4.2 PCM vs FM: Bandwidth, Channel Noise and Output SNR

- Both PCM and FM expand the bandwidth in transmission in order to improve the output SNR. To understand why PCM took over from FM, we should do a quantitative comparison. Same criteria:
  - channel bandwidth
  - channel SNR
  - output SNR

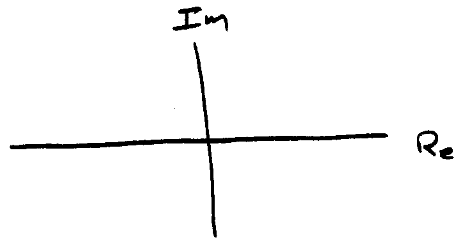
FM Over Threshold Behaviour (Review)

- Transmit $\tilde{s}(t) = A\cos\left(2\pi f_c t + 2\pi k_f \int^t x(\alpha)\,d\alpha\right)$

  so that instantaneous phase is $\theta(t) = 2\pi k_f \int^t x(\alpha)\,d\alpha$

  instantaneous frequency $\Delta f_i(t) = k_f\, x(t)$

$$\tilde{s}(t) = \mathrm{Re}\Big[\underbrace{A e^{j\theta(t)}}_{s(t)}\, e^{j2\pi f_c t}\Big]$$



- Peak frequency deviation $\Delta f_p = k_f\, x_p$

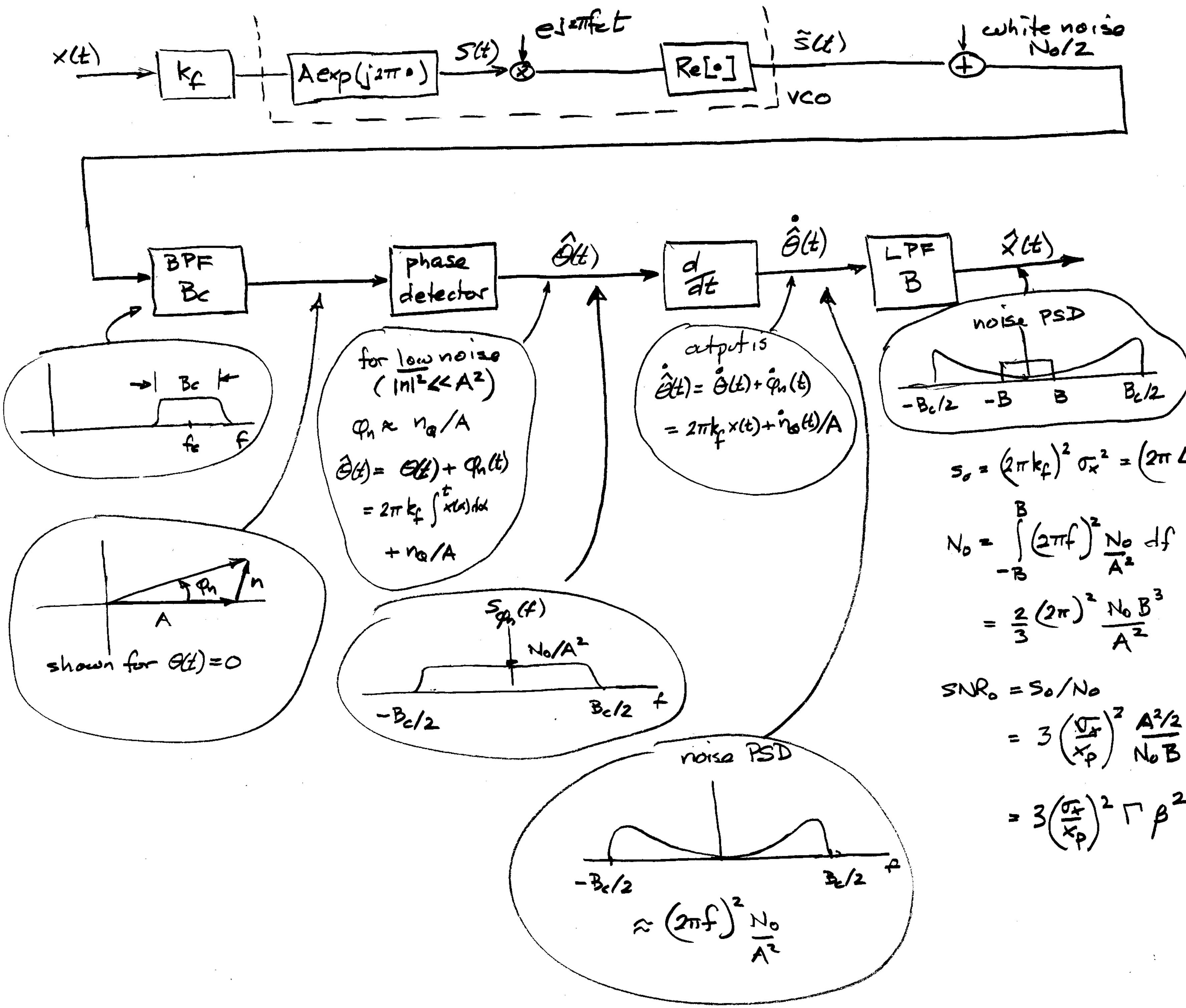
  mean square deviation $\overline{\Delta f_i^{\,2}} = k_f^2\, \sigma_x^2 = \Delta f_p^2 \left(\frac{\sigma_x}{x_p}\right)^2$

