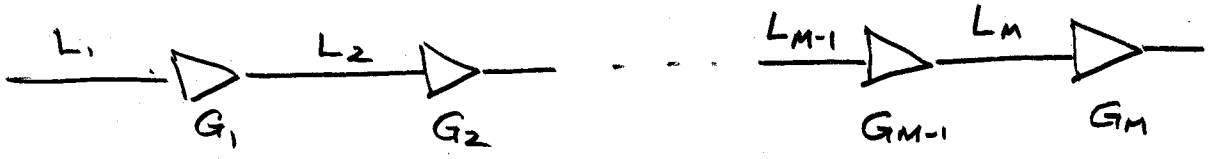
4.3 PCM vs FM: Chain of Repeaters

- Communication over long distances can involve huge attenuation and therefore unuseable SNR values:
  - inverse square law
  - inverse fourth power (mobile communications)
  - exponential (dB/km) in cable or fibre

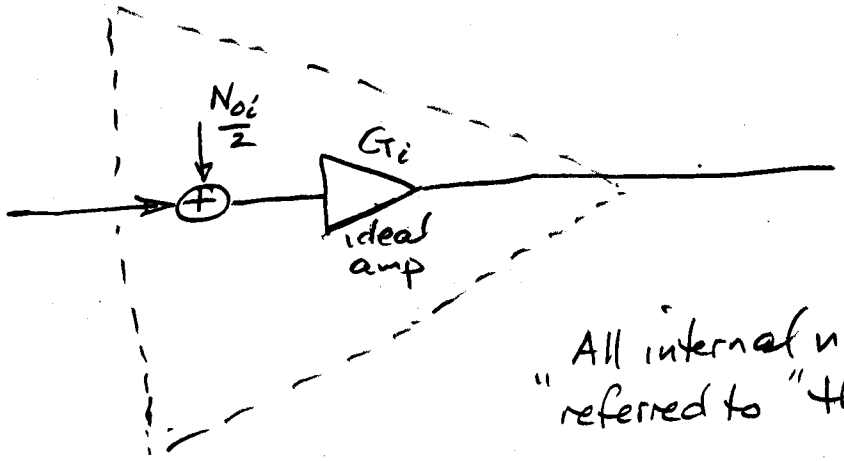
In these cases, it is common to use repeaters to make up the loss in the preceding link



each $G_i = L_i$, unit gain overall.

- The "gotcha"? The signal is so weak at the input to each amp that the amplifier internal noise becomes a consideration.

Model:

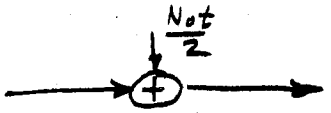


- For FM and other analog modulations, the noise accumulates:

total noise PSD:

$$\frac{N_0}{2} = \frac{1}{2}\sum_{i=1}^{M} N_{0i} G_i$$

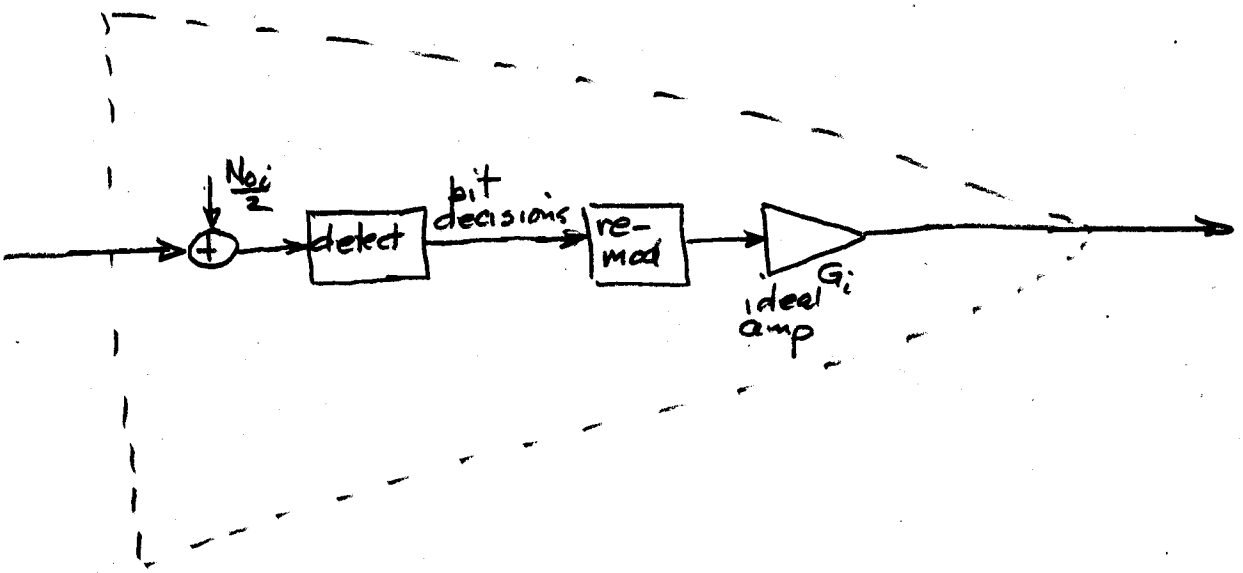
$$= \left(\frac{N_0}{2}\right)_{\text{single}} MG \quad \text{if all units the same } N_0, G, L$$

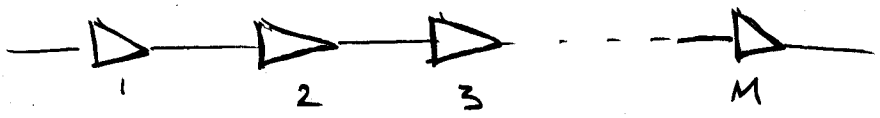


unit gain overall model of tandem link

This noise buildup may cause FM to drop below threshold, so number of links and the amp quality are design issues.

- For PCM, it is more common to use "regenerators" which make bit decisions in each unit and recreate a clean output





There is still a non zero probability of error at each regenerator, and this, too, can accumulate.

Keep it simple — identical units,

Prob a bit is flipped i times as it goes down the chain:

$$f_i(i) = \binom{M}{i} P_e^{\,i} (1-P_e)^{M-i}$$

but only an odd number is of concern:

$$P_{e\,end} = \sum_{\text{odd } i} f_i(i) \approx MP_e \quad \text{for a useable system}$$

- We can compare the performance of FM, PCM in a chain like this:
  - for a given amplifier noise level, what is the optimum number of links?
    Too few makes the attenuation too great. With unity gain overall $N_{ot} = M G_1 N_0$ (G_1: too large)
    Too many makes a lot of noise sources to accumulate $N_{ot} = M_1 G N_0$ (M_1: too large)
  - or for given required SNR_o, how many links are needed?