4.4 Summary of Solution Approaches

- We have seen a variety of detectors in different fading channel situations, and have analysed them in different ways. It's time to step back and compare the two main analytical approaches:
  - quadratic forms
  - averaged instantaneous BER

- The QF is a very general approach, provided the variables are Gaussian (i.e. Rayleigh or Rice fading):
  - accounts for imperfect CSI
  - for complicated situations (e.g. Viterbi equaliser, trellis codes, space-time codes), use the big formulation

    $g = \frac{1}{2} \underline{z}^{+} Q \underline{z}$, calculate $\Pr[g<0]$.

    Use Laplace inversion or Welburn's short cut

- for multichannel structures,

$$g = \sum_i g_i = \sum_i \tfrac{1}{2} \underline{z}_i^{\dagger} Q_i \underline{z}_i \quad , \quad \Pr[g<0]$$

→ if all channels the same

$$Q_i = Q \;\; \forall i, \quad R_i = \tfrac{1}{2}\overline{\underline{z}_i \underline{z}_i^{\dagger}} = R \;\; \forall i$$

and 2×2, then use the special solution in Proakis

→ if all channels different

$$Q_i \neq Q_k, \quad R_i \neq R_k \quad \forall i \neq k$$

then use full solution, but inversion is easy (no differentiation).

- Averaged instantaneous BER

$$\overline{P_e} = \int_0^{\infty} P_e(\gamma)\, p_\gamma(\gamma)\, d\gamma$$

- is intuitive – calculate the BER of a static channel as a function of SNR, then average wrt SNR.
- It is general in allowing non-Gaussian channels – e.g., Nakagami or combined shadowing and fading SNR pdf (Suzuki)

– but does not comfortably extend to imperfect CSI (though possible for some cases) or variation of the SNR across the span of the detector.

- The two methods produce the same results for perfect CSI and slow fading – e.g. the perfect CSI diversity analysis.

inst
$$\overline{P_e} = \left(\frac{1-\mu}{2}\right)^M \sum_{m=0}^{M-1} \binom{M-1+m}{m} \left(\frac{1+\mu}{2}\right)^m \qquad \mu = \sqrt{\frac{\Gamma_b}{1+\Gamma_b}}$$

(Γ_b is per-channel)

QF
$$\overline{P_e} = \frac{1}{1+r^{2M-1}} \sum_{m=0}^{M-1} \binom{2M-1}{m} r^m \qquad r = \frac{1+\alpha}{1-\alpha} = \frac{\sqrt{1+\Gamma_b^{-1}}+1}{\sqrt{1+\Gamma_b^{-1}}-1}$$

superficially different, numerically equal