9.3 Effect of Fading on Detection

- As the channel gain fluctuates, so does the SNR experienced by the receiver and, consequently, so does the BER.

- With proper normalization, instantaneous and average SNR are related by

$$\gamma_b = \Gamma_b r^2 = \Gamma_b z$$

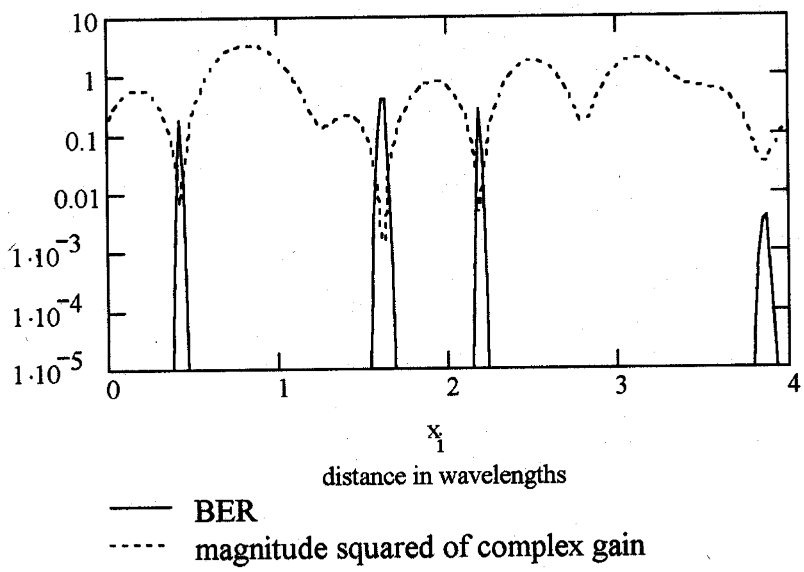
In order for $\bar{\gamma}_b = \Gamma_b$, we need $\bar{z} = 1 \Rightarrow \sigma_g^2 = \frac{1}{2}$

$$p_z(z) = e^{-z}$$

- Example: differential detection (why?), flat fading

$$P_b(z) = \frac{1}{2} e^{-\gamma_b} = \frac{1}{2} e^{-\Gamma_b z}$$

A simulation:



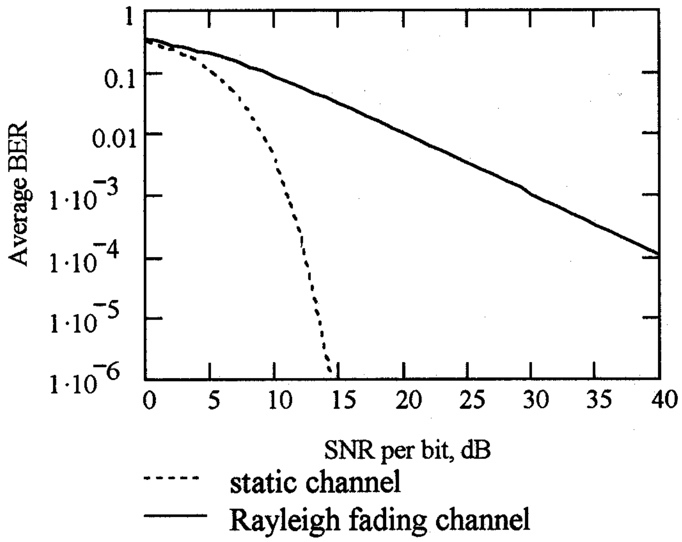
BER as a Function of Position (Time)

- Errors occur only in deep fades
- Errors occur in bursts
- Very different from AWGN channels!

- The bursts have a devastating effect on the average BER

$$\overline{P_b} = E_z\left[P_b(z)\right] = \int_0^\infty \frac{1}{2} e^{-\Gamma_b z} e^{-z}\, dz = \frac{1}{2(\Gamma_b+1)}$$

This is only inverse dependence, not exponential.



- In AWGN, a 3dB increase squares the BER, e.g., $10^{-3} \longrightarrow 10^{-6}$
- In fading, a 3 dB increase only halves the BER, e.g., $10^{-3} \longrightarrow \frac{1}{2} \cdot 10^{-3}$

- Why such a dramatic difference?
  - $P_b(\gamma_b)$ is strongly non linear, so BER at low SNR is much greater than at high SNR
  - When averaging numbers (e.g. BERs) with large dynamic range, such as $10^{-6}, 10^{-7}, 10^{-5}, 10^{-6}, 10^{-2}, 10^{-5}, 10^{-6}$ the sum is determined primarily by the very large numbers and their probability of occurrence.

