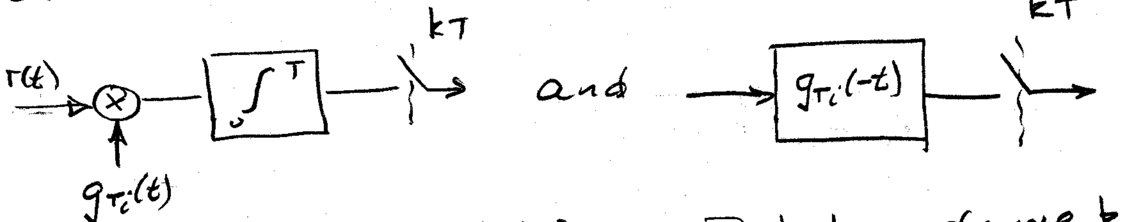
6.3 Synchronization I: Timing Recovery

P+S 8.5

- So far we have assumed that sampling in



takes place at the correct times. But how do we know those correct times? Somehow we have to determine the sampling times from the noisy received signal

$$r(t) = \sum_k a_k \, g_T(t - kT - \tau_0) + n(t) \qquad \text{PAM only}$$

where τ_0 reflects the unknown delay. Usually $|\tau_0| \le T/2$, since integer valued bit slips are taken care of separately by "unique words".

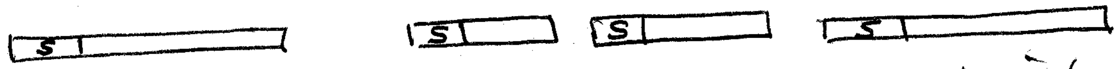
- The required timing accuracy can be gauged from the eye diagram and even analyzed by

$$P_b = \int_{-T/2}^{T/2} P_b(\tau_0)\, f_\tau(\tau_0)\, d\tau_0$$

but generally, $\sigma_{\tau_0}/T \le 0.05$ is an outer limit.

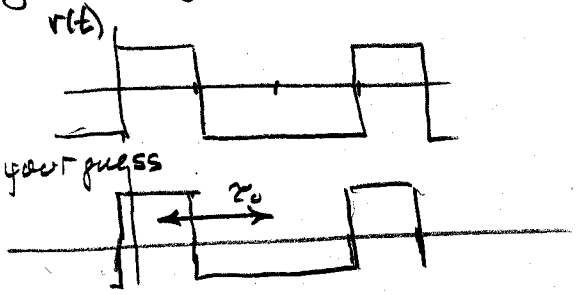
- In one sense, it's easier than bit detection: τ_0 usually drifts <u>very</u> slowly, so you can average over hundreds of bits.

- Burst synch: In some applications, messages are short and appear at arbitrary times, possibly from different users



In these cases, you must obtain an adequate estimate $\hat{\tau}_0$ from a prefix, then flywheel or track to the end of the message. We won't cover this

- It often helps timing recovery if you know what the transmitted data a_k sequence was, so you can get a good timing match



This might occur in a synch word. Such approaches are "data aided," (DA).

If you don't <u>know</u> the data, you can use DA with the decisions (except at useless error rates). This is "decision directed" (DD)

Methods that don't use the data or its estimates are "non data aided" (NDA). Generally noisier or longer time constant, but more robust.

- There are <u>many</u> methods, but we'll look at two:
  - max sample power (NDA)
  - max likelihood (DA or DD)

The ML approach is fertile ground for new methods or for new modulation formats. Often other methods can be interpreted as approximations of ML.

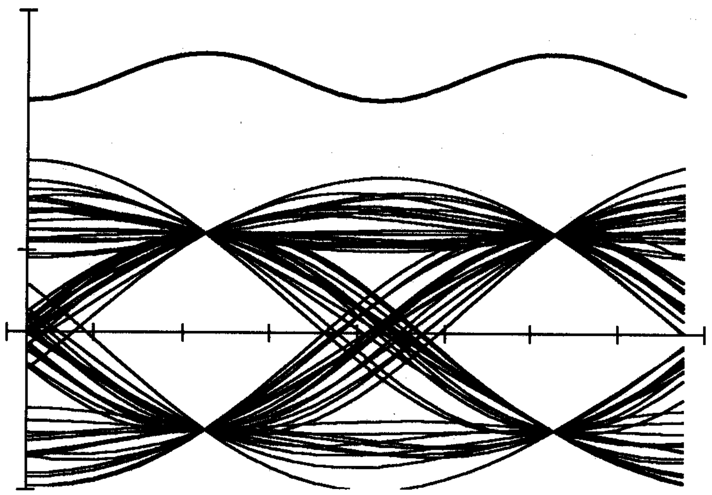
Maximum Sample Power

- Because data signal is cyclostationary, applying a nonlinearity often produces a periodic component (a spectral line) that we can track.

