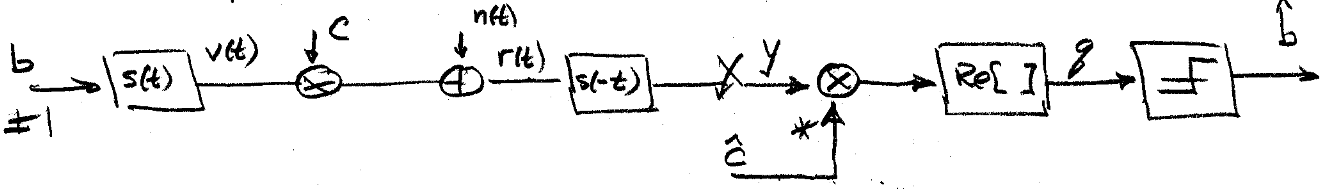
3.6 Imperfect Channel Estimates in Fading and Diversity

- We return to the issue of imperfect CSI. This section demonstrates that:
  - imperfect channel estimates set a BER floor as SNR $\Gamma \to \infty$
  - diversity lowers the floor
  - accuracy of channel estimates can be relaxed as number of diversity branches grows.

3.6.1 Imperfect CSI and the Error Floor

- Consider the BPSK, flat fading system used as an example in Section 3.5



We have $y = cAb + \nu$, with $A = \sqrt{2E_b}$, $\sigma_\nu^2 = N_0$

Channel gain and its estimate have correlation coeff ρ

$$\rho = \frac{\sigma_{c\hat{c}}^2}{\sigma_c \, \sigma_{\hat{c}}}$$

– The correlation coefficient that determines performance is between y and $\hat{c}$:

$$\alpha = \frac{\sigma_{y\hat{c}}^2}{\sigma_y \, \sigma_{\hat{c}}}$$

To obtain it:

$$\sigma_y^2 = \sigma_c^2 A^2 |b|^2 + N_0 = 2\sigma_c^2 E_b + N_0$$

$$\sigma_{y\hat{c}}^2 = \tfrac{1}{2} E[y \hat{c}^*] = \sigma_{c\hat{c}}^2 A b = \sqrt{2E_b}\, \sigma_c \sigma_{\hat{c}} \rho \, b$$

So

$$\alpha = \frac{\rho b}{\sqrt{1 + N_0/2\sigma_c^2 E_b}} \underset{\substack{\sigma_c^2 = \frac{1}{2} \\ b = +1}}{=} \frac{\rho}{\sqrt{1 + \Gamma_b^{-1}}}$$

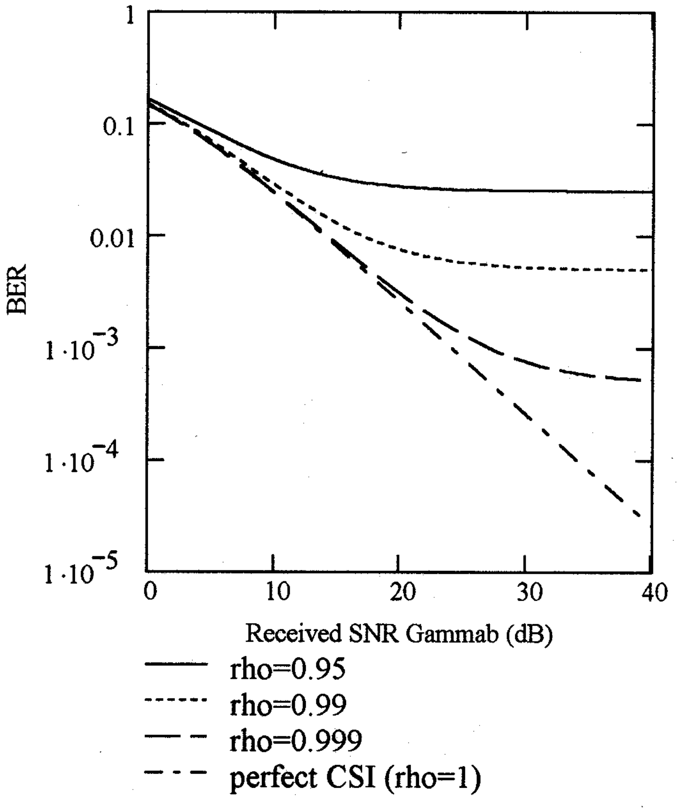
- Then BER is given by (p. 3.5.15)

$$P_e = \frac{1}{2}(1-\alpha) = \frac{1}{2}\left(1 - \frac{\rho}{\sqrt{1+\Gamma^{-1}}}\right)$$

The floor is evident: as $\Gamma \to \infty$, $P_e \to \frac{1}{2}(1-\rho)$, so it is dictated by quality of CSI