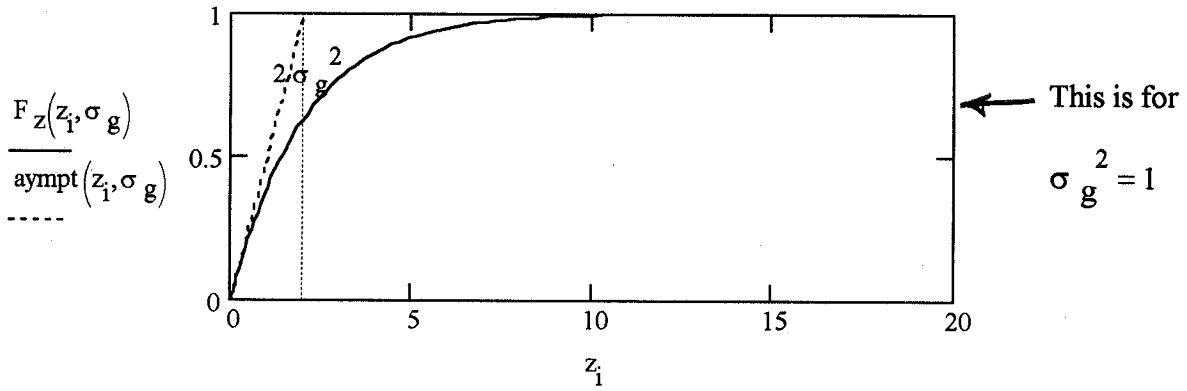
– So deep fades dominate the behaviour and the integrand in

$$\int_0^\infty P_b(z)\, p_z(z)\, dz$$

is significant only for values of z much less than the mean.

– How likely are those low values? Check the cdf of z:

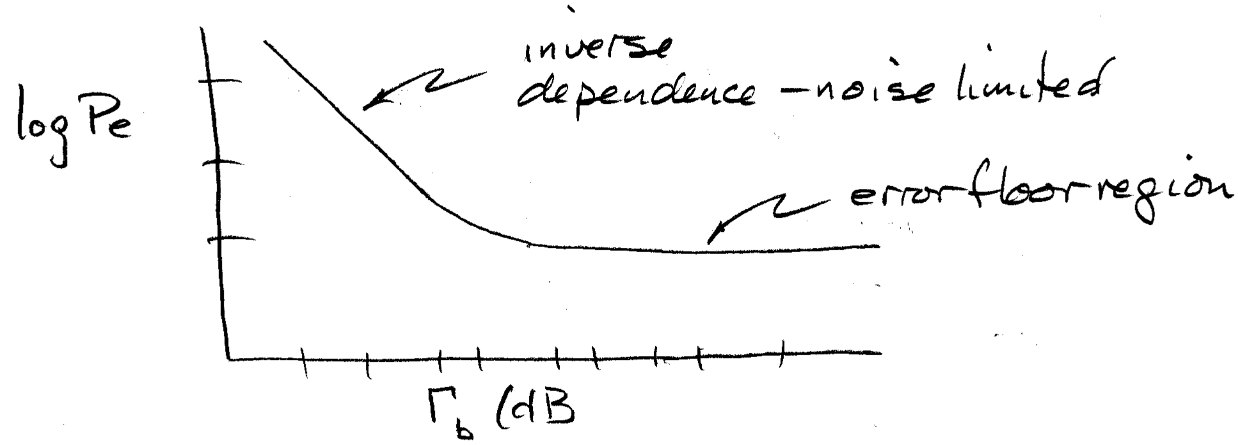
$$F_z(z,\sigma_g) := 1 - \exp\left(-\frac{z}{2\cdot\sigma_g^{\,2}}\right) \qquad \text{aympt}(z,\sigma_g) := \frac{z}{2\cdot\sigma_g^{\,2}}$$



From the asymptote, the probability of fading below some "trouble level" z_t is $\approx \dfrac{z_t}{2\sigma_g^{\,2}}$

which decreases only inversely with $\sigma_g^{\,2}$.

- In addition to their effect on signal strength, deep fades cause abrupt phase reversals. Particularly severe ones can cause errors even if there is <u>no</u> noise! (Consider differential detection). This results in an "error floor".



- Significant delay spread forces use of equalizers, and the time variation of channel means they have to track the impulse response. If poorly, another floor.