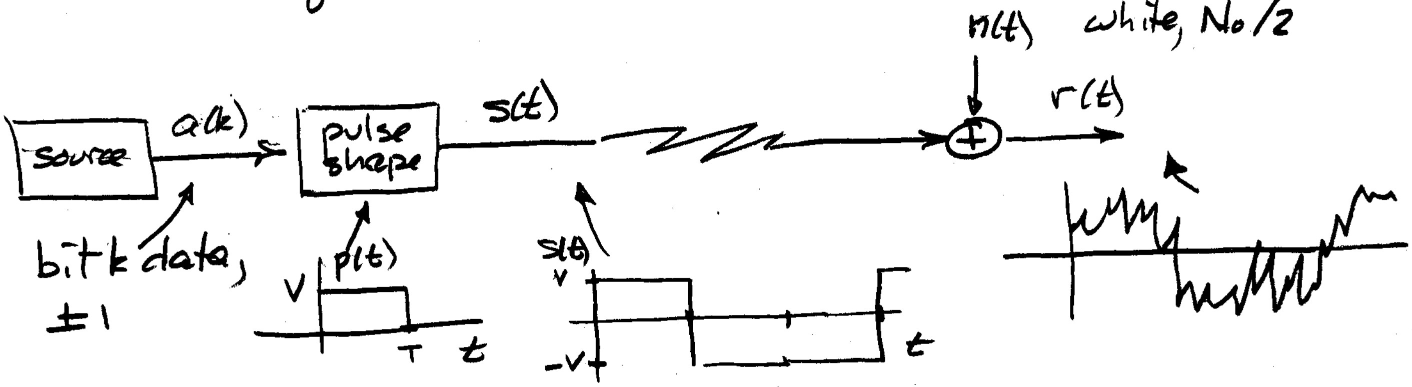
3. FIRST STEP: SINGLE RECTANGULAR PULSE IN WHITE NOISE

- This is a very simple system, but it illustrates many phenomena that occur in more complex systems.

- Send a sequence of NRZ pulses, i.i.d data ±1



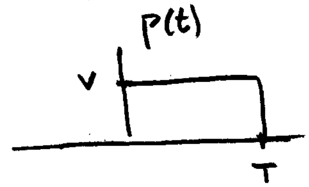
  - data bits indep of each other
  - pulses V volt, T sec
  - noise flat across band, modeled as white

  → what do we do with r(t)??
  → what is signal band?
  → what is the BER performance?

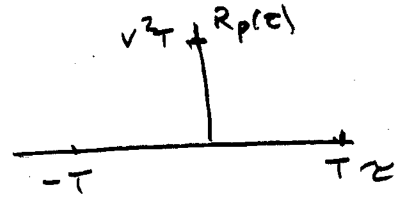
3.1 Signal Band

- From Section 2.4, the autocorrelation function is

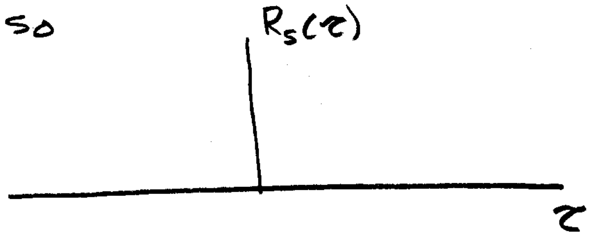
$$R_s(\tau) = \frac{1}{T}\int_{-\infty}^{\infty} p(t)\,p(t-\tau)\,dt$$



$$= \frac{1}{T} R_p(\tau)$$



so



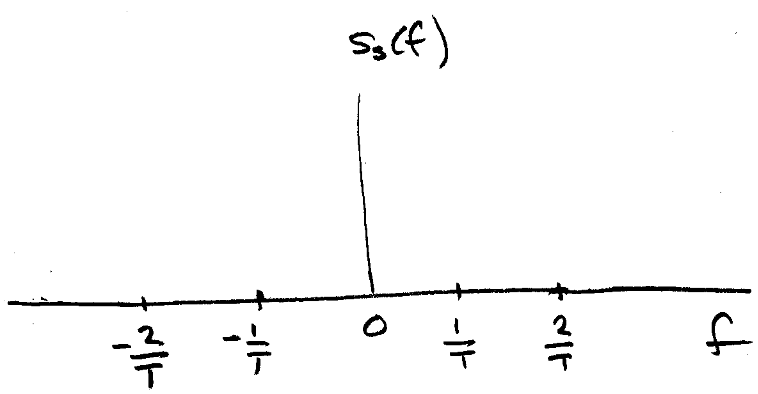
units ?

value of energy per "symbol" (i.e. per pulse):

(variation with bit rate? with power?)