- Results

$$\alpha(\Gamma_b, \rho) := \frac{\rho}{\sqrt{1+\Gamma_b^{-1}}} \qquad P_e(\Gamma_b, \rho) := \frac{1}{2}\cdot(1-\alpha(\Gamma_b, \rho))$$



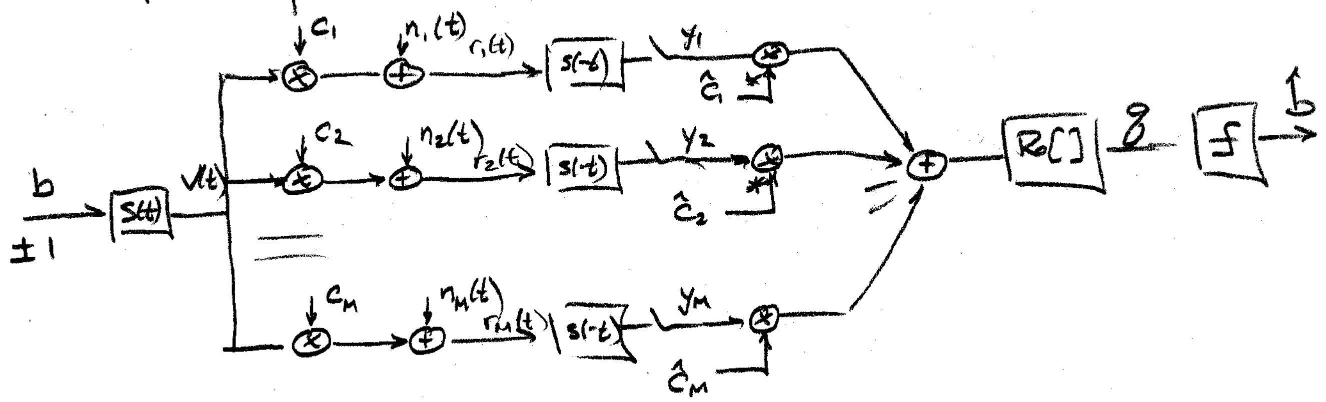
Effect of CSI Quality on BER, M=1

Why a floor? Because, in fades, when $|c|$ is small, the estimation error makes the phase (and amplitude) relationship between y and $\hat{c}$ very erratic — makes errors even without noise.

With a sensible channel estimation procedure (eg PSAM), the corr'n coeff. ρ improves with SNR Γ, so floor is greatly suppressed.

3.6.2 Imperfect CSI and Diversity

- Consider single user diversity, BPSK, flat, as in Example 3 of Section 3.5.1



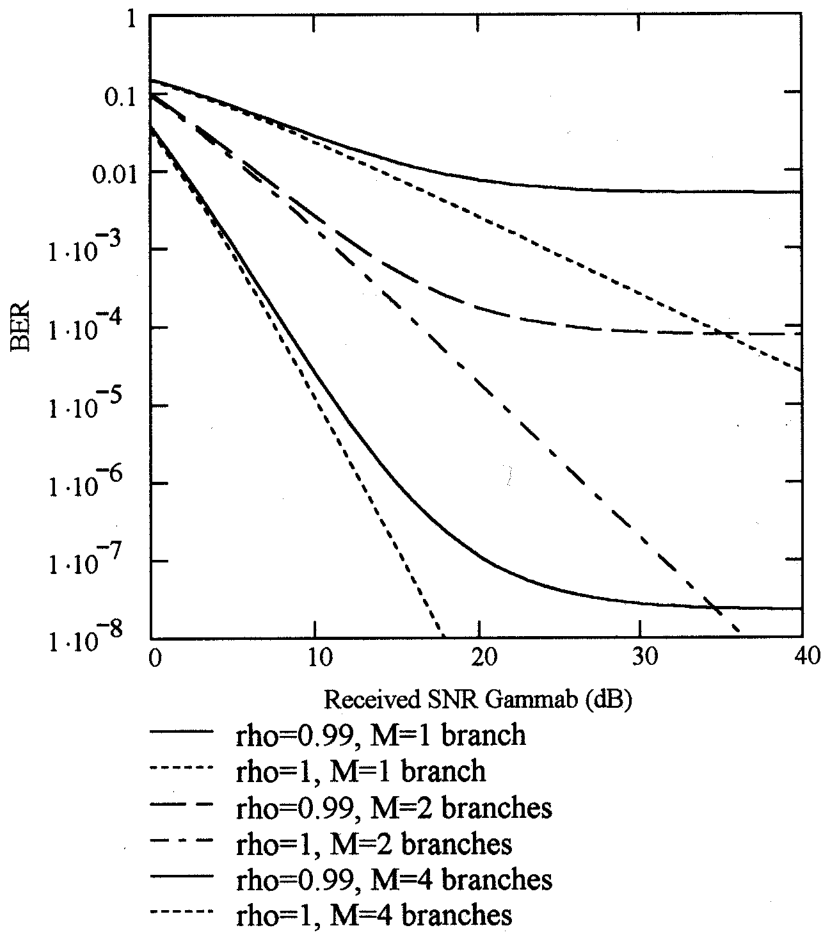
- This is just an M-fold version of our previous example. All branches same, so Proakis results apply.
  - As before, $\alpha = \dfrac{\rho}{\sqrt{1+\Gamma_b^{-1}}}$
  - so pole ratio $v_2/v_1 = \dfrac{1+\alpha}{1-\alpha} \triangleq r$
  - and the BER is (p. 3.5.14)

