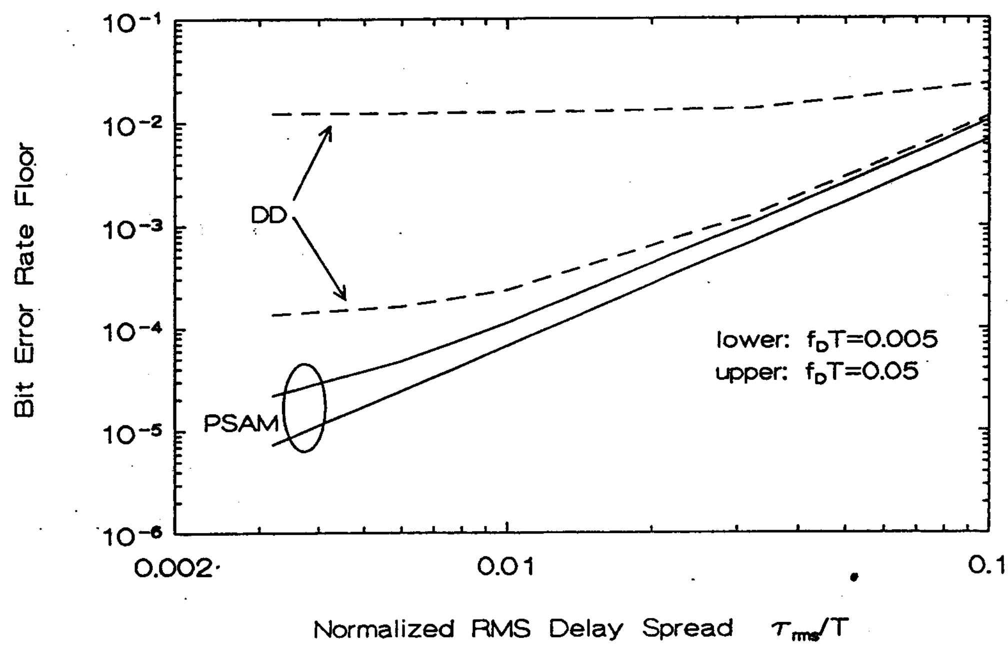
Define $\mathcal{C}_+ = \{\underline{c} \mid c(0) = c(-1)\}$ sequences with $a(0) = 1$

$\mathcal{C}_- = \{\underline{c} \mid c(0) = -c(-1)\}$ sequences with $a(0) = -1$

$P_{e+}(\underline{c}) = \frac{1}{2}\left(1 - \rho(\underline{c})\right) \qquad P_{e-}(\underline{c}) = \frac{1}{2}\left(1 + \rho(\underline{c})\right)$

$$P_e = 2^{-N}\,\frac{1}{2}\left(\sum_{\underline{c}\in\mathcal{C}_+} P_{e+}(\underline{c}) + \sum_{\underline{c}\in\mathcal{C}_-} P_{e-}(\underline{c})\right)$$ N is sequence length.

We now have two contributors to an error floor — pulse to pulse decorrelation from Doppler spread and ISI from delay spread.



$\beta = 0.25$

Figure 3: Error Floors for PSAM and Differential Detection - Effect of Delay Spread

Note • In the ISI-limited region, BER varies as $\left(\frac{\tau_{rms}}{T}\right)^2$

• For very low delay spread, we still have the Doppler floor.

