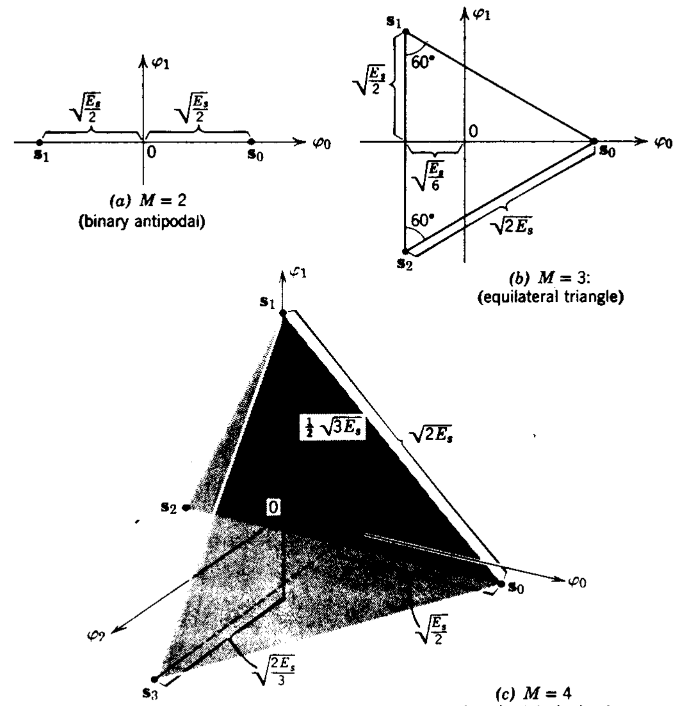
Simplex Signals

- Translate orthogonal signals so the centroid is at the origin

$$\underline{s}_c = \sqrt{E_s}\left[\frac{1}{M}, \frac{1}{M}, \cdots \frac{1}{M}\right]^T$$

$$\underline{s}_i = \sqrt{E_s}\left[-\frac{1}{M}, -\frac{1}{M}, \quad , 1-\frac{1}{M}, -\frac{1}{M}, \cdots -\frac{1}{M}\right]^T$$



(a) $M = 2$ (binary antipodal)

(b) $M = 3$: (equilateral triangle)

(c) $M = 4$ (regular tetrahedron)

Figure 4.38 Simplex signals. All s_i are at distance $\sqrt{E_s(1 - 1/M)}$ from the origin.

- We have reduced the energy by (p. 5.4.2)

$$\|\underline{\ell}\|^2 = \|\underline{s}_c\|^2 = E_s/M$$

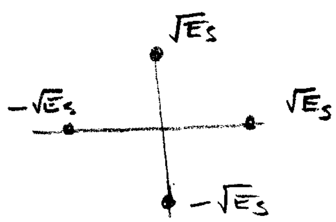
New energy is

$$E_s' = E_s\left(1 - \frac{1}{M}\right)$$

- One less dimension, so less bandwidth or transmit time.

Biorthogonal Signals

- Like orthogonal, but use negative signals, too

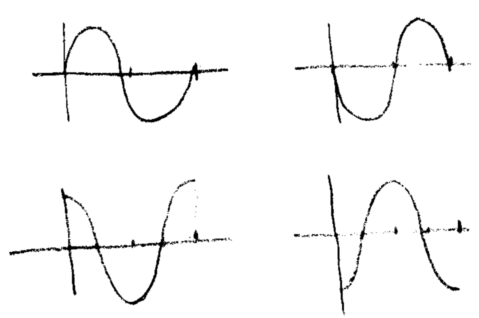


Double the number of signals (one more bit) with no increase in dimensionality.
Negligible increase in error rate.

