6.2 Pulse Shaping for Zero ISI in PAM

$\boxed{\text{P+S 8.2}}$

- We have seen in Section 6.1 that zero ISI is desirable. Achieving it, even for the simple AWGN channel, requires care — and a compromise with the other goal of low bandwidth occupancy.

- This section addresses some of the issues in zero-ISI pulse design for PAM systems. Our model is

$$v(t) = \sum_k a_k \, g_T(t - kT) \qquad \text{and our topic is } g_T(t).$$

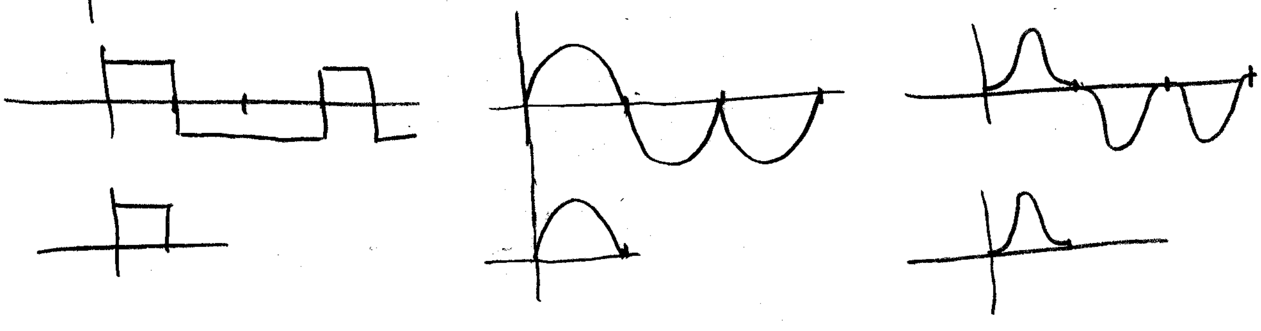
- First, bandwidth occupancy, as determined by the power spectrum $S_v(f)$. For zero mean, iid symbols a_k, we have seen that (p. 2.4.6)

$$S_v(f) = \frac{\sigma_a^2}{T} \, |G_T(f)|^2$$

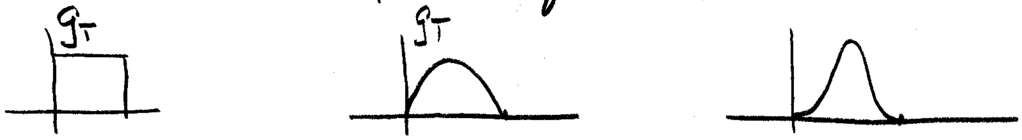
$\boxed{\text{Read P+S 8.1.2}}$ for the more general version with non zero mean. We'll do correlation of a_k later.

Single-Interval Pulses

- The easiest way to guarantee zero ISI (in absence of channel filter) is a pulse limited to a single interval:



- Pay a bandwidth price for this method.
  - the order of derivative containing the first discontinuity determines decay of frequency sidelobes:



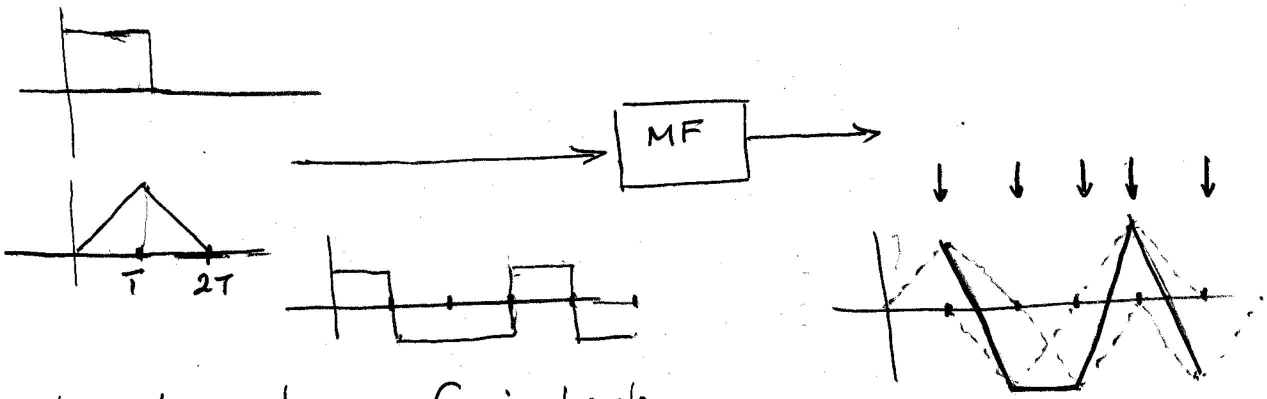
$S_v(f) \sim \text{sinc}^2(fT) \sim \frac{1}{f^2}$