~~4-32~~
~~4-39~~
L9

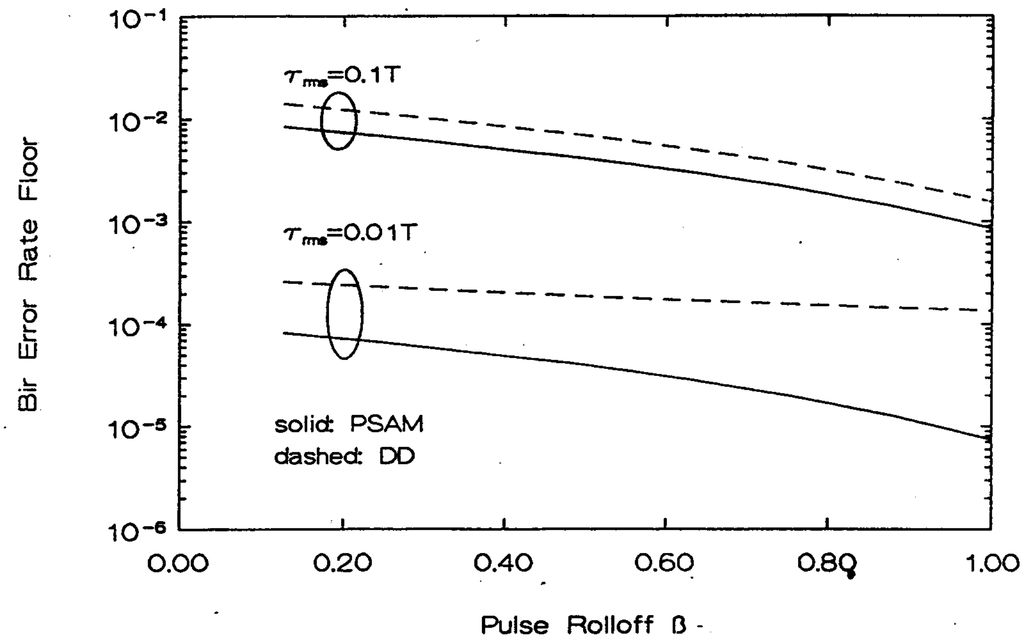


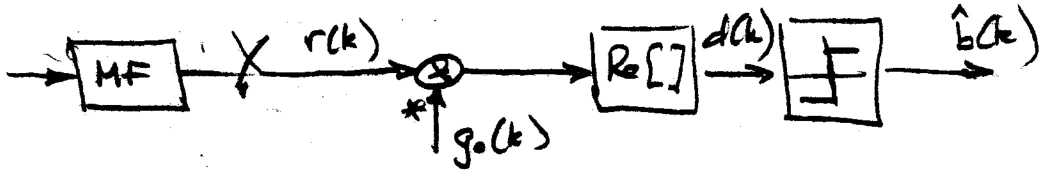
Figure 4: Error Floors for PSAM and Differential Detection - Effect of Pulse Rolloff

Pulses with tight rolloff (small β) have a longer duration and are more vulnerable to ISI.

- Conventional modulation and detection gives unimpressive performance in mobile channels. Next section will look at performance improvement methods.

better clarity the mean delay stuff

- <u>Example</u> - Slow, frequency selective fading with low delay spread. Coherent symbol by symbol detection of BPSK



- model: $r(k) = g_0(k)\,b(k) + g_1(k)\,b(k-1) + n(k)$

  WSS so $g_0(k)$ $g_1(k)$ uncorrelated

- BER analysis: send $b(k) = +1$ so $d(k)$ should be >0.

$$\sigma_r^2 = \sigma_{g_0}^2 + \sigma_{g_1}^2 + \sigma_n^2 \qquad \sigma_v^2 = \sigma_{g_0}^2$$

$$\sigma_{rv}^2 = b(k)\,\sigma_{g_0}^2$$

$$\rho = \frac{b(k)\,\sigma_{g_0}^2}{\sqrt{\sigma_{g_0}^2(\sigma_{g_0}^2 + \sigma_{g_1}^2 + \sigma_n^2)}} = \frac{b(k)\,\eta}{\sqrt{\eta(1+\Gamma_b^{-1})}}$$

where $\eta = \dfrac{\sigma_{g_0}^2}{\sigma_{g_0}^2 + \sigma_{g_1}^2}$ $\qquad \Gamma_b = \dfrac{\sigma_{g_0}^2 + \sigma_{g_1}^2}{\sigma_n^2}$

$$P_e = \tfrac{1}{2}(1-\rho)$$

- Note that the ISI term behaves exactly as Gaussian noise wrt error rate, because $g_1(k)$ uncorrelated with $g_0(k)$. This is unlike static channels, in which we must account for the binary or non Gaussian nature of ISI. The slow fading analysis has to do that, too.

- This detector suffers from ISI. A smarter detector would (1) cancel it, like a DFE and calculate $r(k) - g_1(k)\,\hat{b}(k-1)$; or (2), exploit the ISI, as a second glimpse of $b(k-1)$, in an MLSE or Viterbi equaliser.

- Example : – Slow, frequency selective fading with low delay spread. BPSK, Viterbi equaliser with perfect channel state info ($g_0(k)$, $g_1(k)$).

