3.6 Back to Fading – Second Order Statistics

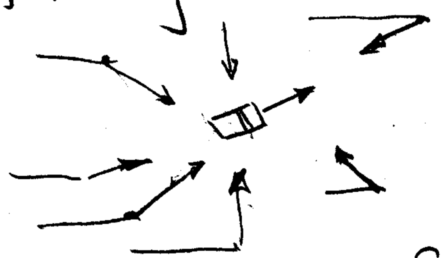
- We have seen that the channel gain can be approximated as a complex Gaussian random variable.

  It depends on time as mobile moves through field
  & frequency, if delay spread

  So it's a random process in t and/or f.

  That means we should have autocorrelation/density transform pairs for both fading and delay spread.

- First, fading...



scatterers reflect signals

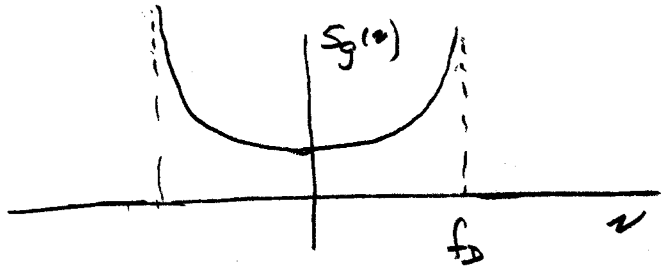
max Doppler shift $f_D = v/\lambda$

- So, if unmodulated carrier transmitted, received complex envelope is random process $g(t)$, and transform $G(\nu)$ is limited to $-f_D \le \nu \le f_D$.
  Shift up by f_D straight ahead, down by f_D behind, zero shift transverse to motion.

- Special case: scattered power is uniform in azimuth, antenna is isotropic.

Then power spectrum

$$S_g(\nu) = \frac{\sigma_g^2}{\pi f_D} \frac{1}{\sqrt{1-\left(\frac{\nu}{f_D}\right)^2}} \qquad (\text{area } \sigma_g^2)$$

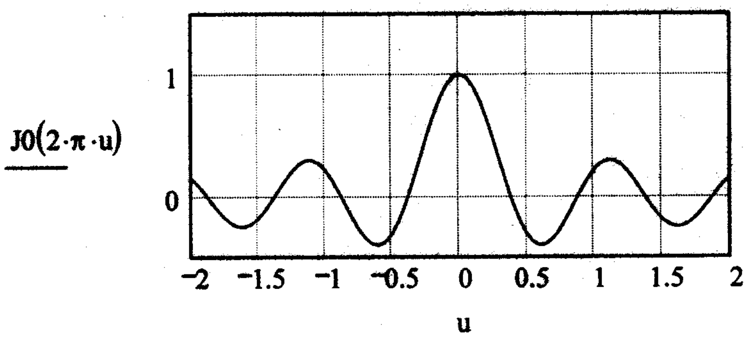


U-shaped

Its transform is the autocorrelation

$$R_g(t) = \tfrac{1}{2}\,\overline{g(\eta)\,g^*(\eta - t)} = \mathcal{F}^{-1}\left[S_g(\nu)\right]$$

$$= \sigma_g^2\, J_0(2\pi f_D t)$$



$u = f_D t$

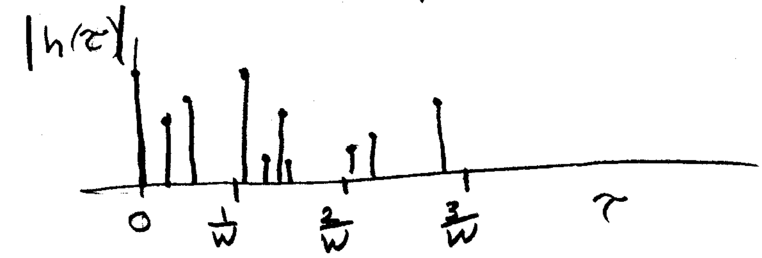
or

$u = x/\lambda$

Complete decorrelation at $x \approx 0.4\,\lambda$

$t \approx 0.4/f_D$

- Second, delay spread . . .



  - Group into bins of duration equal to resolution time of the input signal (about $1/W$). Each bin has a variance (power) for the net gain of scatterers in the bin — so that power depends on which bin; ie. which τ.
  - Consider it to be continuous, so it is a density — the "power delay profile" $P_g(\tau)$.

exponential is a common approximation (good for urban)

The density has area $\sigma_g^2 = \int_0^\infty P_g(\tau)\, d\tau$

  - Its transform $C_g(\Delta f) = \mathcal{F}[P_g(\tau)]$ is the spaced frequency correlation function

$$C_g(\Delta f) = \tfrac{1}{2}\, \overline{G(f)\, G^*(f - \Delta f)}$$

- Pairs

observation time $R_g(t) \longleftrightarrow S_g(\nu)$ Doppler frequency

observation freq $C_g(\Delta f) \longleftrightarrow P_g(\tau)$ delay (echo) domain