$S_v(f) \sim \frac{1}{f^4}$

$S_v(f) \sim \frac{1}{f^6}$ but the narrower pulse broadens the main lobe of $S_v(f)$.

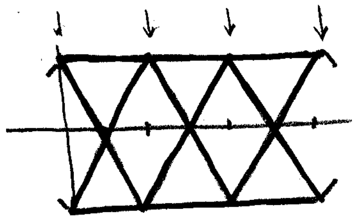
- Why is the ISI zero?

Consider the MF output $R_{g_T}(t)$ (pulse autocorr'n f'n)



The pulse autocorr'n f'n is back to zero for the next sample.

On a scope with trigger set properly:



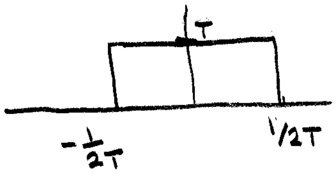
an "eye diagram"

Sinc Pulses

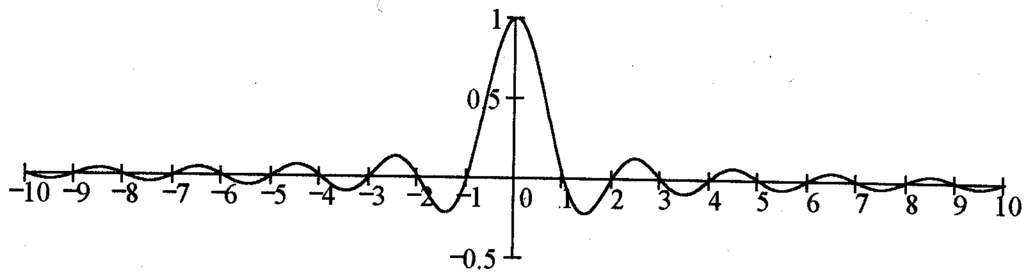
- An inspiration — sinc pulses have regular zero crossings and are bandlimited! Looks like we can have it all!

$$v(t) = \sum_k a_k \,\mathrm{sinc}\left(\tfrac{1}{T}(t-kT)\right) = \sum a_k \,\mathrm{sinc}\left(\tfrac{t}{T}-k\right)$$

