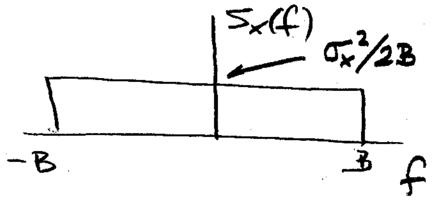
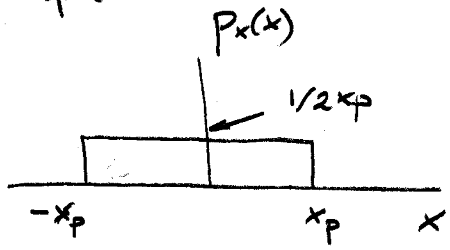
4.4 Companding, DPCM and Delta Modulation

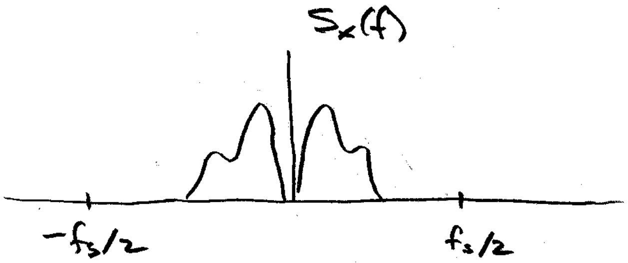
- So far, we have considered only the most straightforward way of sending an analog signal as digital:
  - sample at Nyquist rate
  - convert each sample with a linear (i.e uniform) A/D

This is good for samples from a white (flat) process with a uniform pdf for each sample

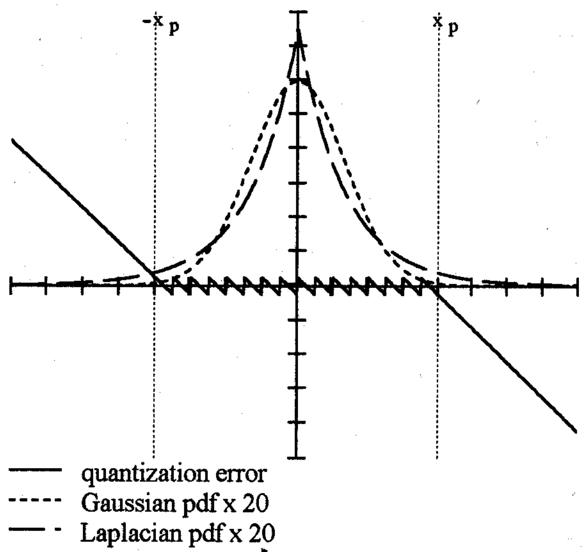




but it's not real life, which is more like



Not flat, sampled faster than Nyquist rate



Uniform Quant, ~~3~~ 4 Bits, 3 Sigma Loading

Not a uniform pdf

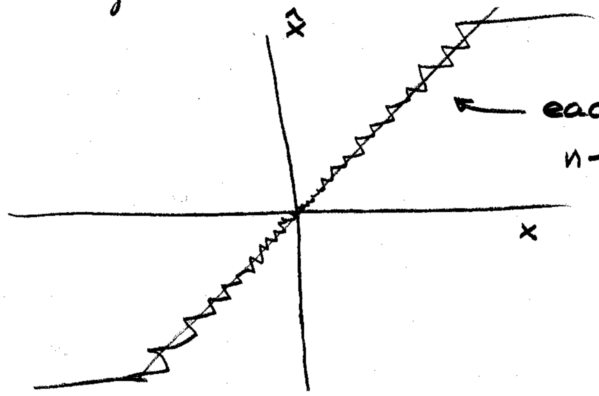
These factors allow us to send the signal with fewer bits, while maintaining equivalent quality. Therefore:

