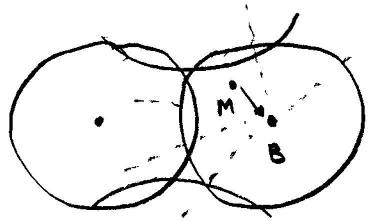
3.3 Diversity Reception – Single User



Ignore MAI or pretend it's white noise.

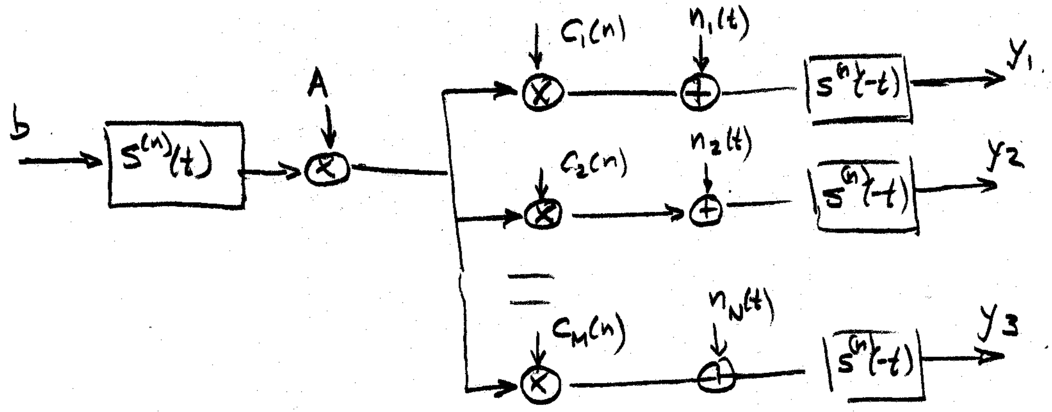
- If receiver can sense the signal in M different ways, independently fading, then the prob that all are deeply faded is $\sim p^M$ where p is prob deep fade on any one.

- Sources of diversity:
  - multipath (Section 3.2)
  - antenna array with wide spacing
  - polarization angles
  - time separation of transmissions
  - frequency separation
  - macrodiversity (geographically separated antennas)
  - etc?

Diversity is a key design element in both single user and multiuser systems.

3.3.1 Antenna Diversity

- Antenna diversity is the easiest to analyze and it exposes many general principles, so that's what we'll focus on.



Again $\underline{y} = R\,\underline{c}\,A\,b + \underline{v}$

where $R = I$, $R_{\underline{v}} = N_0 I$ (follows from Sections 2.2, 2.3)

$$\underline{y} = \underline{c}\,A\,b + \underline{v}$$

- Assume perfect CSI for simplicity. Then combining gives a sufficient stat and it follows from ML.

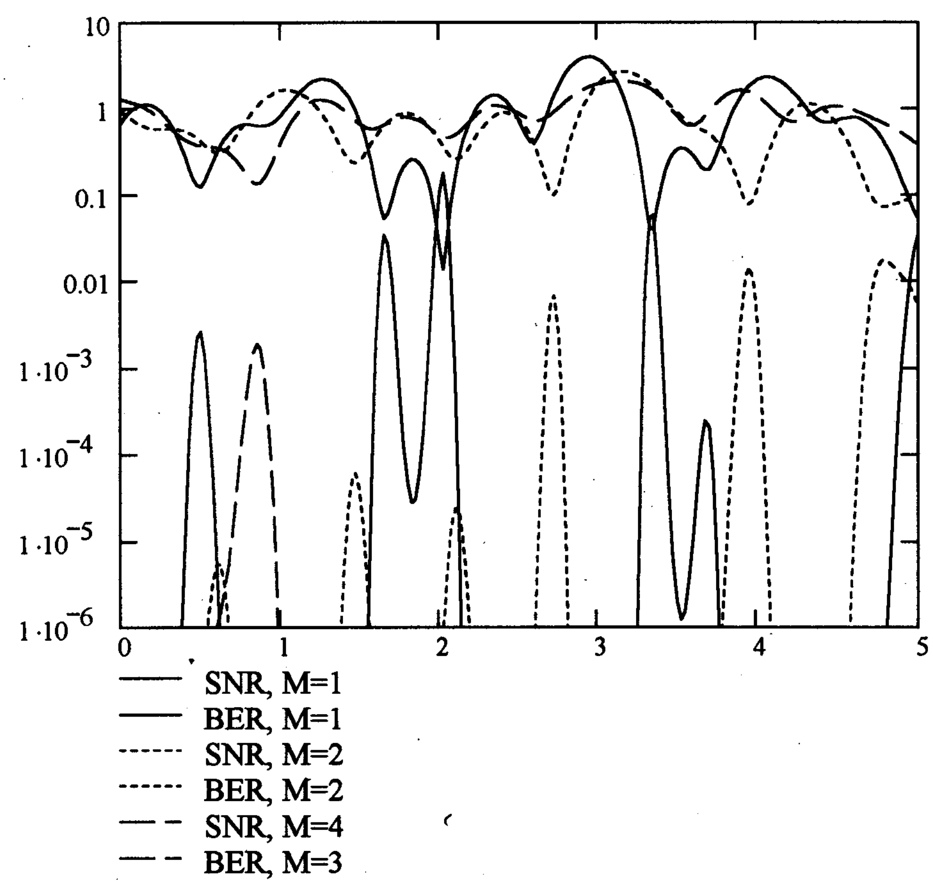
$$\zeta = \underline{c}^{+}\underline{y} = |\underline{c}|^2 A b + \eta \qquad \sigma_\eta^2 = N_0 |\underline{c}|^2$$