- Channel bandwidth

$$B_c \cong 2(\Delta f_p + B) = 2B(\beta + 1)$$

Carson's rule
β = modulation index

- $SNR_o \approx 3\left(\frac{\sigma_x}{x_p}\right)^2 \Gamma \beta^2$ <u>if</u> over threshold (next page)



$$S_o = (2\pi k_f)^2 \sigma_x^2 = (2\pi \Delta f_p)^2 \left(\frac{\sigma_x}{x_p}\right)^2$$

$$N_o = \int_{-B}^{B} (2\pi f)^2 \frac{N_o}{A^2}\, df$$

$$= \frac{2}{3} (2\pi)^2 \frac{N_o B^3}{A^2}$$

$$SNR_o = S_o / N_o$$

$$= 3 \left(\frac{\sigma_x}{x_p}\right)^2 \frac{A^2/2}{N_o B} \cdot \left(\frac{\Delta f_p}{B}\right)^2$$

$$= 3 \left(\frac{\sigma_x}{x_p}\right)^2 \Gamma \beta^2$$

FM Threshold

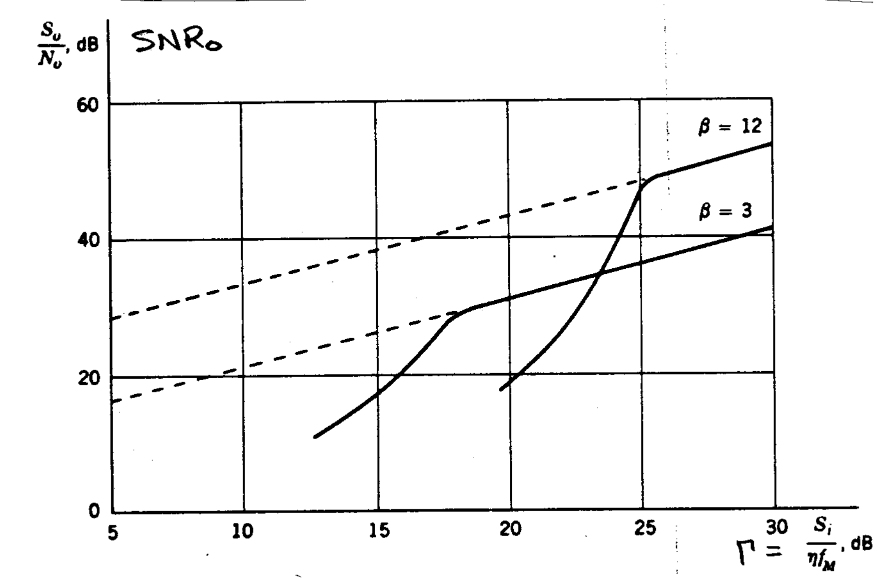
- Output $SNR_o = 3\left(\frac{\sigma_x}{x_p}\right)^2 \Gamma \beta^2$ increases as the square of the modulation index (roughly as the square of the bandwidth expansion factor...

  provided that $\overline{|n|^2} \ll A^2$.

  This low noise approximation is quite good if signal power is at least 10 dB greater than channel noise power (CNR > 10dB)

