SIMON FRASER UNIVERSITY
School of Engineering Science

ENSC 428-4 Data Communications

Assignment 1 Due: 2001 01 18

1. You have designed a unit that selects the maximum value of its N inputs; that is, its output is

$$y = \max\left(x_1, x_2, \ldots, x_N\right)$$

If the inputs $x_1, x_2, \ldots, x_N$ are independent and identically distributed with $f_X(x)$, determine the pdf. of the output $p_Y(y)$. You will find it easiest to work through the cumulative distribution function $F_Y(y)$ in terms of $F_X(x)$.

2. For the bivariate Gaussian distribution as given on page 158 of your text, determine the conditional pdf

$$p_{Y|X}(y\,|\,x)$$

For simplicity, assume that the means are zero, but allow the variates to be correlated with correlation coefficient ρ.

(a) Is the conditional pdf also Gaussian?
(b) Write an expression for the conditional mean of Y, given X (i.e., $m_{Y|X}$). This is very important in estimation of analog values from measurements.
(c) Write an expression for the conditional variance $\sigma^2_{Y|X}$. Is it greater, smaller or the same size as the unconditional variance σ^2_Y?

In this question, you will probably have to resort to tricks like completing the square, as well as use of the normalizing factor on page 2.2.1 of your notes. It's ugly, but you have to do this once in your life.

3. Show that the sum of two independent Gaussian random variables is itself Gaussian. Give them variances σ_1 and σ_2 but keep the means zero, to avoid clutter.

(a) First, use the characteristic function of a Gaussian pdf, (3.1.5) in your text.
(b) Second, set up your proof as a convolution, then contemplate how messy it will get. Don't bother to carry this approach to completion, since you have already gone through most of the steps in Question 2.