- It also maximizes the SNR (Appendix H), hence the term "maximal ratio combining" (MRC)

- Note that, even without diversity considerations, multiple antennas have independent noises, hence opportunity for averaging (i.e. picking up more signal energy). If all gains are equal, then Γ is improved by a factor M, or $10 \log M$ dB.

  Our normalization $\sum_\ell \sigma_\ell^2 = \frac{1}{2}$ tends to conceal this. (we use "received SNR")

- Diversity combination tends to smooth the SNR variation, so fewer deep nulls, and reduce the average BER



all for $\Gamma_b = 15$ dB

- What is the BER with diversity?
  - Follow the same path as for single channel. The instantaneous BER is

$$P_e(z) = Q\left(\frac{|\underline{c}|^2 A}{\sigma_\eta}\right)$$

$$= Q\left(\sqrt{2\Gamma_b \sum_m |c_m|^2}\right) \qquad A = \sqrt{2E_b} \qquad |c_m|^2 = z_m$$

$$= Q\left(\sqrt{2\Gamma_b z}\right) \qquad z = \sum_{m=1}^{M} z_m$$

The effective SNR is the sum of the branch SNRs.

  - The average BER is

$$\overline{P_e} = \int_0^\infty P_e(z)\, p_z(z)\, dz$$

but what is $p_z(z)$? Two easy cases.

  - The average branch SNRs are all the same, so z has a χ^2 pdf with $2M$ d.f. (normalized)

$$p_z(z) = \frac{M^M}{(M-1)!} z^{M-1} e^{-Mz} \quad ; \quad z \geq 0$$

This has mean $\overline{z} = 1$.

- The average branch SNRs are all different. Then (Appendix G):

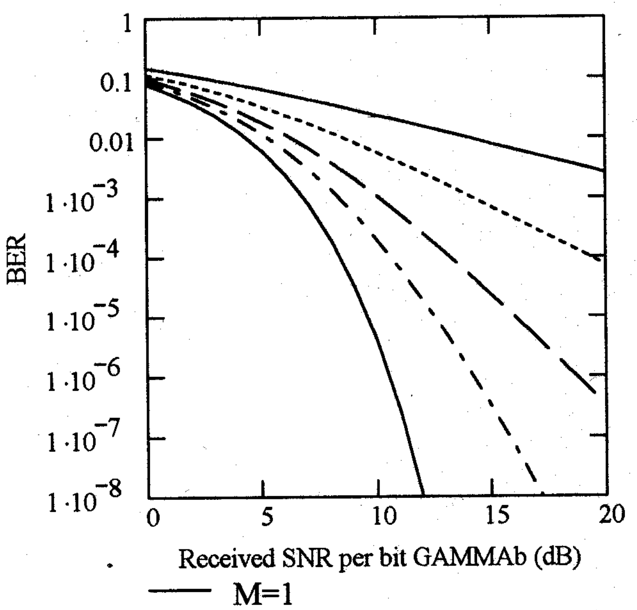
$$p_z(z) = \sum_{m=1}^{M} \frac{e^{-z/\sigma_m^2}}{\sigma_m^2 \prod_{i \neq m} (1 - \sigma_i^2/\sigma_m^2)}$$

- Mixed cases are messy, but see [Welb01].

