6 BASEBAND SIGNALLING WITH PULSE SEQUENCES

- Section 5 established the core theory of detecting one of a few discrete alternative pulses in white noise.

  When we have to transmit a long sequence of bits, the landscape changes, even though much of the basic theory remains as a guide.

- Now we have to deal with:
  - overlapping and mutual interference of successive pulses
  - modulation with memory
  - determining where pulses begin and end (if they do) and when to sample the receiver filter

- You will gain a basic understanding of how to formulate and deal with all of these problems — but there is much more to learn than this course can accommodate

6.1 Statistics and Sufficient Statistics $\boxed{\text{P+S } 8.1.1}$

- To send a sequence of bits, we normally use a sequence of pulses, each carrying a few bits by having been selected from a set of alternatives

$$\text{data} \longrightarrow \boxed{\text{modulator}} \longrightarrow v(t) = \sum_{k=-\infty}^{\infty} a_{i(k)}\, g_{i(k)}(t-kT)$$

T symbol spacing, pulse repetition interval

$g_i(t)$ i^{th} of M alternative pulses

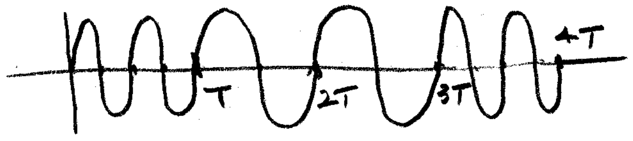
$g_{i(k)}(t-kT)$ the pulse sent in the k^{th} interval

$a_{i(k)}$ the amplitude applied in interval k if the selection was i

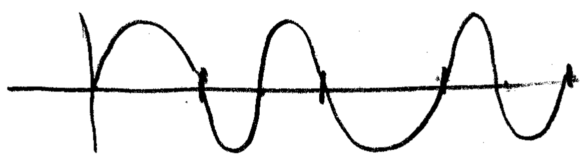
- Includes some easy ones.
  - PAM $\quad v(t) = \sum_{k=-\infty}^{\infty} a_k\, g(t-kT) \quad a_k \in \{-M+1, \ldots -3, -1, 1, 3, \ldots M-1\}$

    same pulse shape every time, amplitude a_k carries the information for interval k

  - 2FSK

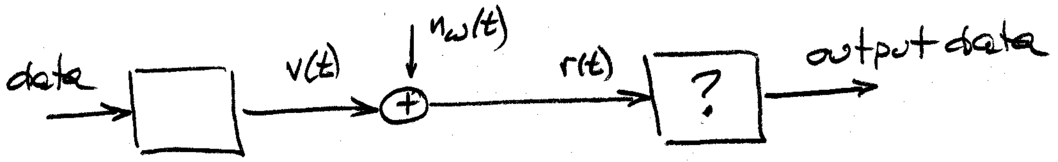


choice of pulse shape carries the information, same choice always



choice of pulse again, amplitude changes for phase continuity.

- How can we detect them in white noise?



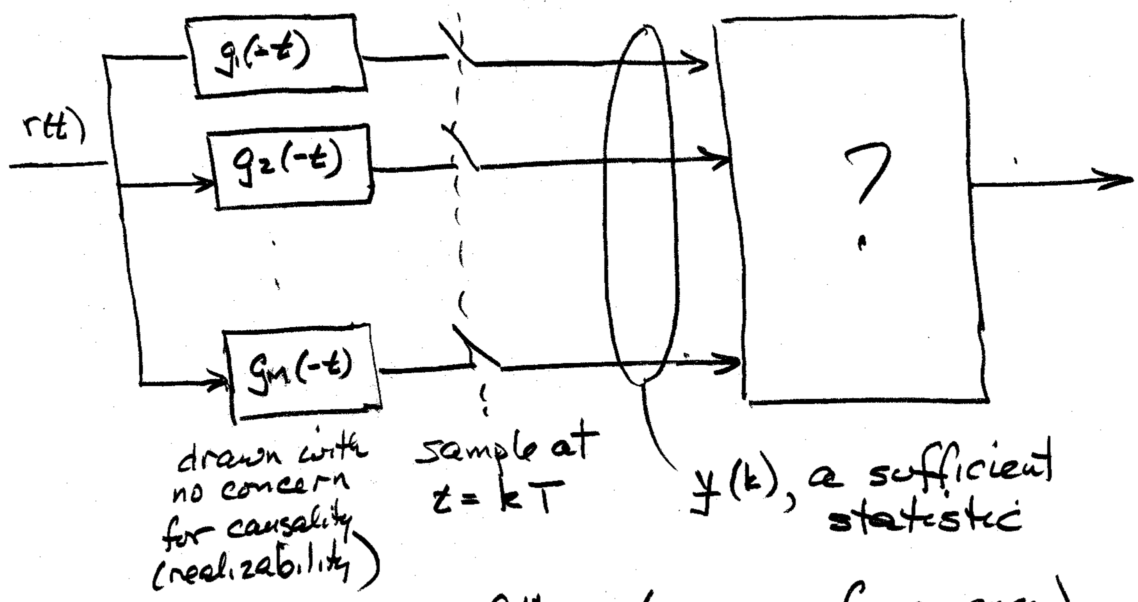
First step – vectorize. The principles of Section 5 help us here.