- less bandwidth expansion
- more energy per bit if same transmit power

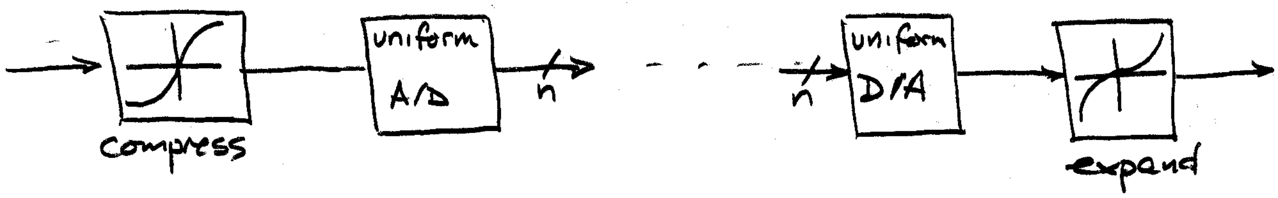
Non-Uniform Quantization (Companding) $\boxed{\text{P+S 4.6.1}}$

- Problem in linear (non-uniform) quantizer:
  - step size large enough to minimize overload error is too coarse at small amplitudes, even when loading factor σ_x/x_p is optimized

Solution:
  - make step size proportional to amplitude in a logarithmic scale



  - It is implemented by "companding"



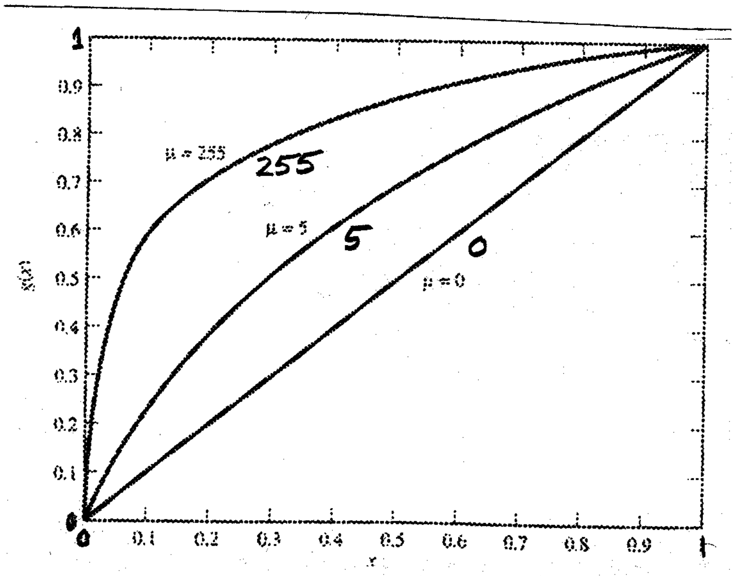
compression: μ law (usually $\mu = 255$)

$$g(x) = \frac{\log(1+\mu|x|)}{\log(1+\mu)}\,\mathrm{sgn}(x), \quad |x| \le 1$$

i.e. $|x/x_p| \le 1$

compression: A law (usually $A = 87.56$)

$$g(x) = \begin{cases} \dfrac{1+\ln(A|x|)}{1+\ln(A)}\,\mathrm{sgn}(x), & \dfrac{1}{A} \le |x| \le 1 \\[2ex] \dfrac{A|x|}{1+\ln(A)}\,\mathrm{sgn}(x) & 0 \le |x| < \dfrac{1}{A} \end{cases}$$



μ law compression g(x)