Solutions for the integrals are in [Proa95, Chapt. 14].

- All branches the same. Repeated integration by parts,

$$\overline{P_e} = \left(\frac{1-\mu}{2}\right)^M \sum_{m=0}^{M-1} \binom{M-1+m}{m} \left(\frac{1+\mu}{2}\right)^m, \quad \mu = \sqrt{\frac{\Gamma_b}{M+\Gamma_b}}$$



Error Rate, BPSK With M-Fold Diversity

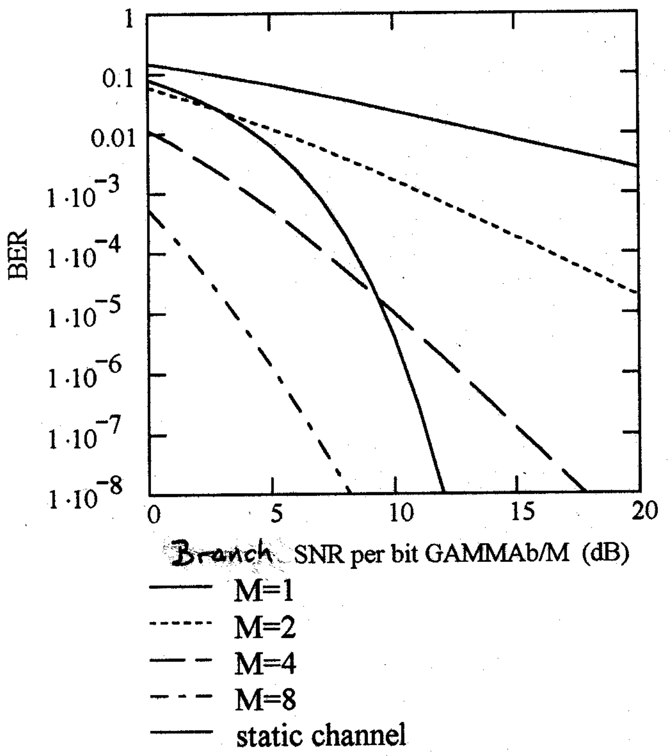
- asymptotic behaviour

$$P_e \sim \binom{2M-1}{M} (4\Gamma_b)^{-M}$$

reflects the fact that all M must be faded to cause an error burst (more interpretation Append G)

- Convergence to static channel
- With superimposed coding?
  - relatively high channel BER
  - less extreme savings
  - interleaving is essential, since bursty and coding induces threshold, but less depth needed with higher diversity (see p 3.3.3 plot)

- Note that we plotted against "received SNR". Our normalization $2\sum_m \sigma_{cm}^2 = \sum_m \bar{z}_m = \bar{z} = 1$ makes the transmitted and received SNRs equal. Convenient, but it conceals the fact that each receive antenna picks up more signal power "for free." Performance is better than the curves suggest; plot against per-branch SNR when $\bar{z} = M$, and there's a big difference.



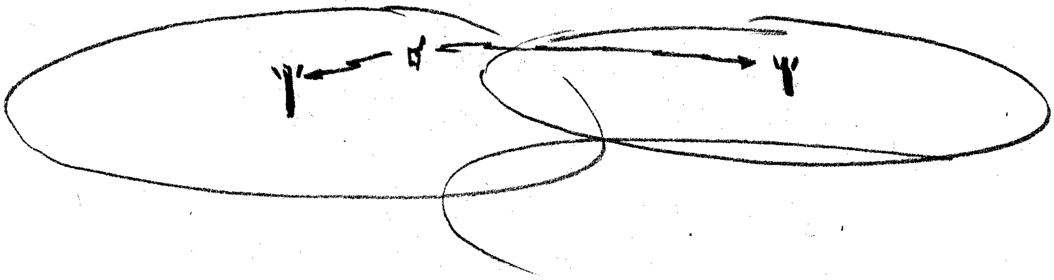
Error Rate, BPSK With M-Fold Diversity

No normalization

- Curves are shifted $10 \log M$ to the left.
- Better than static channel of same Tx power since M times as much energy is received.
- Applies to Rake reception, too.