$$P_e = \frac{1}{1+r^{2M-1}} \sum_{m=0}^{M-1} \binom{2M-1}{m} r^m$$

- Again, a floor is evident. As $\Gamma_b \to \infty$, ratio $r \to \dfrac{1+\rho}{1-\rho}$

- Results show that diversity reduces floor. Not all c_m, $\hat{c}_m$ relationships are erratic, since simultaneous fades are rare.

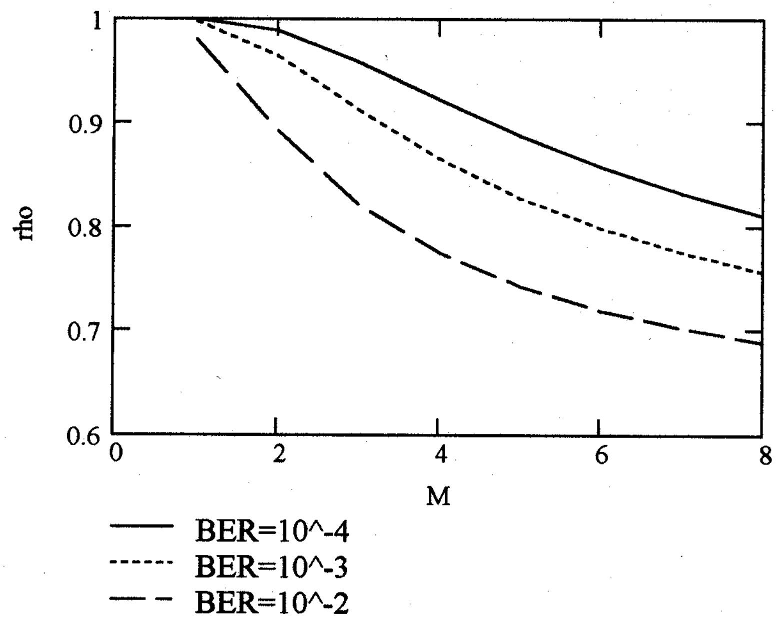
$$r(\Gamma_b,\rho) := \frac{1+\alpha(\Gamma_b,\rho)}{1-\alpha(\Gamma_b,\rho)} \qquad P_e(\Gamma_b,\rho,M) := \frac{1}{1+r(\Gamma_b,\rho)^{2\cdot M-1}} \cdot \sum_{m=0}^{M-1} BC(2\cdot M-1,m)\cdot r(\Gamma_b,\rho)^m$$



Multibranch Diversity, Imperfect CSI

- The required channel estimation accuracy is a big question. To remove SNR from the mix, ask "What corr'n coeff ρ (relating $c, \hat{c}$) is required to give a BER floor of 10^{-3}?"

  Find root of $P_e(\infty, \rho, M) = 10^{-3}$ (or other value)



Required rho drops with more antennas

pretty sloppy estimation!

Also see similar conclusions for multiuser ([Gran98], [Gran00]) and for transmit diversity ([Cave00]).

Exercises

1. Return to the Rake receiver for single user, isolated pulse, with delay spread (pp. 3.2.3 – 3.2.4). Allow perfect CSI. From basic principles, we obtained the decision variable

$$g = bA\,\mathrm{Re}[\underline{c}^{+}\underline{y}], \qquad b = \pm 1$$

Obtain expressions for the BER when the pulse autocorrelation is not perfect (i.e. $r(\tau) \neq 0$). Note that we need the results of Section 3.5.2 to do this. Also see [Mazo91] for another approach

2. Diversity with unequal branch powers when CSI is imperfect. Return to Section 3.3.2 system, but allow different ρ on each branch, since SNRs and qualities of channel estimation differ. A good model here is PSAM, where $\rho_m \approx \sqrt{1 - 0.16/\Gamma_m}$ [Gran98]. You can't use the Proakis results, so revert to Section 3.5.2, using 4×4 matrices. This question does not appear to have been addressed in the literature.