- The post MF signal is

$$y(t) = \underbrace{\sum_k a_k \, x(t - kT - \tau_0)}_{v(t) \text{ (a change of notation)}} + \nu(t)$$

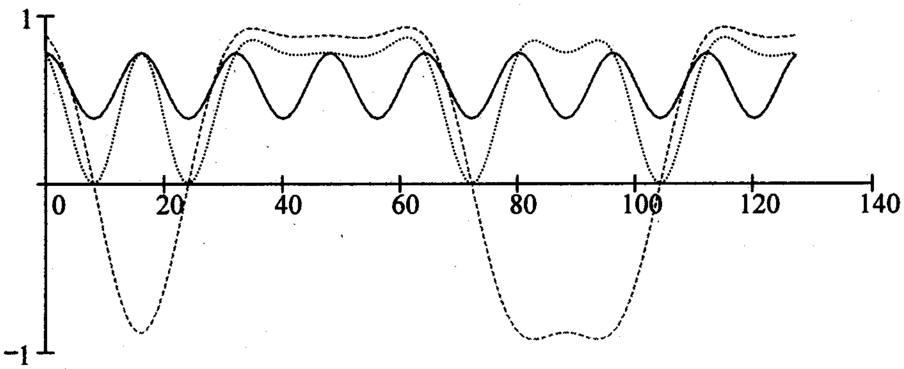
We want to sample where $P_y(t) = E_{\underline{a}}\left[y^2(t) \right]$ is largest.



MF output y(t)
expected power Py(t)
$\alpha = 0.35$

In the absence of channel filtering, the optimum timing is $t = mT + \tau_0$

But we don't observe $P_y(t)$ directly, just $y^2(t)$, so some processing is needed.



The periodic component of $P_y(t)$ is stronger for $\alpha = 1$. It can easily be recovered with a resonant circuit or a PLL.

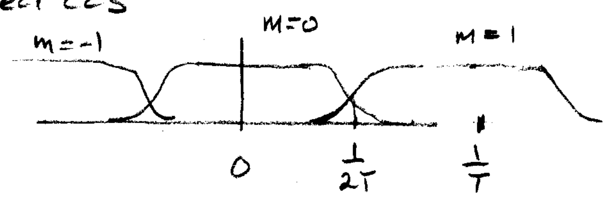
- How big is the ripple in $P_y(t)$? P+S 8.5.1 shows

$$P_y(t) = \frac{\sigma_a^2}{T} \sum_m c_m e^{j 2\pi m (t-\tau_0)/T}$$

where

$$c_m = \int_{-\infty}^{\infty} X(f) X^*\left(f - \frac{m}{T}\right) df$$

For raised cos



so only c_{-1}, c_0, c_1 are non zero and the ripple is sinusoidal

The ripple component is

$$P_y(t) = \frac{\sigma_a^2}{T} c_1 \cos\left(\frac{2\pi}{T}(t-\tau_0)\right)$$

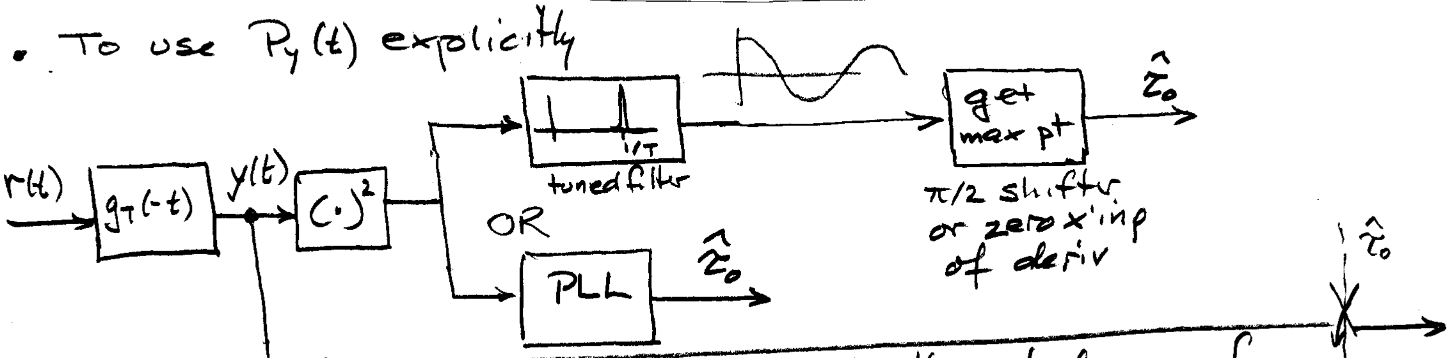
where

$$c_1 = \int_{(1-\alpha)/2T}^{(1+\alpha)/2T} X(f) X\left(f - \frac{1}{T}\right) df = \frac{\alpha T}{8} \text{ after a little trig.}$$

$$P_y(t) = \frac{\sigma_a^2}{8} \alpha \cos\left(\frac{2\pi}{T}(t-\tau_0)\right)$$