- But... it's hard to implement

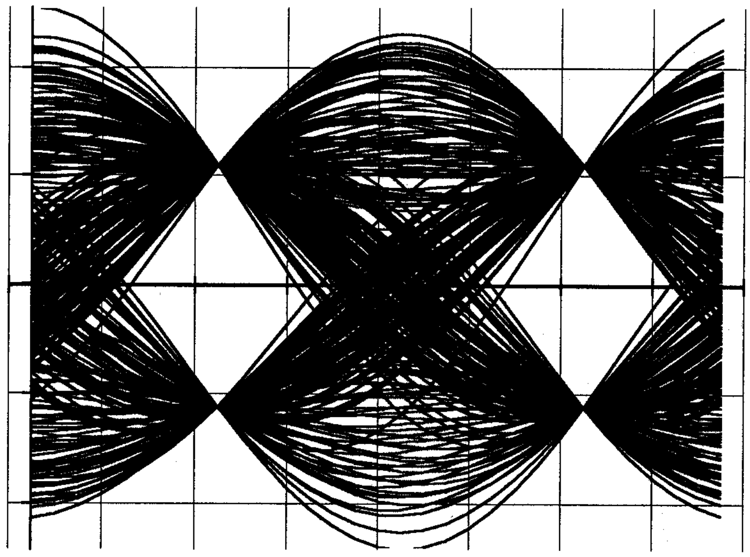


same shape, Tx and Rx



dies out slowly

and the signal can be large between the sample points.



$y_m = a_m x_0 + v_m$

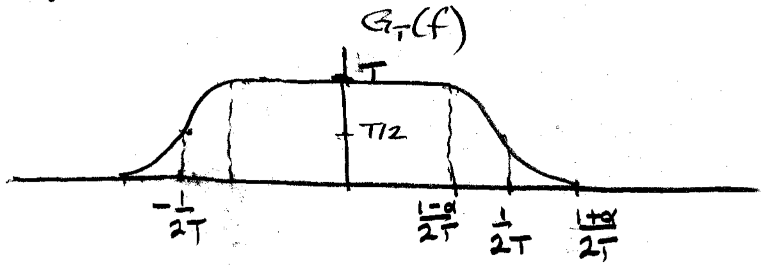
and it's hard to synchronize to sinc pulses,

and a little synch error costs a lot of noise margin

eye diagram for sinc pulses

The Spectral Raised Cosine Family

- This is the most widely-used method in signalling today. DSP made use of it possible.
- Compromise — use a little extra bandwidth to obtain a smooth $G_T(f)$ and a $g_T(t)$ that dies out more quickly.



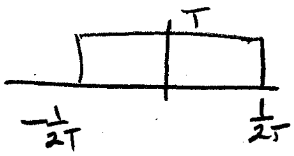
Raised cosine rolloff factor α $0 \le \alpha \le 1$

$$G_T(f) = T \begin{cases} 1 & |f| \le \frac{1-\alpha}{2T} \\ \frac{1}{2}\left(1 - \sin\left(\frac{\pi T}{\alpha}\left(|f| - \frac{1}{2T}\right)\right)\right), & \frac{1-\alpha}{2T} \le |f| \le \frac{1+\alpha}{2T} \\ 0, & |f| \ge \frac{1+\alpha}{2T} \end{cases}$$

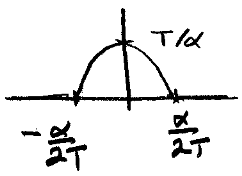
The corresponding $g_T(t)$ can be obtained by inverse transforming to obtain

$$g_T(t) = \operatorname{sinc}(t/T)\,\frac{\cos(\pi\alpha t/T)}{1 - 4(\alpha t/T)^2}$$