3.3.2 Diversity with Unequal Branch Powers

- What if the average power is different at different antennas?
  - It doesn't happen in microdiversity (antennas at the same base site).



  - but it's a good precursor to examination of correlated antennas
  - and it shows that you don't need much power in secondary paths to get good diversity improvement

- From pp. 3.3.4, 3.3.5 the average BER is

$$\overline{P_e} = \int_0^\infty Q\left(\sqrt{2\Gamma_b z}\right) p_z(z)\, dz$$

where $p_z(z) = \sum_{m=1}^{M} \frac{e^{-z/\sigma_m^2}}{\sigma_m^2 \prod_{i \neq m} (1 - \sigma_i^2/\sigma_m^2)}$ (all branches different)

Use the integral on p 3.2.2 repeatedly to obtain

$$\overline{P_e} = \sum_{m=1}^{M} \frac{1}{2\prod_{i \neq m}(1-\sigma_i^2/\sigma_m^2)} \left(1 - \sqrt{\frac{\sigma_m^2 \Gamma_b}{1+\sigma_m^2 \Gamma_b}}\right)$$

(all branches different) [Proa95]

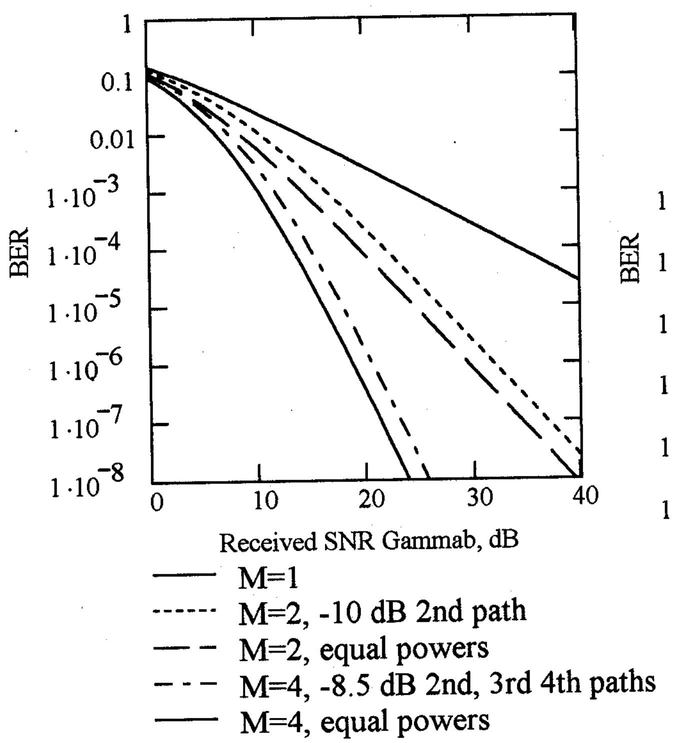
- Plotted below for

$\sigma_1^2 = 0.909$, $\sigma_2^2 = 0.091$ (−10dB)

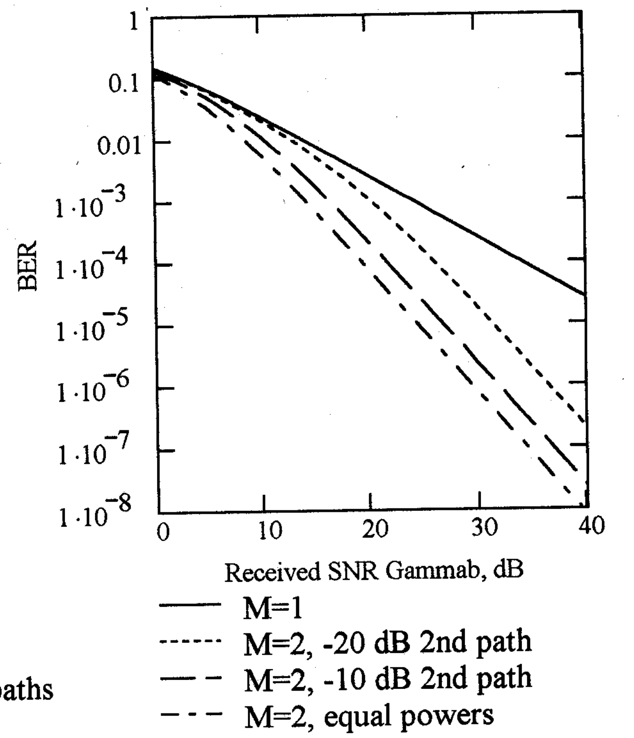
$\sigma_1^2 = 0.7$, $\sigma_2^2, \sigma_3^2, \sigma_4^2 \approx 0.1$ (−8.5dB)