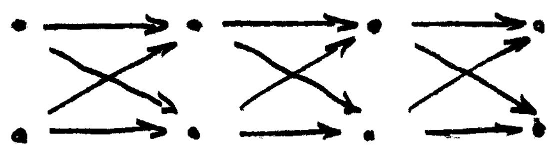
  – Model $$y(k) = g_0(k)\, b(k) + g_1(k)\, b(k-1)$$

  $$r(k) = y(k) + n(k)$$

  – Trellis : $y(k)$ is determined by $b(k)$, $b(k-1)$. Use $b(k-1)$ as the state

+1

–1



k-1 k k+1

state pair $(s(k), s(k-1))$ determines $(b(k), b(k-1))$, hence $y(k)$, in conjunction with CSI

– Metric: $p(\underline{r} \mid \underline{g}\, \hat{\underline{b}})$ is to be max'd wrt $\hat{\underline{b}}$

Since noise samples are white Gaussian,

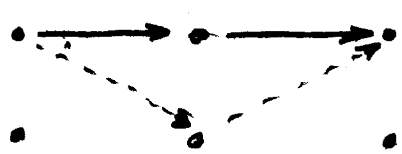
$$p(\underline{r} \mid \underline{g}, \hat{\underline{b}}) \sim \exp\left(- \sum_k \frac{|r(k) - \hat{y}(k)|^2}{\sigma_n^2}\right)$$

Log likelihood gives additive metric per branch

$$M(s(k), s(k-1)) = |r(k) - g_0(k)\hat{b}(k) - g_1(k)\hat{b}(k-1)|^2$$

All set up for the Viterbi algorithm!

– Error rate analysis : union bound is sum of pairwise error event probabilies over all allowable events. Here's one:



——— true path

- - - - erroneous alternative

k-1 k k+1

- Express the metric difference as a quadratic form ~~4-3-12~~ L12

The metric for branch k is

$$M(k) = |r(k)|^2 + |g_0(k)|^2 + |g_1(k)|^2 - 2\,b(k)\,\mathrm{Re}[r(k)\,g_0^*(k)]$$
$$- 2\,b(k-1)\,\mathrm{Re}[r(k)\,g_1^*(k)] + 2\,b(k)\,b(k-1)\,\mathrm{Re}[g_0(k)\,g_1^*(k-1)]$$

and the metric difference between true and erroneous at branch k is

$$\Delta M(k) = \tilde{M}(k) - M(k)$$
$$= 2\,(b(k) - \tilde{b}(k))\,\mathrm{Re}[r(k)\,g_0^*(k)]$$
$$+ 2\,(b(k-1) - \tilde{b}(k-1))\,\mathrm{Re}[r(k)\,g_1^*(k)]$$
$$- 2\,(b(k)\,b(k-1) - \tilde{b}(k)\,\tilde{b}(k-1))\,\mathrm{Re}[g_0(k)\,g_1^*(k)]$$

simplifies because $b(k-1) = \tilde{b}(k-1)$.

At brank $k+1$:

$$\Delta M(k+1) = 2\,(b(k+1) - \tilde{b}(k+1))\,\mathrm{Re}[r(k+1)\,g_0^*(k+1)]$$
$$+ 2\,(b(k) - \tilde{b}(k))\,\mathrm{Re}[r(k+1)\,g_1^*(k+1)]$$
$$- 2\,(b(k+1)\,b(k) - \tilde{b}(k+1)\,\tilde{b}(k))\,\mathrm{Re}[g_0(k+1)\,g_1^*(k+1)]$$

simplifies because $b(k+1) = \tilde{b}(k+1)$

Total metric difference $\Delta M = \Delta M(k) + \Delta M(k+1)$.
Using $b(k-1) = \tilde{b}(k-1)$, $b(k+1) = \tilde{b}(k+1)$, we have

$$\Delta M = (b(k) - \tilde{b}(k))\,\mathrm{Re}[r(k)\,g_0^*(k)] - b(k-1)(b(k) - \tilde{b}(k))\,\mathrm{Re}[g_0(k)\,g_1^*(k)]$$
$$+ (b(k) - \tilde{b}(k))\,\mathrm{Re}[r(k+1)\,g_1^*(k+1)] - b(k+1)(b(k) - \tilde{b}(k))\,\mathrm{Re}[g_0(k+1)\,g_1^*(k+1)]$$

$$= 2\,\mathrm{Re}[r(k)\,g_0^*(k)] - 2\,\mathrm{Re}[g_0(k)\,g_1^*(k)]$$
$$+ 2\,\mathrm{Re}[r(k+1)\,g_1^*(k+1)] - 2\,\mathrm{Re}[g_0(k+1)\,g_1^*(k+1)]$$