Alternatively, note that $G_T(f)$ is convolution of



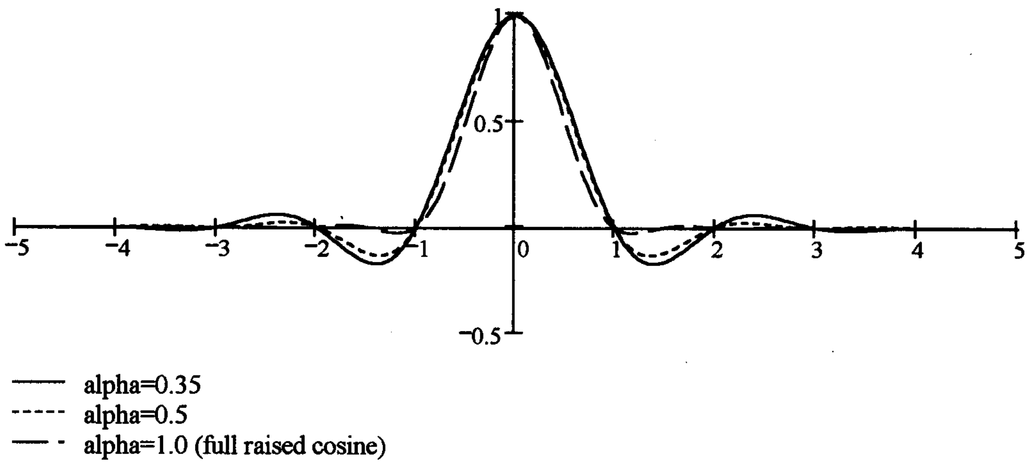
and



$\frac{T}{\alpha}\cos\left(\pi\alpha\frac{t}{T}\right)$

giving

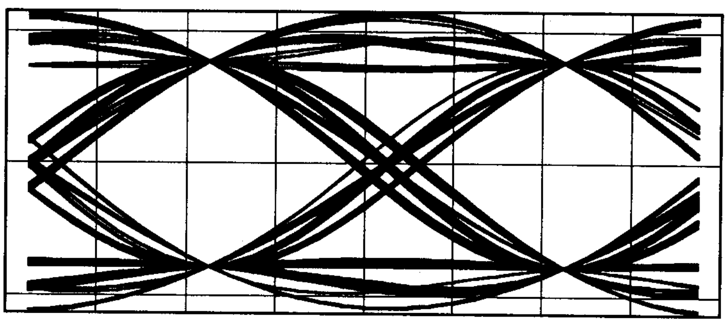
$$g_T(t) = \frac{\pi}{4}\operatorname{sinc}\left(\frac{t}{T}\right)\left(\operatorname{sinc}\left(\alpha\frac{t}{T} + \frac{1}{2}\right) + \operatorname{sinc}\left(\alpha\frac{t}{T} - \frac{1}{2}\right)\right)$$



Raised Cosine Pulses, Different Rolloffs

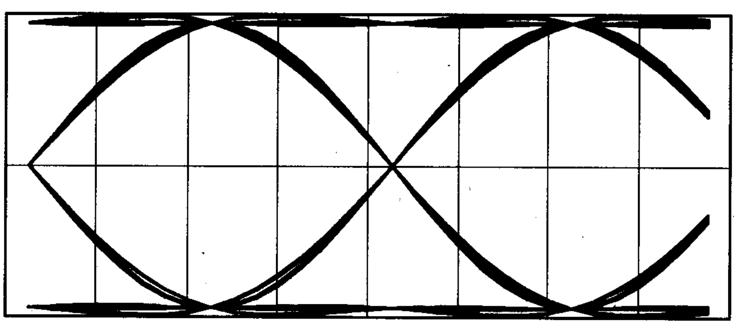
- regular, zero crossings
- dies out quickly especially for $\alpha = 1$, so easier to implement than $\alpha = 0$ (sinc function).

- The eye diagram is cleaner than for sinc pulses:



Rolloff Factor

$\alpha = 0.5$

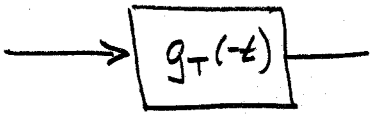


Rolloff Factor

$\alpha = 1$

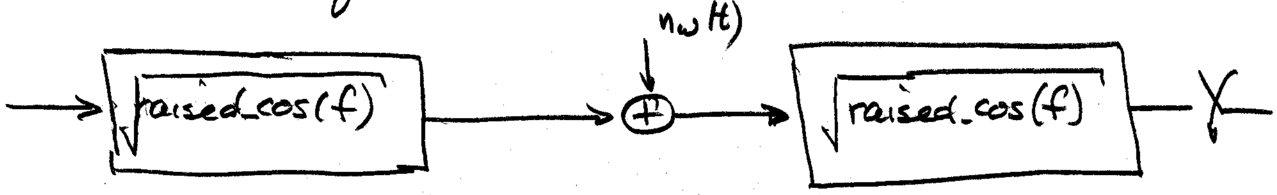
Less dynamic range required, less sensitivity to timing error.