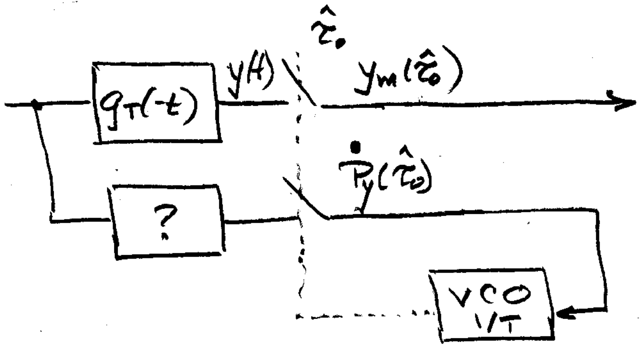
This is proportional to α. Shows why sinc pulses ($\alpha = 0$) are hard to synch to.

• To use $P_y(t)$ explicitly



Both tuned filter and PLL require the whole waveform y(t) continuously in time. They perform the average by combination of their periodic structure and time constant.

• It is more attractive to use only samples of the MF output, so we can use correlators instead, to save computation. Try to form a gradient signal for a tracking loop



$$P_y(t) = E_a\left[y^2(t)\right]$$

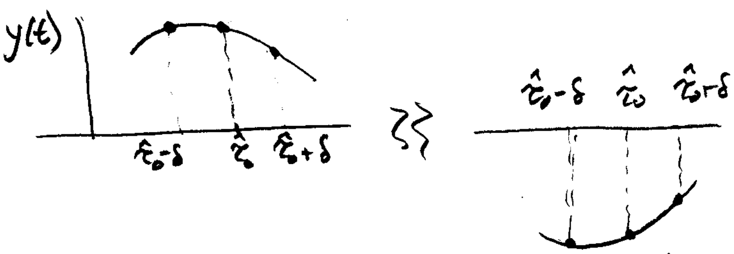
$$\frac{d}{dt}P_y(t) = \dot{P}_y(t) = E\left[y(t)\,\dot{y}(t)\right]$$

a good starting point, two approximate methods below

Early Late Gate $\boxed{P+S\ 8.5.2}$

$$\dot{y}(\hat{\tau}_0) \approx \frac{1}{2\delta}\left(y(\hat{\tau}_0+\delta) - y(\hat{\tau}_0-\delta)\right)$$

$$\dot{P}_y(\hat{\tau}_0) \sim E\left[\, y(\hat{\tau}_0)\left(y(\hat{\tau}_0+\delta) - y(\hat{\tau}_0-\delta)\right)\right]$$ three correlators! (but we need one anyway)



Note that mult by $y(\hat{\tau}_0)$ corrects polarity as well as weights. Simplify — multiply by sign of $y(\hat{\tau}_0)$ only, or just take

$$P_y(\hat{\tau}_0) \approx E\left[\,|y(\hat{\tau}_0+\delta)| - |y(\hat{\tau}_0-\delta)|\,\right]$$ early late gate

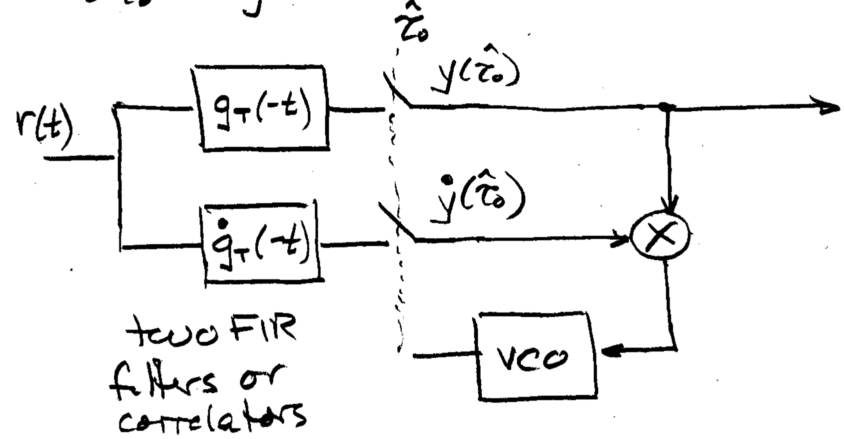
or mult by $\hat{a}_m$ to make DD.

