SIMON FRASER UNIVERSITY
School of Engineering Science

ENSC 428 Data Communications

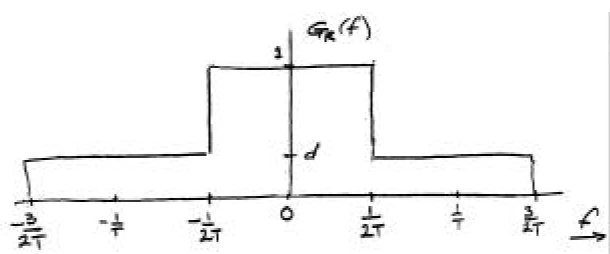
Assignment 4 **Due: 2001 03 15**

1. How Important is a Matched Filter?

We have spent considerable effort in determining the optimum filtering in a receiver. But is it really that sensitive? What if implementation considerations make an approximation desirable? In this question, you'll look at two examples, one posed in time, the other in frequency.

(a) Suppose the transmitted pulse shape is $g_T(t) = A\sin(\pi t/T), 0 \le t \le T$ and you use a binary antipodal constellation. The signal arrives in white noise with PSD $N_o/2$. Rather than implement a matched filter, you are tempted simply to integrate the received signal over the bit duration; in effect, you use a filter with a rectangular impulse response. Determine the BER as a function of the bit SNR γ_b for this receiver and compare to the matched filter receiver. What did it cost in dB?

(b) Now suppose the transmitted pulse shape is $g_T(t) = A\,\mathrm{sinc}(t/T)$ - not a good choice, for reasons we have discussed in class, but this is just a question on an assignment. At the receiver, you want to save computation, so you truncate the impulse response of the matched filter. Unfortunately, you pick up sidelobes by doing so (as well as a little distortion in the main lobe, which you can ignore here). The result is modeled crudely by the frequency response shown below, in which the amplitude of the sidelobes is a factor $d < 1$ times that of the main lobe. Determine the BER as a function of the bit SNR γ_b for this receiver and compare to the matched filter receiver? What did it cost in dB, as a function of d ?

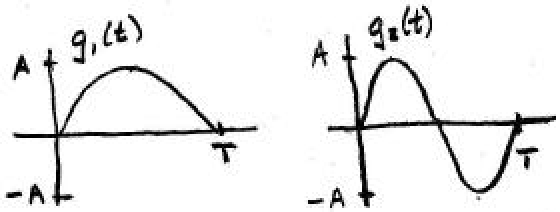


To put your results in perspective, recall that the only issue you considered here was white noise, not intersymbol interference or adjacent channel interference.

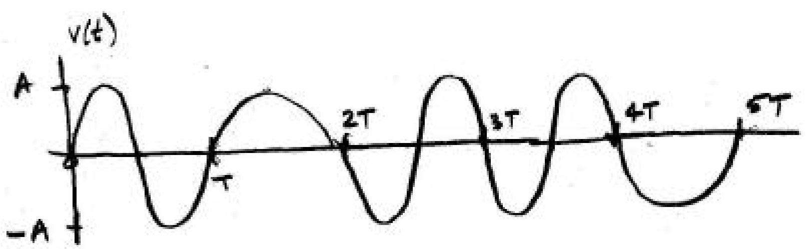
2. Viterbi Receiver for FSK

In this question, you will derive the Viterbi receiver for a form of FSK. It differs from the PAM discussed in the notes, because there is more than one pulse shape. It's easier than it sounds, though – just follow the basic principles.

First, the signals. Here are the two pulse shapes:



When strung together with continuous phase, they produce a signal that looks like this:



Frequency f_1 causes the starting slope of the next pulse to be reversed, and frequency f_2 keeps it the same. The signal is received in white noise with PSK $N_o/2$, so that

$$r(t) = v(t) + n_w(t)$$

(a) How do you "vectorize" the received signal; i.e., project it onto the signal subspace? Give an expression for the resulting $\mathbf{r}(k)$ for symbol k.

(b) The state summarizes the entire past history of the signal insofar as it affects the next signal to be transmitted. Define the state at the start of a symbol as 0 if the signal is to start with a positive slope and 1 if it starts with negative slope. Associate logic 0 with f_1 and logic 1 with f_2. Draw a state diagram and a trellis diagram to represent the signaling method. As a check, the state *transition* should determine what you would receive in the absence of the noise. Another check: your diagrams should generate any allowable sequence and only allowable sequences.

(c) The branch metrics are functions of the received $\mathbf{r}(k)$ and the signal associated with that transition. Frequently, they are the log or the negative log of the conditional pdf of $\mathbf{r}(k)$, given the associated signal. Write expressions for each of the branch metrics. How simple can you make them?

(d) What is the probability of the most likely error event, and how many bit errors does it produce?