- Wait a minute! We still have to put the pulse through a matched filter

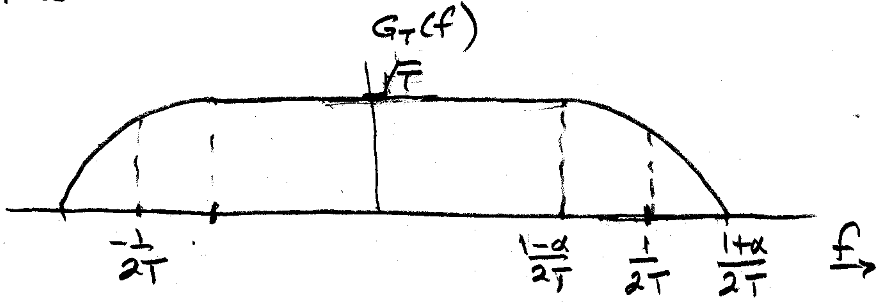


and $g_T(t) \circledast g_T(-t)$ loses its zero ISI property.

Solution: the square root raised cosine pulse shape



This is matched, for good noise performance, and has a composite pulse shape that equals the raised cos.

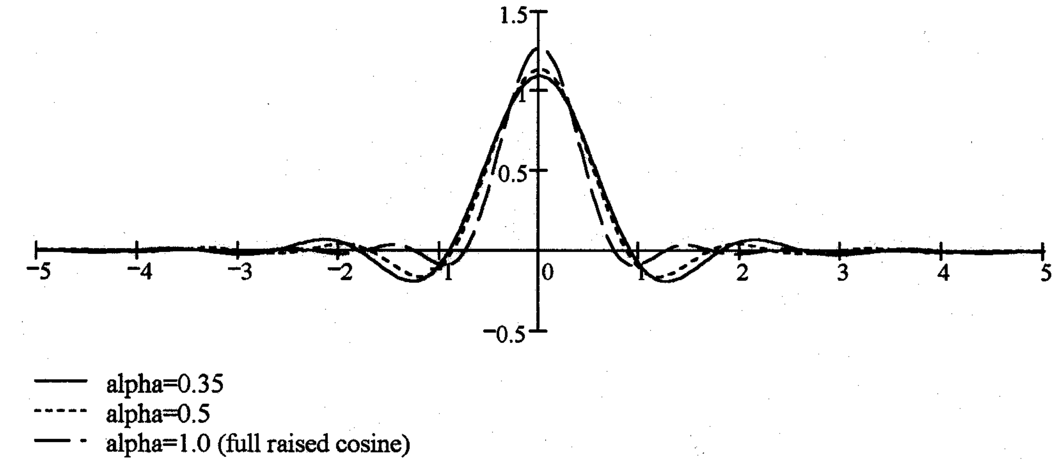


$$G_T(f) = \sqrt{T} \begin{cases} 1, & |f| \le (1-\alpha)/2T \\ \cos\left(\frac{\pi T}{2\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right), & \frac{1-\alpha}{2T} \le |f| \le \frac{1+\alpha}{2T} \\ 0, & |f| \ge (1+\alpha)/2T \end{cases}$$

for which [S. Chennakeshu, VTC'91]

$$g_T(t) = \frac{1}{\sqrt{T}} \begin{cases} 1-\alpha+\frac{4}{\pi}\alpha, & t = 0 \\ \frac{\alpha}{\sqrt{2}}\left[\left(1+\frac{2}{\pi}\right)\sin\left(\frac{\pi}{4\alpha}\right)+\left(1-\frac{2}{\pi}\right)\cos\left(\frac{\pi}{4\alpha}\right)\right], & |t| = \frac{1}{4\alpha} \\ \dfrac{\sin(\pi(1-\alpha)|t|) + 4\alpha|t|\cos(\pi(1+\alpha)|t|)}{\pi|t|\left(1-(4\alpha t)^2\right)} & \text{elsewhere} \end{cases}$$