- The transmitted power spectrum is

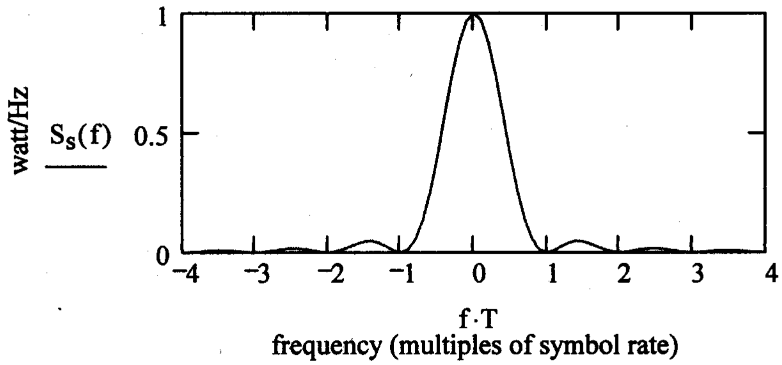
$$S_s(f) = \mathcal{F}\left[R_s(\tau)\right] =$$

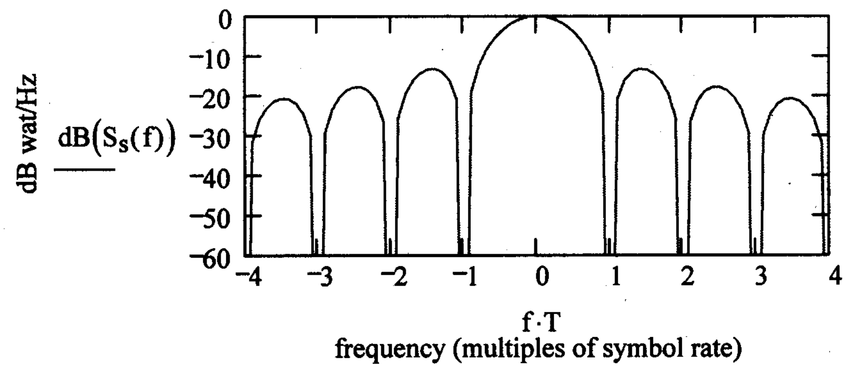


units ?

value at $f=0$?

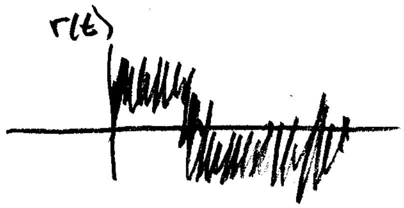
$T := 1 \qquad V := 1 \qquad S_s(f) := V^2 \cdot T \cdot \mathrm{sinc}(f \cdot T)^2$





3.2 How to Process the Received Signal

- First observation: the bits are independent, and white noise decorrelates instantly, so...

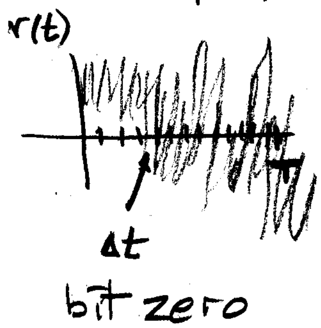


but with infinite noise power, instant decorr

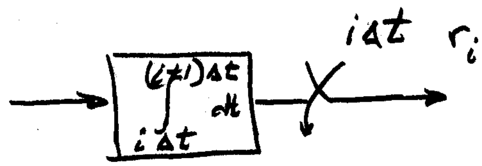
Only $r(t)$ in $[kT, (k+1)T]$ is useful for detecting the k^{th} bit $a(k)$.

Do them one at a time.

- Next, conceptually convert to discrete time by integrating over subintervals of width Δt (we'll do it properly later).

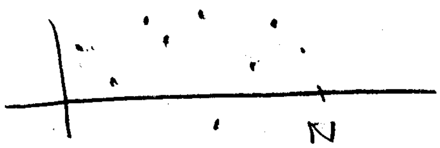


$N \Delta t = T$



New question — how to process the N samples in the bit?

- Start: what are the stats?
  - signal component $\pm V\Delta t$ this is the mean
  - noise component has variance $\frac{N_0}{2}\Delta t$ (Section 2.5)
  - same for all samples.



We want the mean value to see if it's + or −, so sum or average the samples.

- Decision variable

$$d_{sum} = \sum_{i=1}^{N} r_i \quad \text{or} \quad d_{av} = \frac{1}{N\Delta t}\sum_{i=1}^{N} r_i$$