$\sigma_1^2 = 0.909$, $\sigma_2^2 = 0.091$ (−10 dB)

$\sigma_1^2 = 0.99$, $\sigma_2^2 = 0.0099$ (−20dB)



Diversity With Unequal Branch Powers



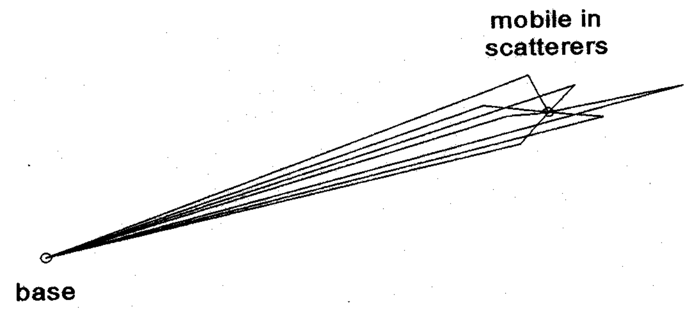
Diversity With Unequal Branch Powers

It doesn't take much to produce a big improvement!

3.3.3 Diversity With Correlated Antennas

- What if the channel gains to different antennas are correlated?
  - A genuine issue in microdiversity (though not in macrodiversity).
  - We'll see that even highly correlated paths produce satisfactory diversity.

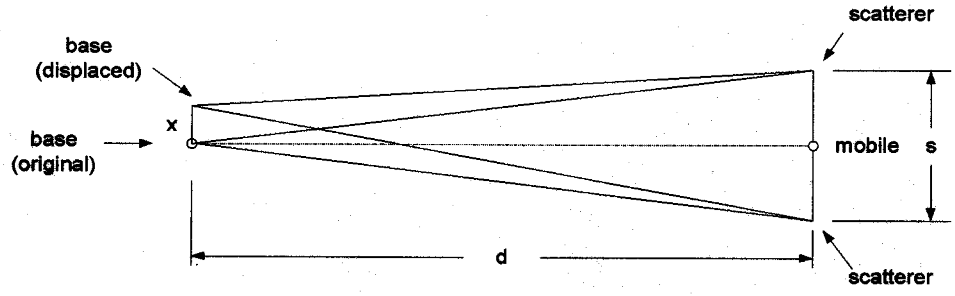
- Consider the difference between base and mobile scattering environments



[Cave00]

- A small change in mobile position (portion of a wavelength) causes different phase shifts in the rays, hence significant difference in resultant — some 10's of dB.
- A radial displacement of the base (toward or away from mobile) produces a more or less common phase shift in all rays, almost no change in amplitude of resultant.
- A transverse displacement of base antenna produces some differential phase change, so slow change in amplitude of resultant.

• Examine transverse displacements further:

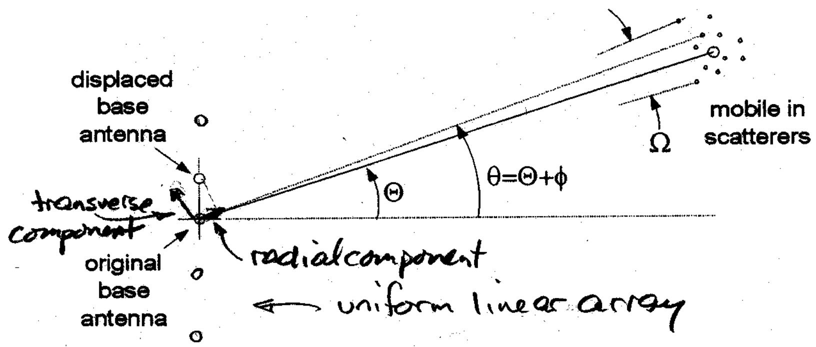


- No phase diff between rays in original position
- Phase diff at new position is

$$\Delta\phi \approx \frac{x}{\lambda}\,\frac{s}{d} = \frac{x}{\lambda}\,\Omega$$