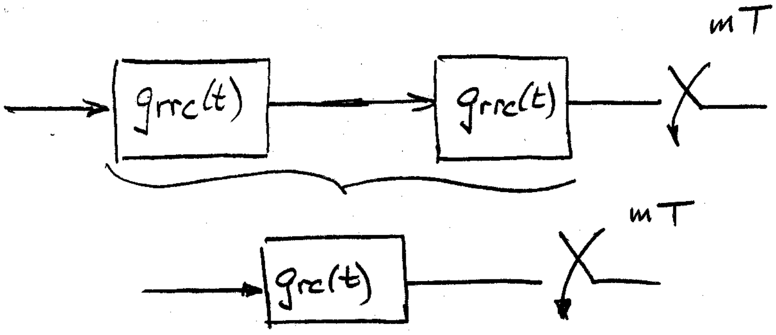
and here it is, the most widely used pulse shape in communications.



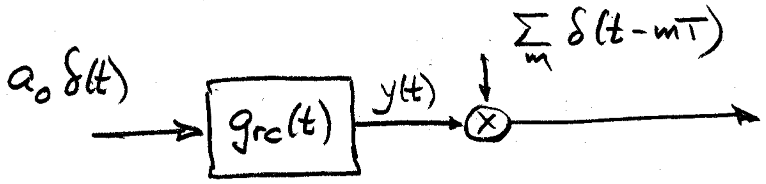
Square Root Raised Cosine Pulses

Nyquist's Criterion

- What was it about the raised cos pulse that gave zero ISI?



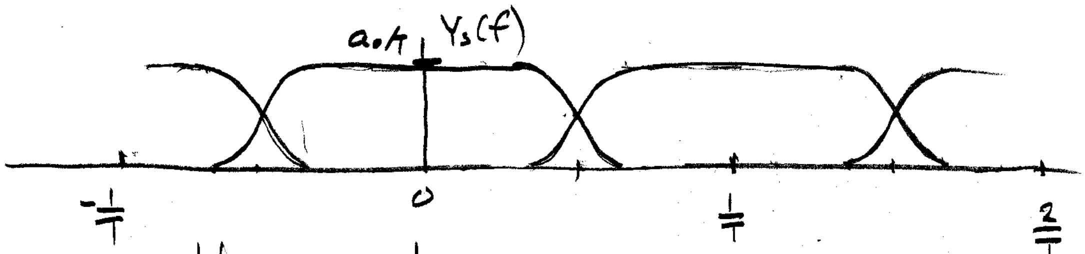
- Model as impulse weighted data, impulse sampling



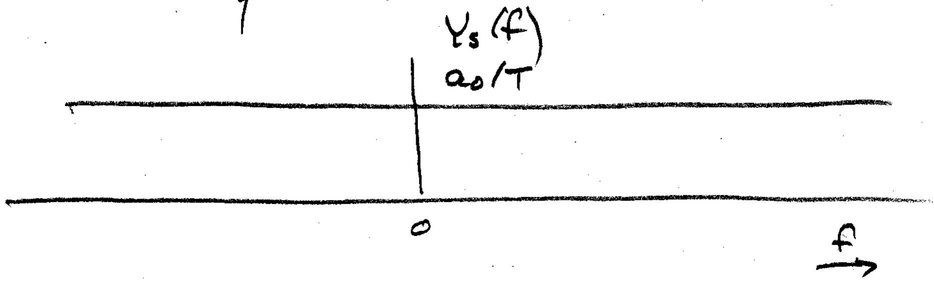
$$y_s(t) = a_0\, g_{rc}(t) \sum_m \delta(t-mT)$$

From sampling theory,

$$Y_s(f) = \frac{a_0}{T} \sum_k G_{rc}(f - k/T)$$



they sum to



because of the complementary symmetry

so $y_s(t) = \frac{a_0}{T} \delta(t)$ and no ISI

$\boxed{\text{Read P+S 8.2.1}}$