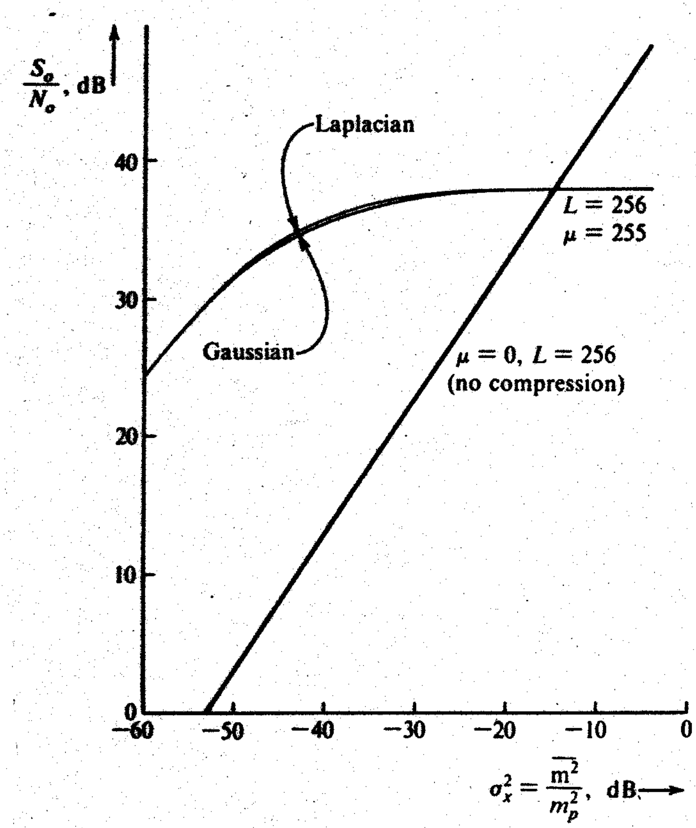


Figure 6.17 PCM performance with and without companding.

(from Lathi)

- Performance improvement:
  - 8-bit μ-law ($\mu = 255$) usually considered to be subjectively equivalent to 12-bit uniform
  - standard PCM in NA is 7 bits, $\mu = 255$. Companding improves performance by about 24 dB.

Differential PCM

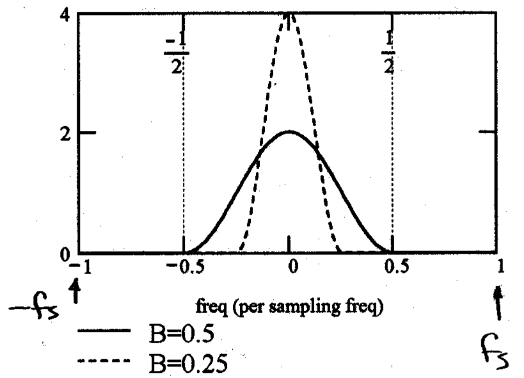
P+S 4.6.2

- When the input spectrum is not flat, successive samples become correlated. Here's an example.

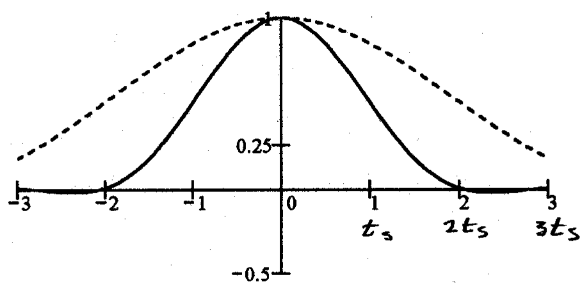
A special case of the raised cosine spectrum. Here the bandwidth is selected to be a fraction of the sampling frequency B/f_s.

$$S_x(f,B) := \frac{1}{2\cdot B}\cdot\left(1+\cos\left(\pi\cdot\frac{f}{B}\right)\right)\cdot(|f| \le B)$$

$$R_x(t,B) := \frac{\sin(\pi\cdot t\cdot B)\cdot\cos(\pi\cdot t\cdot B)}{\pi\cdot B\cdot t\cdot\left(1-4\cdot t^2\cdot B^2\right)}$$

