2. REVIEW OF SIGNAL THEORY

2.1 Representation of Bandpass Signals

- In radio communications, we routinely deal with bandpass signals. Represent them by the complex envelope:

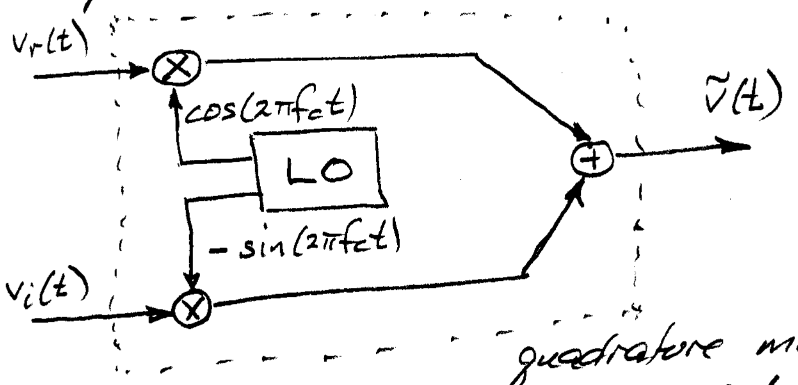
$$\tilde{v}(t) = \mathrm{Re}\left[v(t)\, e^{j2\pi f_c t}\right] \qquad f_c \text{ carrier freq.}$$

$$= v_r(t)\cos(2\pi f_c t) - v_i(t)\sin(2\pi f_c t)$$

$$= |v(t)|\cos(2\pi f_c t + \arg(v(t)))$$

Think of it as a time-varying phasor.

- To create a bp signal with specified complex envelope



quadrature modulator
or IQ modulator

- To recover the complex envelope of a bandpass signal:
  - For easy analysis, use this identity.

$$\mathrm{Re}[\alpha]\,\mathrm{Re}[\beta] = \tfrac{1}{2}\,\mathrm{Re}\left[\alpha\beta^* + \alpha\beta\right]$$

  - So $\tilde{v}(t)\cos(2\pi f_c t)$

$$= \mathrm{Re}\left[v(t)e^{j2\pi f_c t}\right]\mathrm{Re}\left[e^{j2\pi f_c t}\right]$$

$$= \tfrac{1}{2}\,\mathrm{Re}\left[v(t) + v(t)e^{j4\pi f_c t}\right]$$

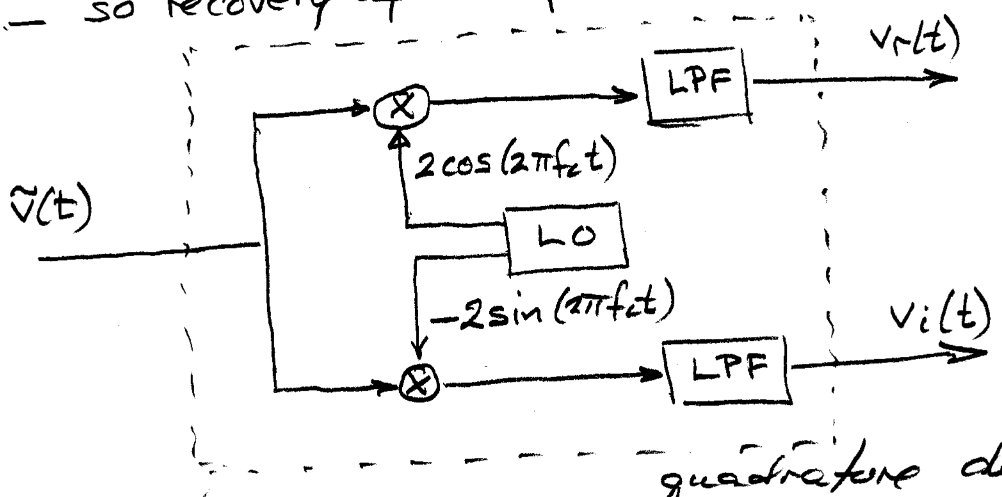
$$\xrightarrow[\text{lopass}]{} = \tfrac{1}{2}\,v_r(t)$$

  - and $\tilde{v}(t)\sin(2\pi f_c t) = \mathrm{Re}\left[v(t)e^{j2\pi f_c t}\right]\mathrm{Re}\left[-j\,e^{j2\pi f_c t}\right]$

$$= \tfrac{1}{2}\,\mathrm{Re}\left[j\,v(t) - j\,v(t)e^{j4\pi f_c t}\right]$$

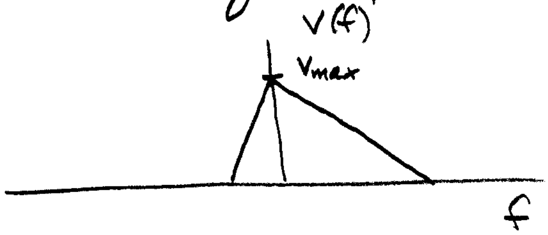
$$\underset{\text{lopass}}{=} \; -\tfrac{1}{2}\,v_i(t)$$

  - so recovery of complex envelope by



quadrature demodulator

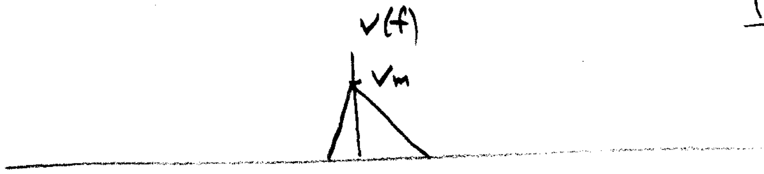
- Seen in frequency domain . . .