- The signal is a linear combination of basis functions (not necessarily orthonormal)

  $g_i(t),\ g_i(t-T),\ g_i(t-2T) \dots \qquad i = 1, \dots M$

  How many? $M \times$ number of pulses.

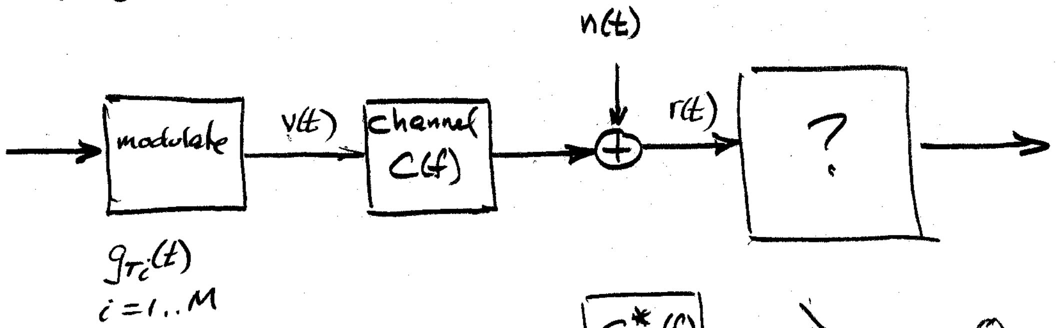
- So take the inner products, discarding noise outside signal space. No loss wrt detection
- If MF implementation,

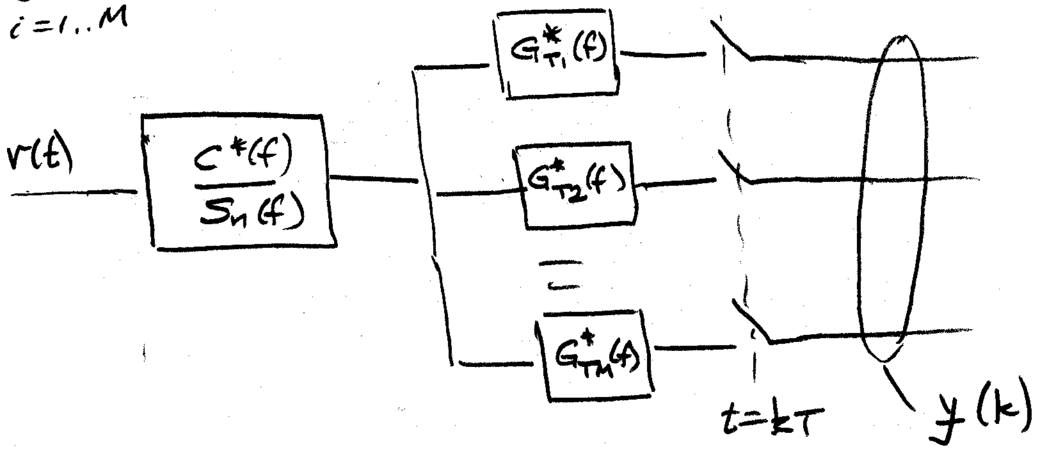


- In M-FSK, use M filters (one per frequency)
  In PAM, use 1 filter

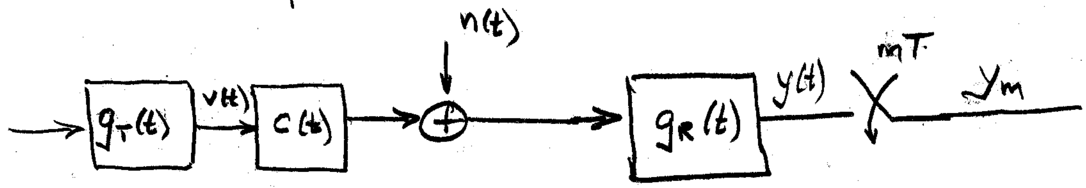
- Correlators a more common implementation.
  Just stagger them.