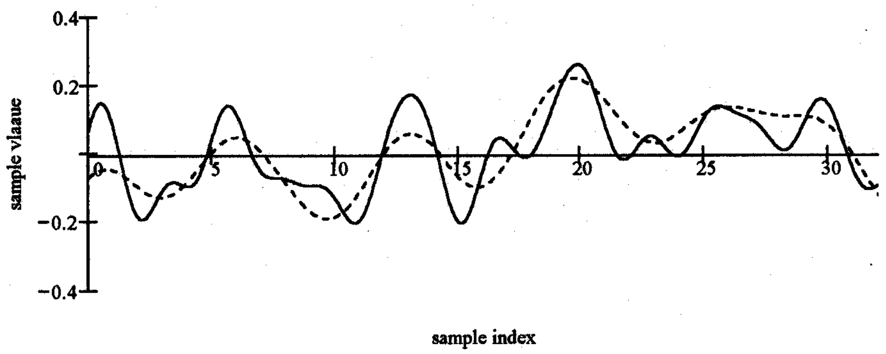


—— B=0.5
----- B=0.25



—— B=0.5
----- B=0.25

Autocorrelation Function

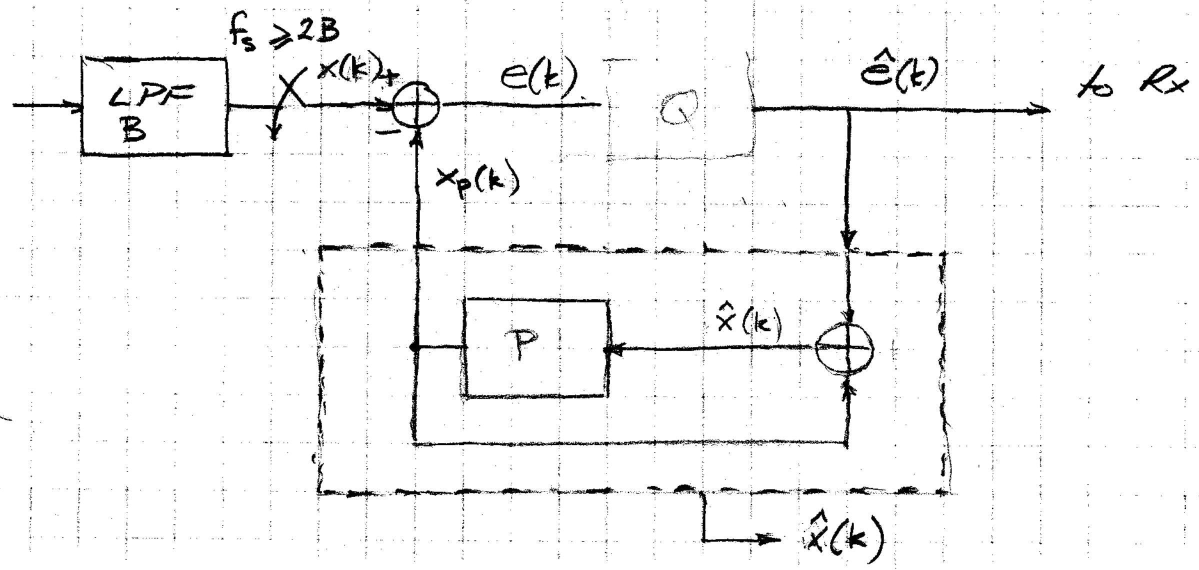


—— B=0.5
----- B=0.25

Typical Sample Functions

Two Different Filters, same White Noise Input

- To reduce the number of bits needed, predict the value of the sample from the previous samples. The receiver can do this, too, so just quantize and send the prediction error — it has lower variance than the sample.



- Principles:
  - The quantizer can be uniform or companded.
  - Given a prediction $x_p(k)$, the sample itself is just
    $x(k) = x_p(k) + e(k)$ or $\hat{x}(k) = x_p(k) + \hat{e}(k)$ (Rx version)
  - The prediction is based on samples reconstructed from the quantized $\hat{e}(k)$, so that Tx and Rx don't drift apart.
  - Rx is a direct copy of the dotted box above.
  - Because prediction error variance σ_e^2 is smaller than σ_x^2, we can use fewer bits in quantizer to get same quality of $\hat{x}(k)$ or more quality with same number of bits

- The predictor can take a variety of forms:
  - zero order predictor $x_p(k) = \hat{x}(k-1)$
  - first order predictor $x_p(k) = \frac{R_x(t_s)}{R_x(0)} \hat{x}(k-1)$
  - two point or higher predictor