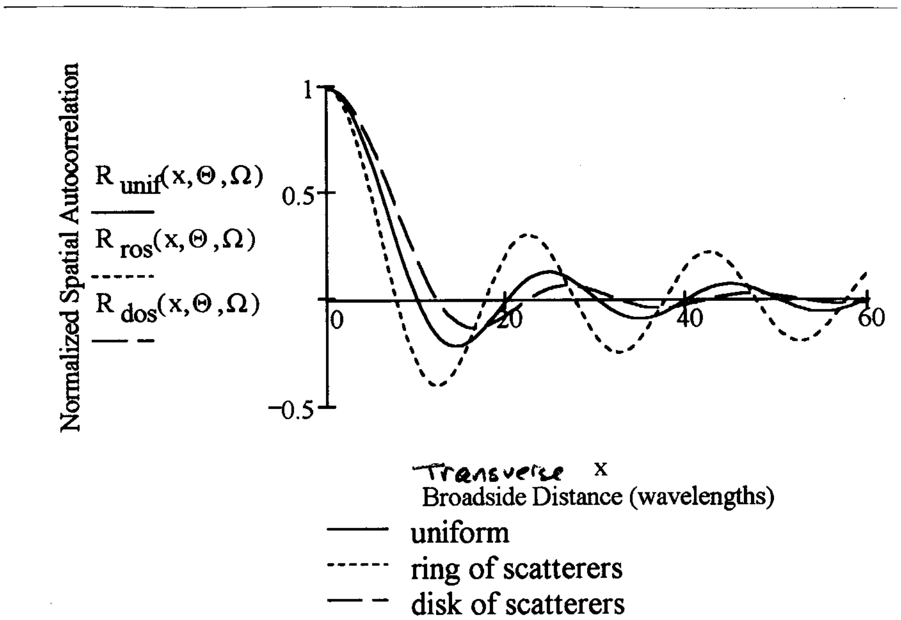
(Ω angle spread)
x transverse distance

- Greater phase difference, hence greater change in resultant with greater angle spread, greater transverse x
- This geometry is the basis of both diversity and spatial smoothing in eigenmethods.



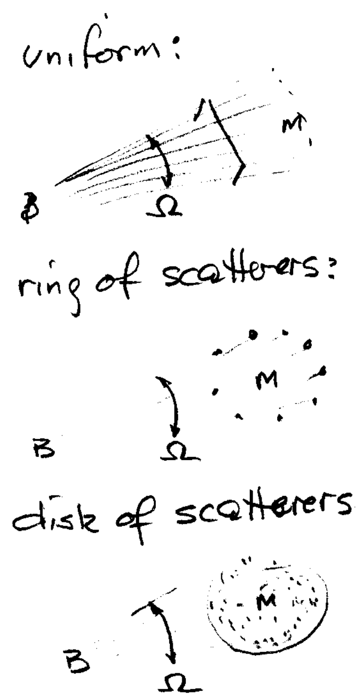
Only transverse displacements affect the magnitude of the resultant.

- If we allow a stochastic model of the resultant complex gain, there are expressions for correlation coefficient between antenna elements [Cave00]. For a max angle spread of $\Omega = 0.1$ rad (5.73 degrees) we see this:



[Cave00]

Autocorrelation at Base (Broadside)



A useful quadratic approx. for small displacements:

$$R_c(x) \approx 1 - \frac{1}{2}\left(2\pi \frac{x}{\lambda} \Omega_{rms}\right)^2$$

Ω_{rms} is rms angle spread

for all models of scatterer distribution.

