5.6 Section Summary

This section introduced several important concepts — and a lot of detail! Here is a summary of the conceptual "stepping stones" to help you keep an eye on the forest.

- Many mathematical structures can be described in the same terms, equivalent to vectors in Euclidean space.
- If a message consists of one of a finite number M of alternative waveforms, and if it is received in AWGN, it can be vectorized to a set of sufficient statistics by calculating the inner products of $r(t)$ with
  - the set of waveforms $s_m(t)$, $m = 1..M$
  - a set of orthonormal basis waveforms of the signal space $\psi_i(t)$, $i = 1..N$
  - any basis of the signal space. $v_k(t)$, orthonormal or not.

  The vectors so obtained are related by simple linear transformations, e.g.

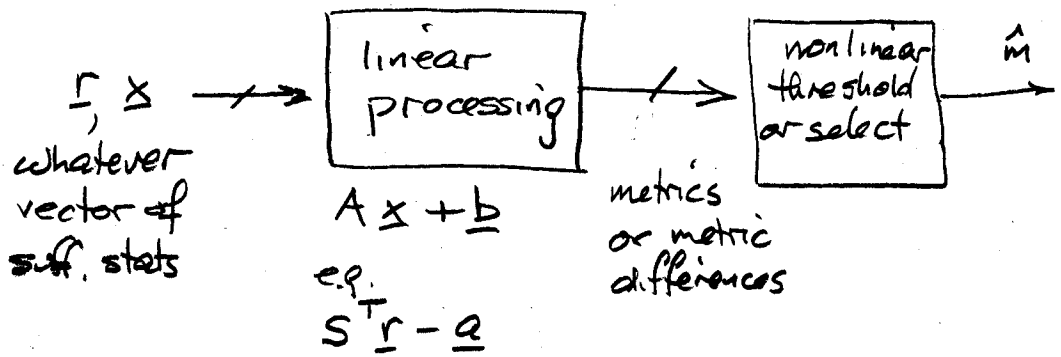
  $$\underline{r} = A\,\underline{x}$$ where neither dimension of A is less than N

- The vector $\underline{r}$ of components w.r.t. some orthonormal basis set has the properties:
  - Its length $\sqrt{\sum_i r_i^2}$ equals $\sqrt{\text{energy}}$ of corresponding signal
  - The noise components are i.i.d. Gaussian, variance $N_0/2$
- The inner products can be implemented with correlators or matched filters.

- MAP decision making with the vector components

  $$\max_i f_{S|r}(\underline{s}_i|\underline{r})$$

  is always equivalent to $\max_i f_{\underline{r}|\underline{s}_i}(\underline{r}|\underline{s}_i)P(\underline{s}_i)$.
  Different types of channels (AWGN, coloured noise, fading, photonic, etc) have their own $f_{\underline{r}|\underline{s}_i}(\underline{r}|\underline{s}_i)$.
- For AWGN, MAP decision making always has the structure



- For equiprobable signals, MAP (and ML) processing (AWGN) is equivalent to selecting the signal vector $\underline{s}_m$ nearest $\underline{r}$.

- Different constellations and basis waveforms have different error performance and bandwidth or transmit time requirements.

- Rotation and translation of a constellation has no effect on error probability, but translation may affect average transmit energy.
- If $M>2$ then distinguish between P_s and P_b, γ_s and γ_b. Mapping of bit k-tuples to constellation points affects BER.
- There are roughly $2W$ dimensions/sec available from bandwidth W; roughly $2T$ dimensions/Hz available in T seconds; roughly $2WT$ dimensions in W and T.