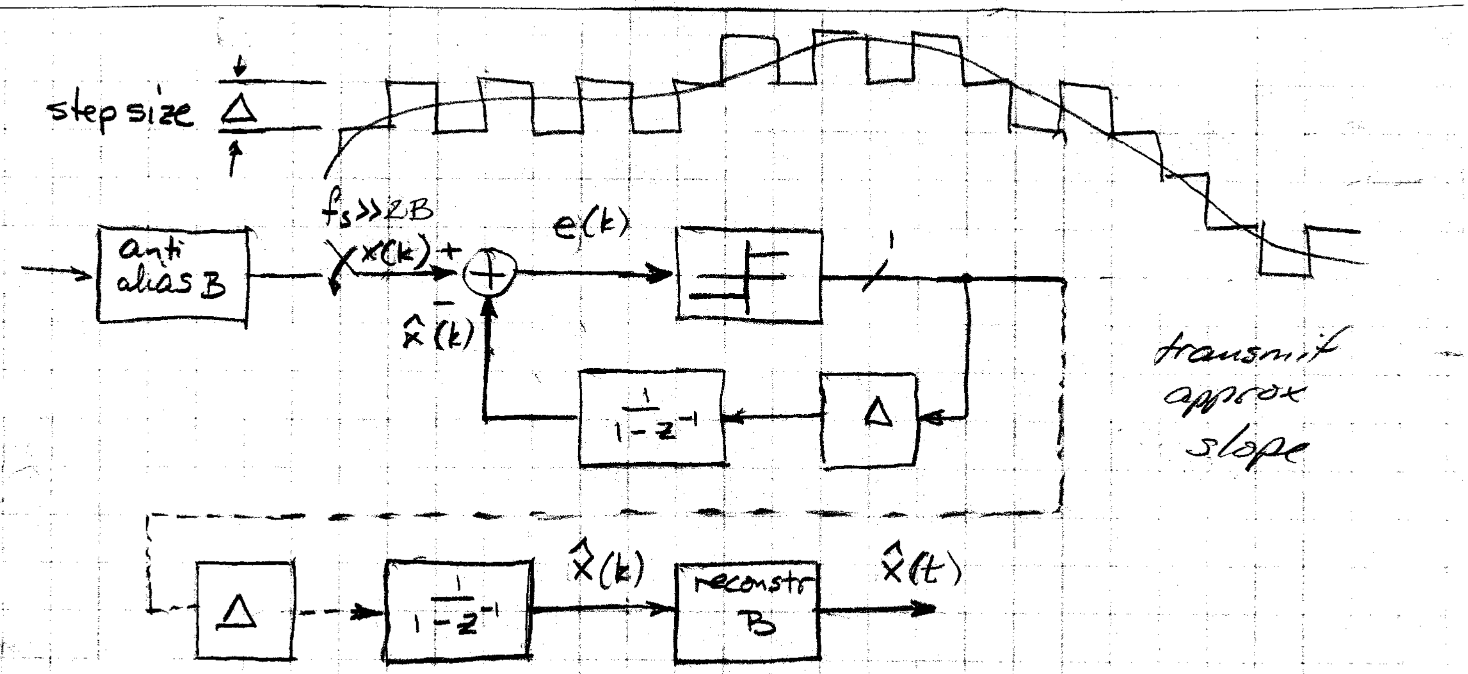
    $$x_p(k) = a_1 \hat{x}(k) + a_2 \hat{x}(k-2) + \cdots + a_n \hat{x}(k-n)$$

    Coefficients selected to minimize conditional variance (mean squared error) of prediction error.
    "Yule Walker equations"
- DPCM is widely used for coding speech

Delta Modulation

P+S 4.6.3

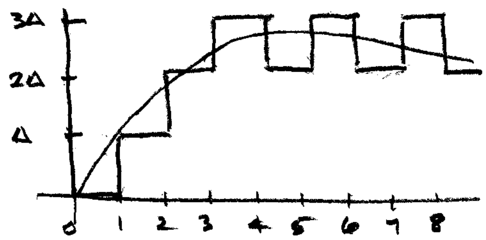
- In the same spirit as DPCM, we encode sample-to-sample differences — but we sample much faster than Nyquist and use only a 1-bit (two level) quantizer. The result is a staircase-like approximation.



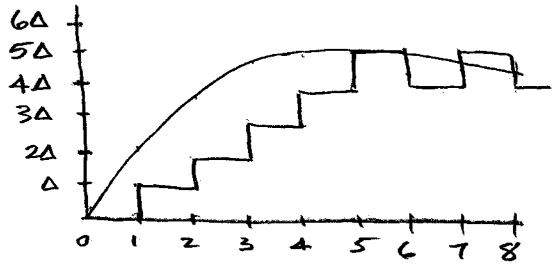
Note coder generates a replica of the signal reconstructed by the decoder (ignoring channel errors), as in DPCM

ΔM gives "toll quality" (64 kb/s PCM quality) at 32 kb/s.

- Choice of step size Δ is important:



Δ too large: coarse representation, especially in low slope and dithering regions



Δ too small: slope overload.

- Adaptive Δ mod adjusts the step size in response to changes in slope. The principle again: the coder and decoder both base adaptation on the <u>transmitted</u> signal in order not to drift apart.