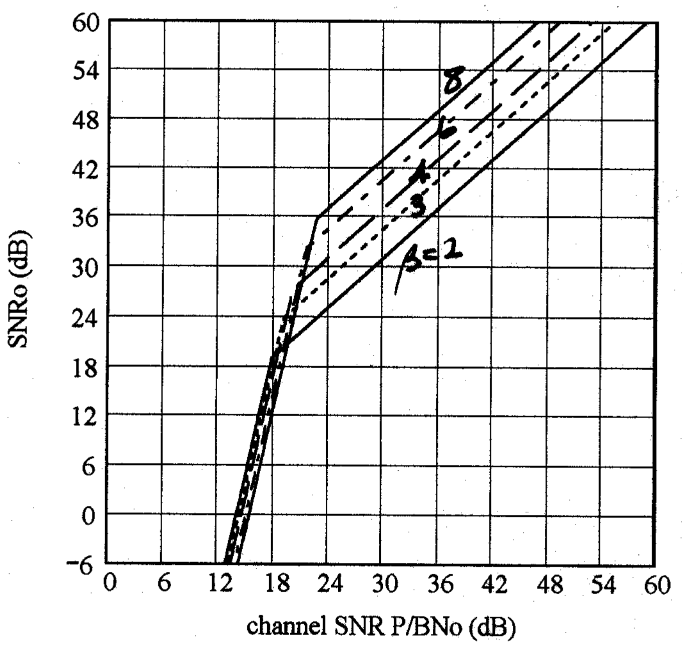
$$\frac{A^2/2}{N_o B_c} = \frac{A^2/2}{N_o B} \cdot \frac{1}{2(\beta+1)} = \frac{\Gamma}{2(\beta+1)} \geq 10$$

- Performance degrades rapidly below this threshold



β	$20(\beta+1)$	$dB(20(\beta+1))$
12	260	24.15 dB
3	80	19.0 dB

Fig. 10.6-2 Output SNR of an FM discriminator when demodulating an FM signal which is sinusoidally modulated.



Analysis of behaviour under threshold is challenging and approximate at best. It's also not of much interest. Here it's approximated as Γ^5 dependence.

Operate at threshold?

- Recall over threshold behaviour

  PCM: $SNR_o \sim e^{2\ln 2 (B_c/B)}$ FM: $\sim (B_c/B)^2$

They don't improve indefinitely if you simply increase B_c/B.
Suppose fixed transmit power and noise PSD.

PCM: higher bit rate means less energy per bit, hence higher BER

FM: higher modulation index means greater bw of signal and front end BPF, so more noise power enters discriminator and low noise approx becomes less valid.

In both cases, we encounter a threshold on bw expansion, beyond which performance doesn't improve — it deteriorates!

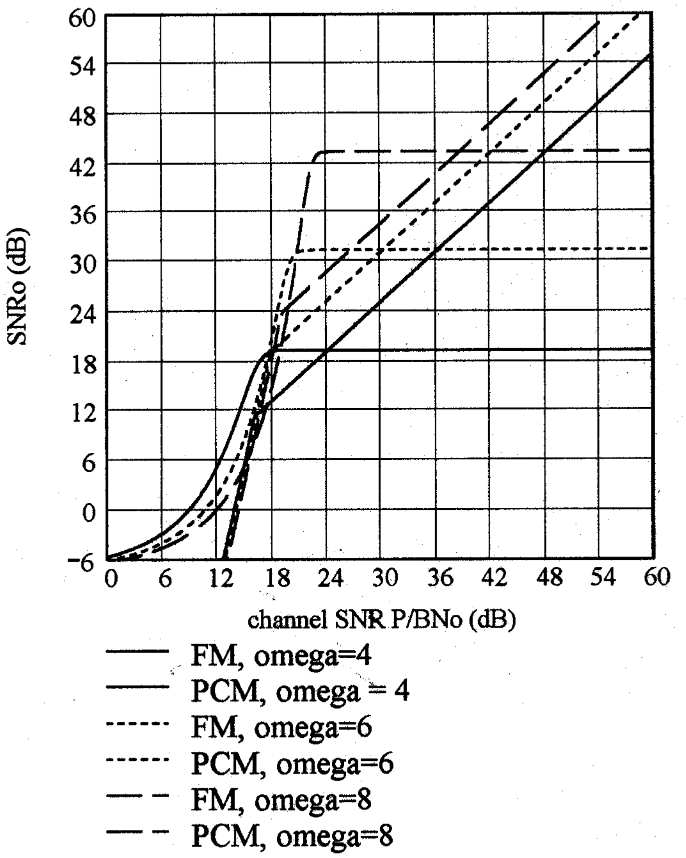
The Comparison

- Recall the issues: Γ, SNR_o, B_c/B.

To compare on the same footing, use

$$\Omega_e = B_c/B = n \text{ (PCM)} = 2(\beta+1) \text{ (FM)}$$

b.w. expansion factor



PCM vs. FM, b.w. expansion as param

- For a given b.w. expansion, increasing Γ eventually lets FM win.
- But would we operate there? For $\Omega_e = 8$, the crossover point is at Γ 16 dB higher than PCM threshold. That is, PCM can achieve same SNR_o with 40 times less power.
- On the other hand, if you need that extra SNR_o and don't have the freedom to increase Ω_e, looks like FM is the way, since PCM is in a box.
- Or is it? See compression schemes Sec 4.4.