- The average error rate with correlation sounds hard, but it's not. In any bit, we have

$$\underline{r}(t) = \begin{bmatrix} s(t) & & & 0 \\ & s(t) & & \\ & & \ddots & \\ 0 & & & s(t) \end{bmatrix} \begin{bmatrix} c_1 & & 0 \\ & \ddots & \\ 0 & & c_M \end{bmatrix} A\, b + \underline{n}(t)$$

Correlator output

$$\underline{y} = \underline{c}\, A b + \underline{\nu} \qquad \text{where} \quad \tfrac{1}{2} E[\underline{c}\,\underline{c}^\dagger] = R_c$$

An invertible transformation will not change the information content, so use eigenvectors of R_c.

$$R_c = \Phi \Lambda \Phi^\dagger, \quad \Phi = [\underline{\phi}_1 | \cdots | \underline{\phi}_M] \text{ is unitary, since } R_c \text{ Hermitian}$$

Form

$$\underline{y}' = \Phi^\dagger \underline{y} = \Phi^\dagger \underline{c}\, A b + \Phi^\dagger \underline{\nu}$$
$$= \underline{c}' A b + \underline{\nu}'$$

We now have uncorrelated branches, since

$$R_{c'} = \tfrac{1}{2} E[\underline{c}'\underline{c}'^\dagger] = \tfrac{1}{2} E[\Phi^\dagger \underline{c}\,\underline{c}^\dagger \Phi] = \Phi^\dagger R_c \Phi = \Lambda$$

$$R_{\nu'} = \Phi^\dagger R_\nu \Phi = N_0 I$$

but the branches have unequal power $\sigma_{c'_m}^2 = \lambda_m$.

This is what we analyzed earlier, so

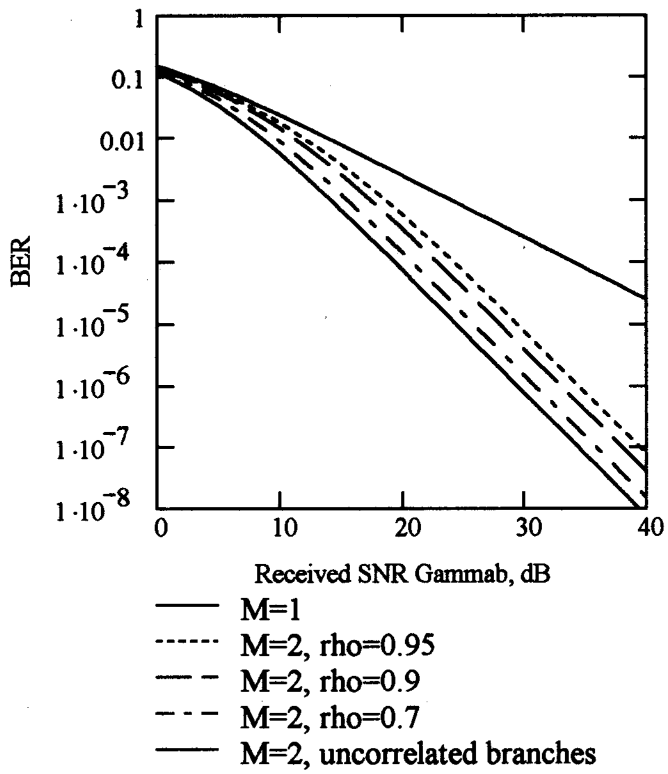
$$\overline{P_e} = \sum_{m=1}^{M} \frac{1}{2 \prod_{i \neq m} (1 - \lambda_i/\lambda_m)} \left(1 - \sqrt{\frac{\lambda_m \Gamma_b}{1 + \lambda_m \Gamma_b}}\right) \qquad \text{correlated branches}$$

Note normalization requires $\sum_m \lambda_m = \mathrm{tr}[R_c] = 1.$

- For numerical example, choose $M=2$ and

$$R_c = \frac{1}{2}\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \qquad \lambda = \tfrac{1}{2}+\rho,\ \tfrac{1}{2}-\rho$$

for normalization



Dual Diversity, Correlated Branches of Equal Power

It doesn't take much decorrelation to improve performance!

(Caveat: these results, especially for low BERs, depend on perfect CSI)

- It would be interesting to combine this result with the correlation plots on p 3.3.11 to obtain diversity gain of various antenna configurations.

Section 3.3 Summary

- Maximal ratio combining (MRC) can be deduced from sufficient statistics, from max likelihood and from max SNR. It is optimum for
  - single user
  - in white noise
  - with perfect CSI
- The BER is asymptotic to const. Γ_b^{-M}.
- If branches have unequal power (e.g. Rake), it doesn't take much power in the secondary paths to produce a big diversity gain.
- Branches can be highly correlated and still produce big diversity gain.