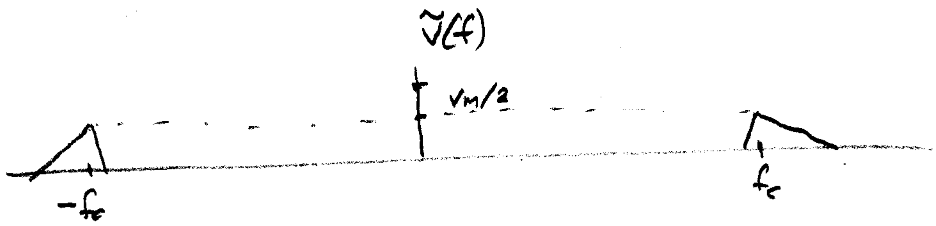


$V(f)$ may be complex

- $V(f)$ is not <u>necessarily</u> conjugate symmetric
  $(V(-f) = V^*(f)$ may not hold$)$ $\boxed{\text{Why not ?}}$

- Bandpass counterpart <u>is</u> conjugate symmetric,
  $\tilde{V}(-f) = \tilde{V}^*(f)$ $\boxed{\text{Why?}}$





$\frac{1}{2} V^*(-(f+f_c))$ $\qquad$ $\frac{1}{2} V(f-f_c)$

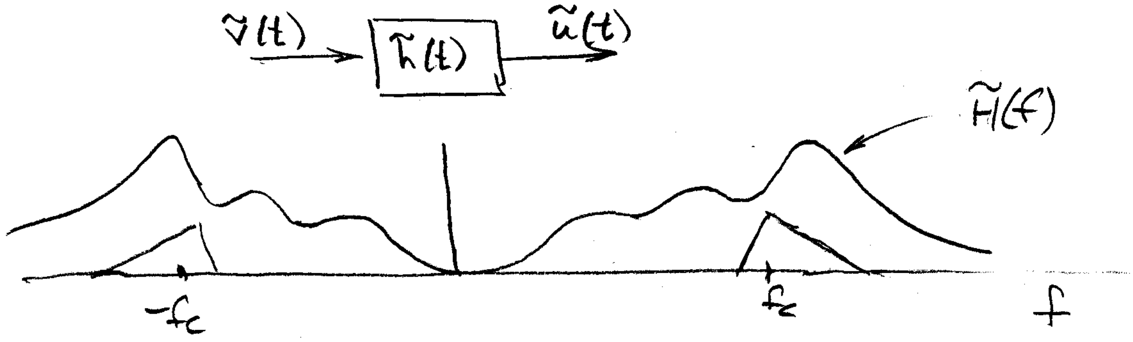
$$\tilde{V}(f) = \frac{1}{2} V(f-f_c) + \frac{1}{2} V^*(-(f+f_c))$$

- Recovery:

$$V(f) = \left(2\,\tilde{V}(f+f_c)\right)_{\text{lopass}}$$

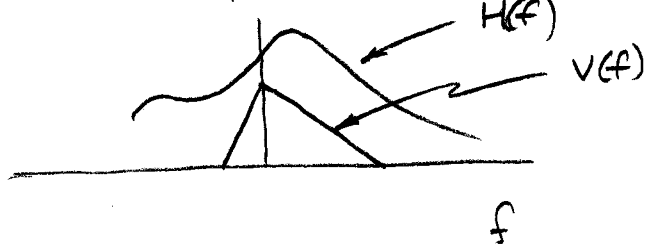
or $$v(t) = \left(2\,\tilde{v}(t)\,e^{-j2\pi f_c t}\right)_{\text{lo pass}}$$

- Bandpass filtering...
  - The real bp signal $\tilde{v}(t)$ passes through a real filter $\tilde{h}(t)$



so $\tilde{U}(f) = \tilde{H}(f)\,\tilde{V}(f)$

  - Effect on complex envelope



$$U(f) = H(f)\,V(f)$$

where $H(f) = \left(\tilde{H}(f+f_c)\right)_{\text{lo pass}}$

  - and, in time,

$$u(t) = \int v(t-\tau)\,h(\tau)\,d\tau$$

- For details on complex envelope, see Appendix B1.
  Also, Section 7.1 in ENSC 428