This is a quadratic form:

$$\underline{x} = \left[r(k), g_0(k), g_1(k), r(k+1), g_0(k+1), g_1(k+1) \right]^T$$

$$F = 2 \left[\begin{array}{ccc|ccc} 0 & 1 & 1 & & & \\ 1 & 0 & 0 & & 0 & \\ 1 & 0 & 0 & & & \\ \hline & & & 0 & 1 & 1 \\ & 0 & & 1 & 0 & 0 \\ & & & 1 & 0 & 0 \end{array} \right]$$

$$R = \left[\begin{array}{ccc|ccc} \sigma_0^2+\sigma_1^2+\sigma_n^2 & \sigma_0^2 & \sigma_1^2 & \mu(\sigma_0^2+\sigma_1^2) & \mu\sigma_0^2 & \mu\sigma_1^2 \\ \sigma_0^2 & \sigma_0^2 & 0 & \mu\sigma_0^2 & \mu\sigma_0^2 & 0 \\ \sigma_1^2 & 0 & \sigma_1^2 & \mu\sigma_1^2 & 0 & \mu\sigma_1^2 \\ \hline \mu(\sigma_0^2+\sigma_1^2) & \mu\sigma_0^2 & \mu\sigma_1^2 & \sigma_0^2+\sigma_1^2+\sigma_n^2 & \sigma_0^2 & \sigma_1^2 \\ \mu\sigma_0^2 & \mu\sigma_0^2 & 0 & \sigma_0^2 & \sigma_0^2 & 0 \\ \mu\sigma_1^2 & 0 & \mu\sigma_1^2 & \sigma_1^2 & 0 & \sigma_1^2 \end{array} \right]$$

where $\mu = J_0(2\pi f_D T)$

note R depends on true sequence $\underline{b}$

- We know how to express the pairwise prob in terms of residues and eigenvalues.
- We have to do this for every pairwise event!
  - only do the shortest (dominant) ones, length 2, 3 for example.
  - do it more systematically.
  - numerical evaluation of residues,
- each bit has two chances to affect metric, once through g_0, again through g_1, as can be seen from metric expression.
- BER is <u>better</u> than if no ISI, all power in single tap, because risk of fade is spread out over two taps (diversity)

- The improvement due to delay spread, uncorrelated scattering, is well known. This presupposes an appropriate receiver, such as
  - DFE
  - Viterbi equaliser

- In [R. D'Avella, L. Moreno & M. Sant'Agostino "An Adaptive MLSE Receiver for TDMA Digital Mobile Radio" IEEE J Sel Areas Commun" Jan '89] we see that performance first improves due to implicit diversity as τ/T increases, then degrades, since 32 state VE cannot represent CIR well enough. Need $M>5$, more states, more complexity; exponential growth.

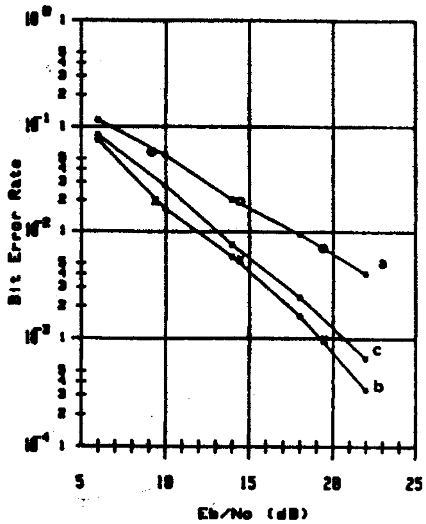


Fig. 6. Bit error rate in a two-ray Rayleigh channel. Simulation results: (a) echo delay $\tau = 0$ μs, (b) $\tau = 10$ μs, (c) $\tau = 16$ μs. Experimental results [3]: (o) $\tau = 0$ μs, (x) $\tau = 10$ μs.

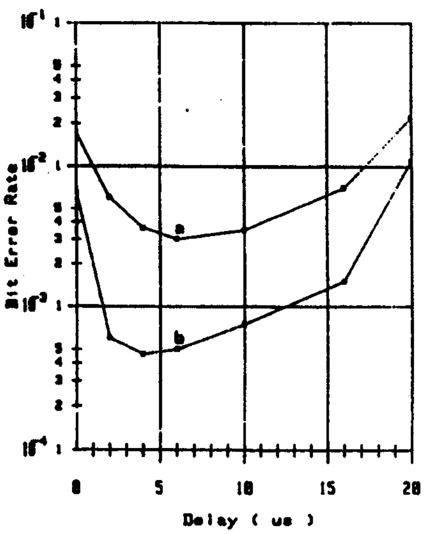


Fig. 7. Bit error rate in a two-ray Rayleigh channel as a function of echo delay: (a) $E_b/N_0 = 10$ dB, (b) $E_b/N_0 = 20$ dB.