$\underline{s}_1$	$\underline{s}_2$	$\underline{s}_3$	$\underline{s}_4$
$\sqrt{E_s}$	$-\sqrt{E_s}$	0	0
0	0	$\sqrt{E_s}$	$-\sqrt{E_s}$

example

4-PSK



5.4.15

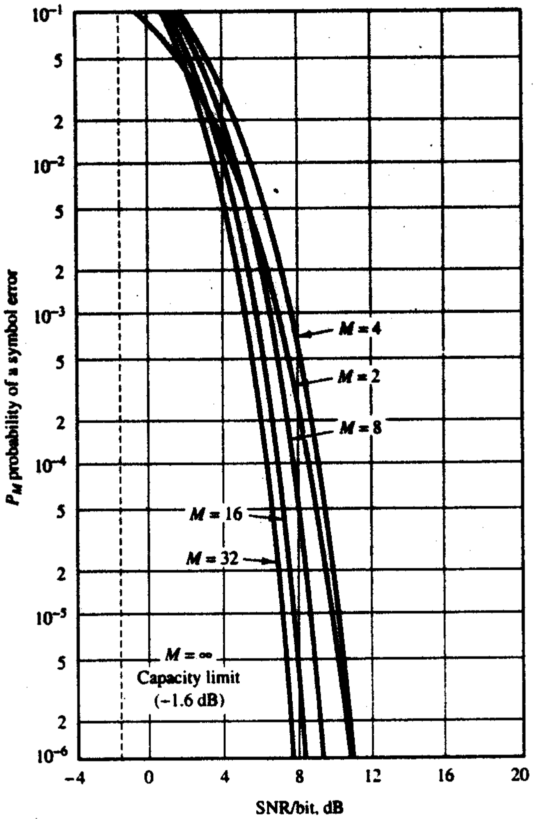
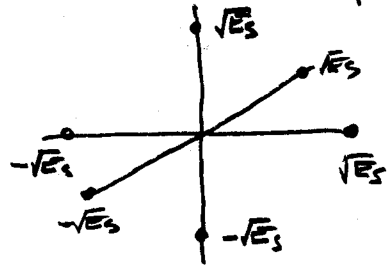


FIGURE 7.46. Probability of symbol error for biorthogonal signals.

The probability of error is messy (see P+S), but its union bound is easy.



vertices of an octohedron

A signal is equidistant from all other signals but its own complement:

$$P_s \le (M-2)Q(\sqrt{\gamma_s}) + Q(\sqrt{2\gamma_s})$$

$$\approx (M-2)Q(\sqrt{\gamma_s}), \text{ large SNR}$$

Vertices of a Hypercube

- At last - an easy one.

$$\underline{s} = \begin{bmatrix} \pm\sqrt{E_s/N} \\ \pm\sqrt{E_s/N} \\ \vdots \\ \pm\sqrt{E_s/N} \end{bmatrix} \qquad M = 2^N$$

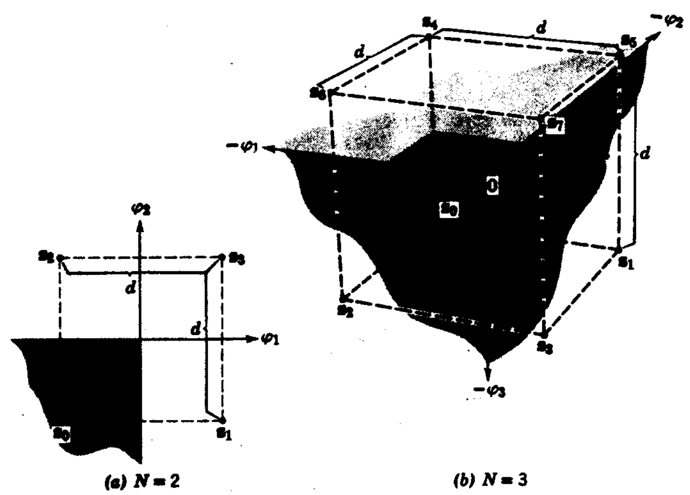
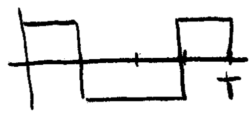
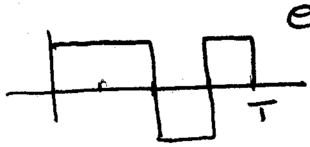


