SIMON FRASER UNIVERSITY
School of Engineering Science

ENSC 428-4 Data Communications

Assignment 3 **Due: 2001 02 23**

1. Unequal Prior Probabilities

Consider binary antipodal signals in which $P(s_1) \neq P(s_2)$.

(a) Sketch the conditional pdfs $f_{r|s_1}(r \mid s_1)$ and $f_{r|s_2}(r \mid s_2)$ and the marginal pdf $f_r(r)$.

(b) Where is the decision boundary?

(c) Give expressions for the conditional error rates $P_b(s_1)$ and $P_b(s_2)$ and the average error rate P_b.

2. Translation of Signal Constellations

Consider an arbitrary constellation of M points in N-dimensional space. You have seen in class that "rigid body" translation (i.e., translation that maintains the *relative* locations of the points) does not affect the error rate, but may affect the average energy. Show that translating the centroid of the constellation to the origin minimizes the energy. [Hint: it's easiest to do it backwards, starting with a centred constellation.]

3. Union Bound

Here you will use the M=4, N=2 "vertices of a hypercube" constellation to develop some additional understanding of the union bound. To get started, assume that $\boldsymbol{s}_1$, the signal in upper right quadrant, was transmitted, and sketch the constellation and the decision boundaries for region R_1.

(a) There is a symbol error if the noise takes the received $\mathbf{r}$ over either or both of the decision boundaries. Upper bound the symbol error probability by the probability of the union of these two events. Write an expression for that union bound.

(b) What region of the plane was counted twice in your bound? What is the probability that the received value $\mathbf{r}$ lies in it? For this constellation, you can write an exact expression for the probability.

(c) Why does the effect of double counting that region become negligible compared with the bound as the SNR becomes large? (In other words, why does the bound converge to the true symbol error rate as the SNR becomes large?) Use two forms of argument: one based on the probability of the double-counted region from part (b); and a more approximate one based simply on sketches of the received signal probability surface $f_{\mathbf{r}|\mathbf{s}_1}(\mathbf{r}\,|\,\mathbf{s}_1)$.

(d) Another version of the union bound is frequently used for more complex situations. If you transmit $\boldsymbol{s}_1$ then you make a symbol error if **r** is closer to $\boldsymbol{s}_2$ or **r** is closer to $\boldsymbol{s}_3$ or … or **r** is closer to $\boldsymbol{s}_M$. ; the symbol error event is the union of the *pairwise* error events. The probability of pairwise event *m* is the pairwise probability of error $P_2(\mathbf{s}_1, \mathbf{s}_m)$. The pairewise events are not mutually exclusive, so we can overbound the probability of symbol error for $\boldsymbol{s}_1$ by the sum of pairwise events: $P_s(\mathbf{s}_1) \le \sum_{m=2}^{4} P_2(\mathbf{s}_1, \mathbf{s}_m)$.

Sketch the three pairwise decision boundaries. Is this union bound tighter or looser than the one in part (a), which was based on knowledge of the nearest decision boundaries? Why do the bounds converge to the same value as SNR increases?

4. Gram-Schmidt

Use the Gram-Schmidt procedure on the monomials $v_0(t) = 1$, $v_1(t) = t$ and $v_2(t) = t^2$ over the interval [-½ , ½] to produce three orthonormal polynomials.