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### **ENSC 428 Data Communications**

### **Assignment 5**

Due: 2001 03 29

## 1. Linear Equalizer

A particular data link uses binary antipodal signals with square root Nyquist pulses. The receiver filter is matched to the transmitted pulse and sampled at the symbol rate. Unfortunately, there is a spoiler, in the form of some filtering in the transmission link, and it makes the sampled impulse response at the matched filter output equal

 $x_0 = 0.1$   $x_1 = 1$   $x_2 = -0.3$ so that the samples are  $y_k = \sum_{i=0}^{2} a_{k-i} \cdot x_i + v_k$ 

The signal values  $a_k$  belong to  $\{+1,-1\}$  and the noise variance is  $\sigma_v^2$ .

#### (a) Give an expression for the BER.

(b) One remedy is a simple linear equalizer; i.e., an FIR filter. With 5 coefficients, its impulse response is  $w_j$ , j=0..4. Write the normal equations that determine the MMSE coefficients, making sure to show all elements of the Gram matrix and the cross-correlation vector, as well as the coefficient values, so that your equations aren't accidentally in reverse order.

#### 2. Carrier Recovery

Your company is developing a satellite link for data transmission in which the received signal at the earth terminals is at 2.4 GHz. You have already solved the problem of ensuring that your local oscillators are at the right frequency, but the carrier phase remains a problem - so now you must design a DSP-based carrier phase recovery algorithm.

The signal format is DQPSK with square root Nyquist pulses, and the timing recovery algorithm has already been designed and tested. From the perspective of your algorithm, therefore, you receive symbol-spaced samples of the matched filter output, each a phase-rotated complex signal with superimposed complex Gaussian noise. You must process them with a decision-directed algorithm to estimate the phase offset  $\theta$ , and then use that phase to detect the data.

To test your algorithm in simulation, you use a file of complex samples generated by the code below:

Generate the Data

$$Qdata(x) := exp\left(j \cdot \frac{\pi}{2} \cdot floor(rnd(4))\right)$$
 generates random QPSK data, x dummy

number of symbols to be	index from 0	index from 1
generated		
N <sub>sym</sub> := 50	n0 := 0 N <sub>sym</sub>	n1 := 1 N <sub>sym</sub>

 $a_{n1} := Qdata(n1)$  the random QPSK data

 $\mathbf{b}_0 \coloneqq \mathbf{1}$   $\mathbf{b}_{n1} \coloneqq \mathbf{b}_{n1-1} \cdot \mathbf{a}_{n1}$  the differentially encoded symbols

Set up Noise Generation

 $\gamma_{bdB} := 5$  dB  $\gamma_b := 10^{0.1 \cdot \gamma_{bdB}}$   $\gamma_b = 3.162$  the SNR cgauss(x) :=  $\sqrt{-2 \cdot \ln(rnd(1))} \cdot \exp(j \cdot rnd(2 \cdot \pi))$  generates complex Gaussian noise, unit variance, x dummy

"unit variance": if *n* is generated by cgauss(x), the  $\frac{1}{2} \cdot E[(|n|)^2] = 1$ 

Generate the Samples at the Matched Filter Output

$$\theta := rnd(2 \cdot \pi)$$
 random phase shift  
 $r_{n0} := e^{j} \cdot \theta \cdot \sqrt{2} \cdot b_{n0} + \frac{1}{\sqrt{\gamma} b} \cdot cgauss(n0)$ 

# Save to a File

save the samples to a file	save the data for comparison
rsave := augment( $Re(r)$ , $Im(r)$ )	asave := $augment(Re(a), Im(a))$

I can't seem to make a pdf file when the procedures for file write are included. Essentially, they just create a text file with each row containing the real part and the imaginary part, separated by a space.

# What You Do

Describe your algorithm and use it on the samples in the file asst5Rx.txt. Give:

- \* your estimate of  $\theta$ ;
- \* your decisions for the *a* data using that estimate of  $\theta$ ;
- \* an estimate of the SNR (may not be the same as above).

Compare your *a* decisions with the true data in the file asst5adata.txt as a plausibility check on your processing, but do not use the latter file to help you estimate  $\theta$  - you must use a decision-directed algorithm, not a data-aided one. Why do errors seem to occur in pairs?

Note that, because you are using DSP, you have the luxury of being able to go over the data record more than once; you could estimate  $\theta$  in one pass, and use it in the next.

For the numerical work, you can use whatever package you like, such as C, Matlab or Mathcad. If you don't have any of them right now, download Mathcad Explorer and use the blank worksheet on the website (but you won't be able to print or save the results).