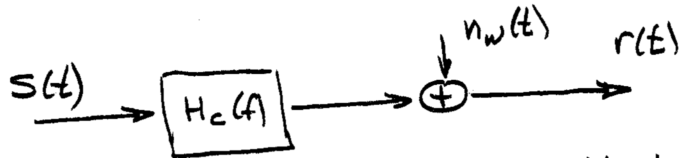
5.5 Additional Topics

- Before we rush into detecting sequences of bits, we should look at some interesting (and less detailed) topics regarding isolated pulses. Payoff: more insight.

Detection in Coloured Noise, Channel Filtering

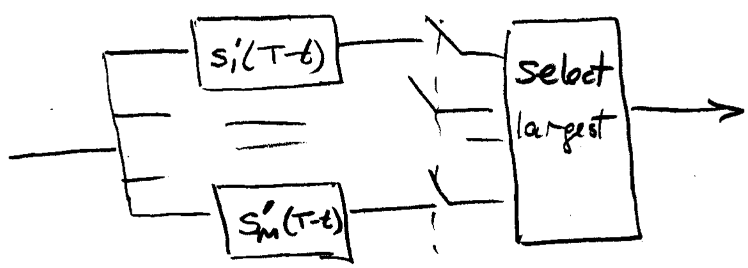
- All our results so far deal only with AWGN channels: no distortion, white noise. They are the basics, but let's revisit.
- Start with channel filtering.



The received signals don't look like the transmitted ones. What to do? Answer: the same thing.

Receive signals $s_m'(t) = s_m(t) \circledast h_c(t) \quad m = 1, .., M$

So match to $s_m'(t)$



signal:

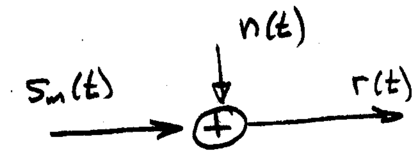
$$S_m'(f) = S_m(f) H_c(f)$$

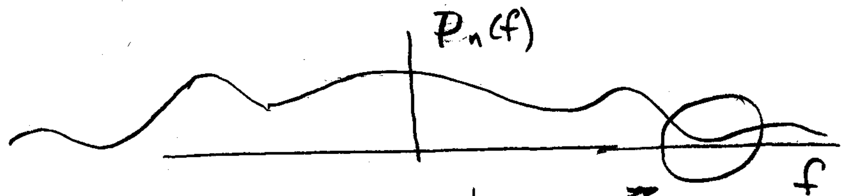
matched filters:

$$S_m^*(f)\, H_c^*(f) \longleftrightarrow h_c(-t) \circledast s_m(-t)$$

- Now coloured noise:

PSD $P_n(f)$ not necessarily $\frac{N_0}{2}$

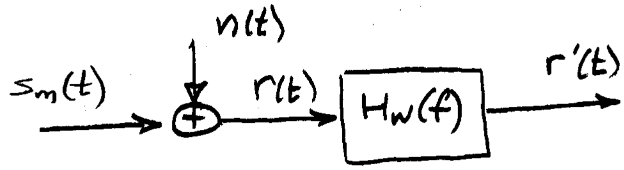




What to do?

weak noise – more weight here?

Answer: Whiten the noise first.

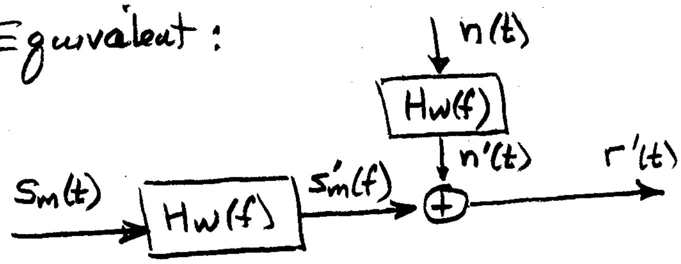


If $H_w(f)$ is invertible, no information is lost.

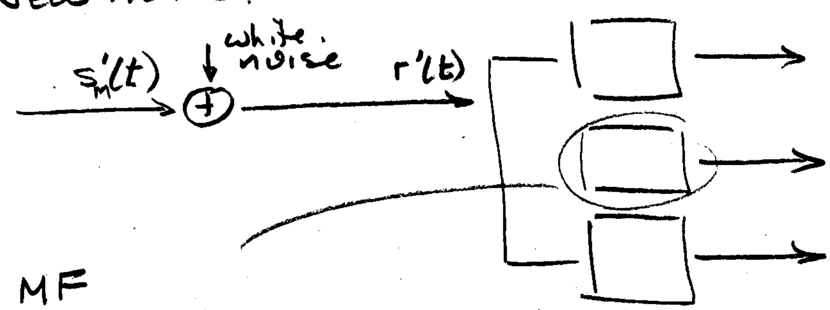
Choose $H_w(f) = \frac{1}{\sqrt{P_n(f)}} e^{j\Theta(f)}$

$e^{j\Theta(f)}$ an arbitrary phase factor.