Figure 4.35 Signals on the vertices of two- and three-dimensional cubes: (*a*) $N = 2$; (*b*) $N = 3$. The decision regions I_0 are shaded.

examples

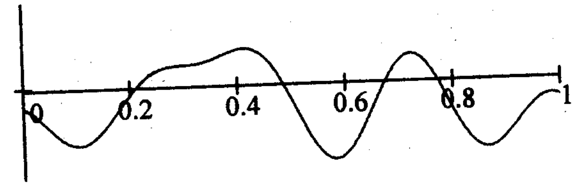


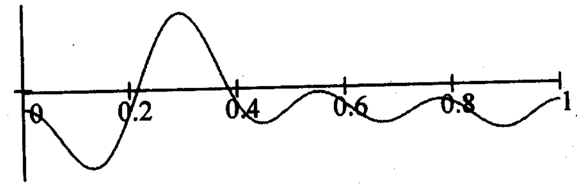
etc



$$s(t) = \sum_{i=0}^{N-1} b_i \, p(t - i\tfrac{T}{N})$$

$b_i = \pm 1$





$$s(t) = b_0 + \sum_{i=1}^{(N-1)/2} b_i \cos\left(2\pi i \tfrac{t}{T}\right) + \sum_{j=1}^{(N-1)/2} b_j' \sin\left(2\pi \tfrac{jt}{T}\right)$$

- The mapping of bit N-tuples to constellation points is straightforward: the k^{th} ± 1 bit multiplies the k^{th} dimension (i.e. basis function).
- Bits can be detected independently, since what happens in one dimension is unrelated to the others. Decide bit k using only $r_k = \pm\sqrt{E_s/N} + n_k$

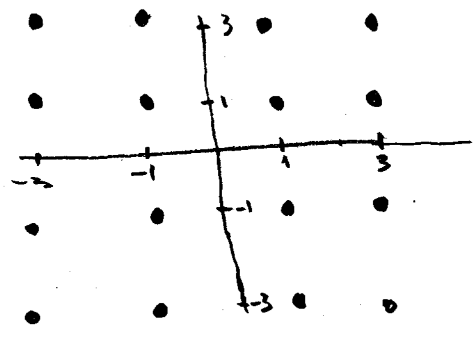
$$P_b = Q\left(\frac{\sqrt{E_s/N}}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\tfrac{2}{N}\cdot \gamma_s}\right) = Q\left(\sqrt{2\gamma_b}\right)$$

- Symbol error not usually an issue, but:
  - symbol error if any bit is in error (union bound?)
  - more sensible to say symbol correct if all bits correct, and they are independent, so

$$P_s = 1 - P(\text{symb. correct}) = 1 - \left(1 - Q\left(\sqrt{2\gamma_b}\right)\right)^N$$

QAM

- common if $N=2$ and bases are sin, cos.



16 QAM

~~It's~~ ASK applied independently on each axis. Grey code independently, too.

Gives high transmission rate without a bandwidth increase.

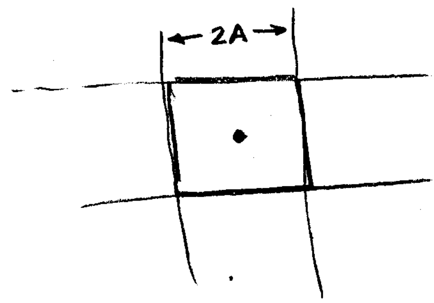
Penalty in error performance.