- With channel distortion and coloured noise





- Statistics of the samples for PAM



for MF,
$g_R(t) = g_T(t) \circledast c(t)$

- Signal component

$$v(t) = \sum_k a_k\, g_T(t-kT)$$

so

$$y(t) = \sum_k a_k\, x(t-kT) \qquad \begin{aligned} x(t) &= g_T(t) \circledast c(t) \circledast g_R(t) \\ X(f) &= G_T(f)C(f)G_R(f) \end{aligned}$$

and samples are

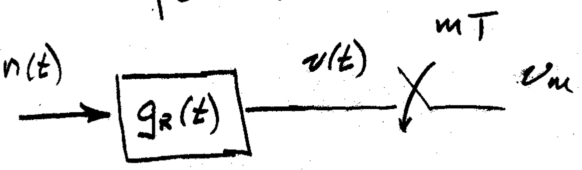
$$y(mT) = \sum_k a_k\, x((m-k)T) \quad \text{or} \quad y_m = \sum_k a_k\, x_{m-k}$$

$$x_i = \int G_T(f)C(f)G_R(f)\, e^{j2\pi i T} df$$

$$= \int |G_T(f)|^2 |C(f)|^2 e^{j2\pi i T} df \text{ if matched.}$$

The expression $y_m = \sum_k a_k x_{m-k}$ shows that sample m is determined, not just by a_m, but by other symbols. We have recovered the signal components optimally, but we can have ISI.

– Noise component



$R_v(0) = ?$

$v(t)$ is a stationary process with power spectrum

$$S_v(f) = S_n(f)\,|G_R(f)|^2 = S_n(f)\,|G_T(f)|^2\,|C(f)|^2 \quad \text{if matched}$$

$$R_v(iT) = \int S_v(f)\, e^{j2\pi f iT}\, df = \left[R_n(t) \circledast g_R(t) \circledast g_R(-t)\right]_{t=iT} = \frac{N_0}{2}\, x(iT) \quad \text{if matched}$$

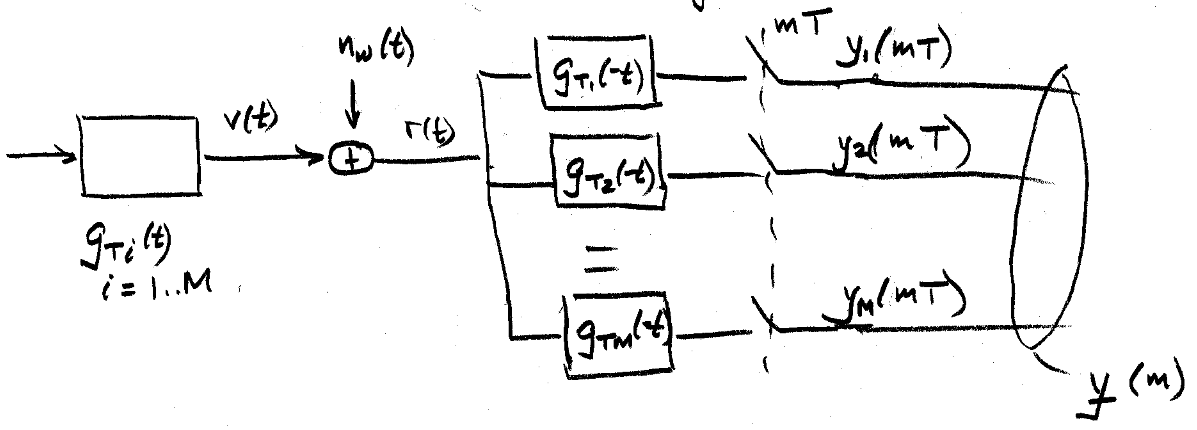
So successive samples of noise may be correlated. Like ISI, it suggests that subsequent processing may have to span several (all?) samples to recover any of the data bits.

– We have $y_m = a_m x_0 + \sum_{k \neq m} a_k x_{m-k}$

If $x_i = x_0 \delta_{i0}$ then no ISI, and if $g_R(t)$ is matched, as well, then successive noise samples are independent.

$y_m = a_m x_0 + v_m$ — like the isolated pulse, with error rates as determined in Section 5

- Stats for multidimensional signals (AWGN channel only)



– Signal components
If $g_{Ti}(t)$ was sent in interval 0, then component k of $\underline{y}(m)$ is $x_{ki}(mT) = \left[g_{Ti}(t) \circledast g_{Rk}(-t) \right]_{t=mT}$

So we may get filter k responding to transmitted pulse i (normal) but also may get "cross ISI" at other times, as well as usual ISI on one filter.

– Noise components:

$$R_{\nu k}(mT) = \frac{N_0}{2} \int |G_R(f)|^2 e^{j2\pi f mT} df$$ temporal correlation

and correlated across filters too (normal)

$$R_{\nu ki}(mT) = \frac{N_0}{2} \int G_{Rk}(f) G_{Ri}^*(f) e^{j2\pi f mT} df$$ but spatio-temporal correlations

– Because of the complexity, we often try to ensure pulses and channels such that there is no ISI and no temporal correlation. If so,

→ like the isolated pulse receiver, with error rates as determined in Section 5