Equivalent:



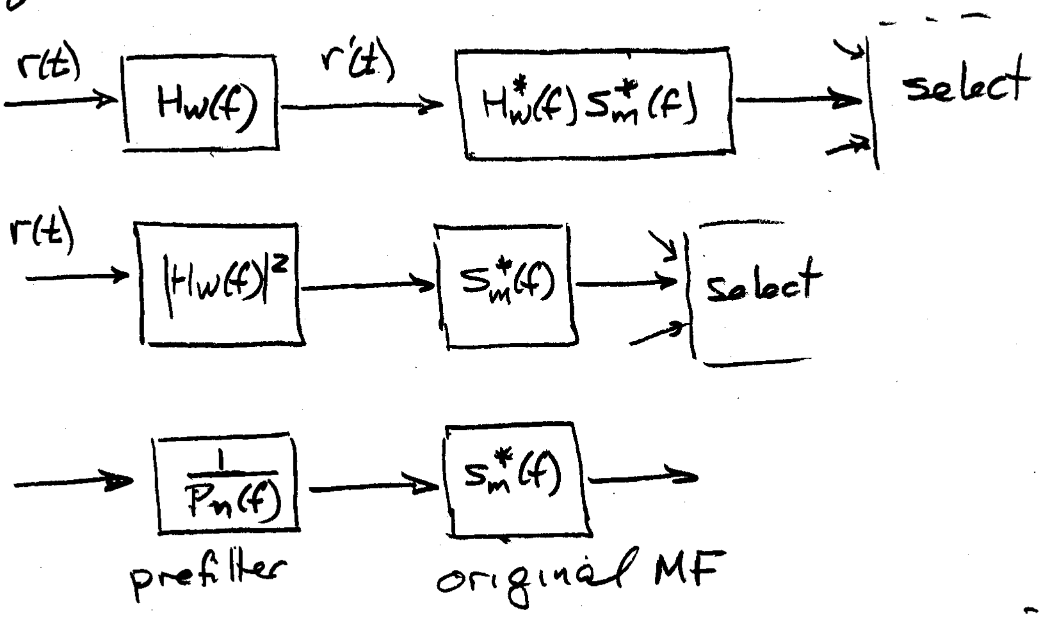
New noise PSD $P_{n'}(f) = P_n(f)\,|H_w(f)|^2 = 1$



MF

$S_m^*(f)\,H_w^*(f)$

The equivalent processing is

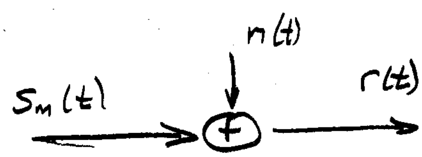


What if the noise PSD is zero over some interval?

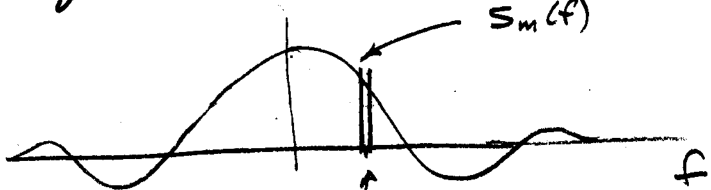
Interpretation of Matched Filter

- Here's a frequency domain interpretation that links to SNR maximization.

- Transmit a known signal $s_m(t) \longleftrightarrow S_m(f)$, receive in noise with PSD $P_n(f)$.

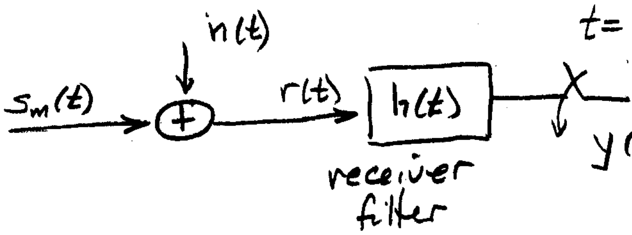


In frequency, it looks like this



In bin df at f, we get:
- signal amplitude $S_m(f)\,df$
- noise var $P_n(f)\,df$

- the noises in different bins are uncorrelated.



$$y(0) = \int r(t)h(-t)\,dt = \int R(f)H(f)\,df$$

Think of the filter as applying weights to different frequencies and summing (integrating). It's similar to $\sum w_i x_i$ where $x_i = m_i + n_i$

If the noise variances are equal ($P_n(f)$ is flat) then best weights are $w_i = m_i^*$

or $H(f) = S_m^*(f)$ the MF!

If noise variances are unequal, $w_i = \dfrac{m_i^*}{\sigma_i^2}$

or $H(f) = \dfrac{S_m^*(f)}{P_n(f)}$ the MF for coloured noise!

Maximum Likelihood Detection

- We have been making MAP decisions, which balance the channel statistics $f_{\underline{r}|\underline{s}_i}(\underline{r}|\underline{s}_i)$ and the a priori signal statistics $P(s_i)$:

$$\max_i f_{\underline{r}|\underline{s}_i}(\underline{r}|\underline{s}_i)P(\underline{s}_i)$$

- What if we don't know the prior probabilities $P(s_i)$ or if they don't have a statistical characterization?

  Maximum Likelihood criterion:

$$\max_i f_{\underline{r}|\underline{s}_i}(\underline{r}|\underline{s}_i)$$

  Choose the signal that is most likely to have produced our measurement. Equivalent to a uniform distribution for $P(s_i)$ — a common, but not universal, case.

- For Gaussian, from p. 5.3.7

$$\max_i -|\underline{r}-\underline{s}_i|^2 \quad \text{or} \quad \min_i |\underline{r}-\underline{s}_i|$$

  Choose signal point closest to our measurement.