- Sum: $m_d = \pm N V \Delta t = \pm V T$ (just make one big integral

$$\sigma_d^2 = N \frac{N_0}{2}\Delta t = \frac{N_0}{2} T$$

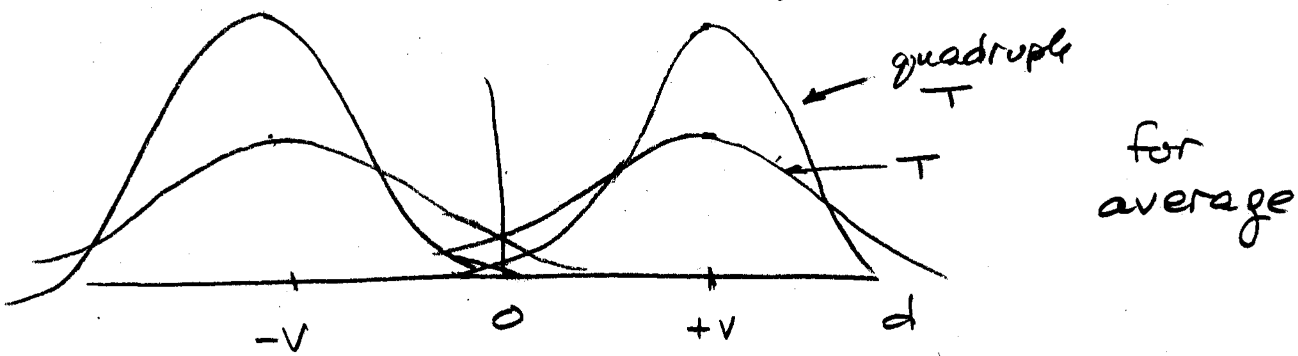
$$\sigma_d = \sqrt{\frac{N_0}{2} T}$$

- Average: $m_d = \pm \frac{1}{N\Delta t} N V \Delta t = \pm V$

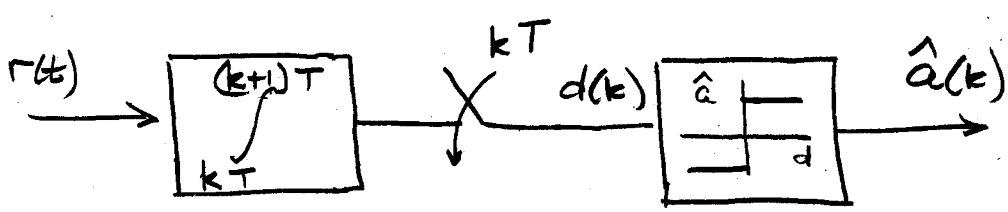
$$\sigma_d^2 = \frac{1}{(N\Delta t)^2} N \frac{N_0}{2}\Delta t = \frac{N_0}{2}\cdot\frac{1}{T}$$

$$\sigma_d = \sqrt{\frac{N_0}{2}\cdot\frac{1}{T}}$$

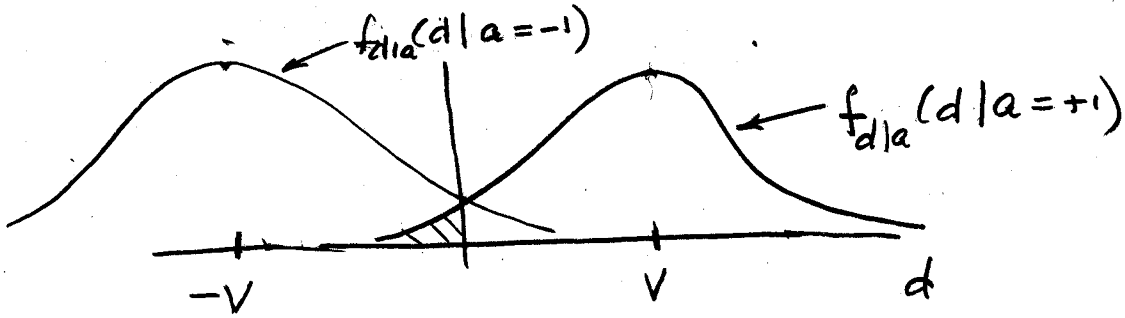
- Behaves like summing (averaging) random variables.



- The processing is equivalent to



3.3 BER Performance



- The probability that d falls on the wrong side of the decision threshold is the probability of error

$$P_e = Q\left(\frac{m_d}{\sigma_d}\right) = Q\left(\frac{V}{\sqrt{\frac{N_0}{2}\cdot\frac{1}{T}}}\right) = Q\left(\sqrt{\frac{2T}{N_0}}\,V\right)$$

dist measured in std dev'ns

- A useful interpretation:

energy/bit $E_b = V^2 T$ units?

$$P_e = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) = Q\left(\sqrt{2\gamma_b}\right)$$

$$\gamma_b = \frac{E_b}{N_0} = \text{SNR}$$ units?

- Observations:
  - For a given BER, increasing the data rate by decreasing T requires increasing the transmit power in proportion.
  - $\frac{m}{\sigma}$ increases as $\sqrt{T}$ for const V
  - γ_b incr as T for const V
  - a simple bound:

$$P_e = Q(\sqrt{2\gamma_b}) \leq \frac{1}{2} e^{-\gamma_b}$$

decreases exponentially with SNR

what does exponential decrease imply?

$$Q_{bnd1}(x) := \frac{1}{2} \cdot \exp\left(\frac{-x^2}{2}\right) \qquad Q_{bnd2}(x) := \frac{1}{\sqrt{2 \cdot \pi} \cdot x} \cdot \exp\left(\frac{-x^2}{2}\right)$$