Easy bit detection

BER is that of ASK. SER union bounded by

From ASK (p 5.4.6)

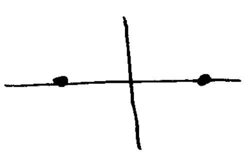
$$E_s = \frac{2}{3} A^2 (M^2 - 1)$$



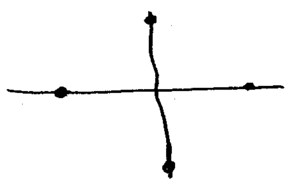
4 boundaries around a cell.

M-PSK

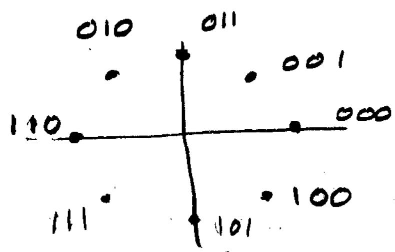
- $N=2$, cos and sin bases



BPSK



4 PSK

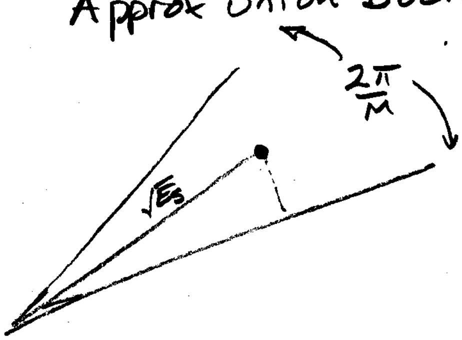


8 PSK

Grey code around the circle

High transmission rate, constant envelope.

Approx union bound on SER



$$SER \lesssim 2\,Q\left(\frac{\sqrt{E_s}\,\sin(\pi/M)}{\sqrt{N_0/2}}\right)$$

$$= 2\,Q\left(\sin(\pi/M)\sqrt{2\gamma_s}\right)$$

5.4.18

Summary

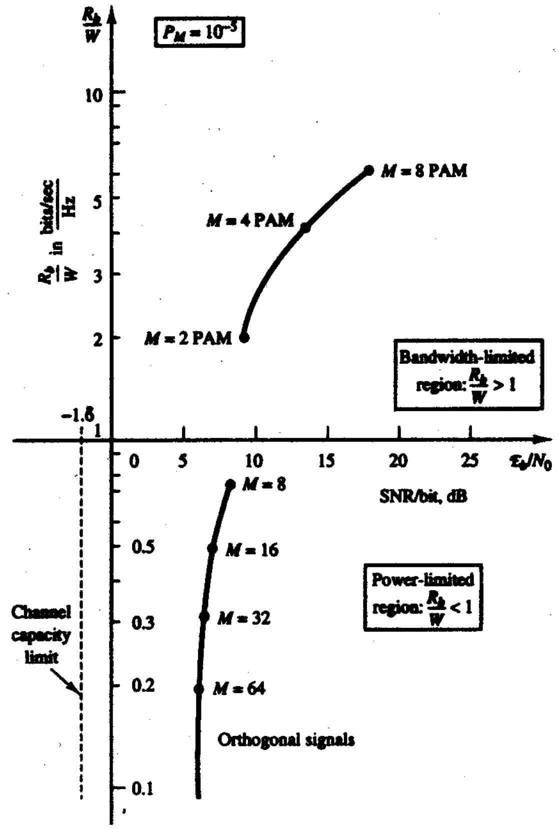


FIGURE 7.49. Comparison of several modulation methods at 10^{-5} symbol error probability.

This type of presentation is often used.

For a given BER (10^{-5}) what combinations of

γ_b SNR

$\frac{R_b}{W}$ b/s/Hz

are needed?

Note: #bits $k = \log_2 M$

Note: Sometimes "bits/dimension" instead of b/s/Hz:

$$\frac{\text{bits}}{\text{dim}} = \frac{k}{N} = \frac{k}{T} \cdot \frac{T}{N} = R_b \frac{1}{2W} = \frac{1}{2} \frac{R_b}{W} \qquad N \le 2WT$$

$$\frac{R_b}{W} = 2 \frac{k}{N}$$