Explicit Derivative

To create $\dot{y}(\hat{\tau}_0)$, note that

$$y(\tau_0) = \int r(\alpha)\, g_R(\tau_0-\alpha)\, d\alpha = \int r(\alpha)\, g_T(\alpha-\tau_0)\, d\alpha$$

$$\frac{d\,y(\tau_0)}{d\tau_0} = -\int r(\alpha)\, \dot{g}_T(\alpha-\tau_0)\, d\alpha$$



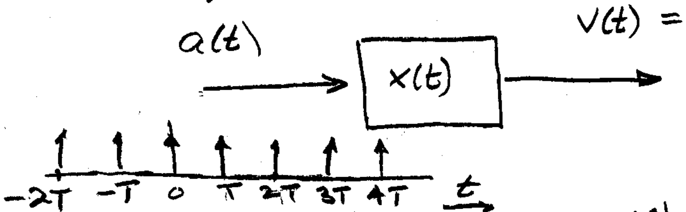
Again, we could use polarity alone of $y(\hat{\tau}_0)$.

Suitable if filters have an internal sampling rate of 4 samples/sym or better, otherwise too coarse.

The Importance of Transitions

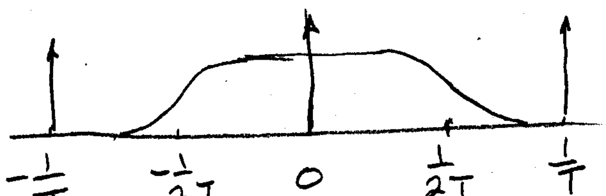
- Data transitions ($+1 \to -1$, $-1 \to +1$, for example) are critical for timing recovery, as can be seen from the sketch on p. 6.3.4.
- What if there were no transitions? Suppose $a_k = 1$ always? Model the system:

$$v(t) = \sum_k a_k\, x(t-kT) = \sum_k x(t-kT)$$



Transforming, $V(f) = A(f)\,X(f) = \left(\frac{1}{T}\sum_i \delta(f - i/T)\right) \cdot X(f)$

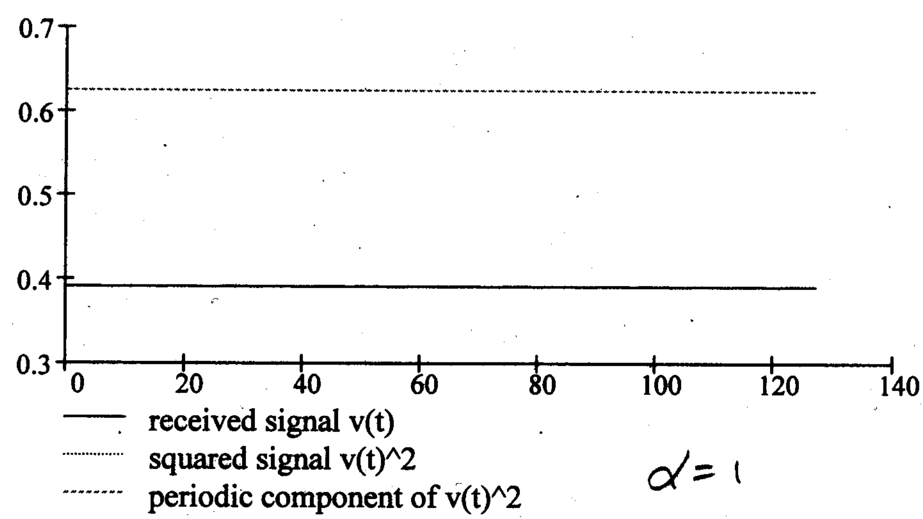
If $X(f)$ is bandlimited,



then $v(t) = \frac{1}{T} X(0)$ a constant.

No ripple, no timing.

This is the counterpart of graph p. 6.3.4 when no transitions



$\alpha = 1$

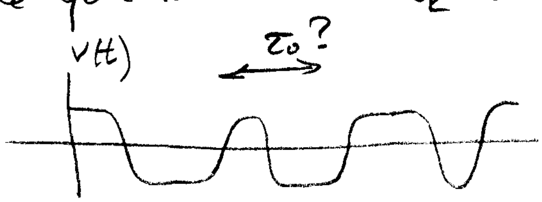
Maximum Likelihood Timing Recovery | P+S 8.5.4

- The ML approach produces optimum clock recovery methods. When you are working with a new or unusual signaling scheme, start here. Simplify the result to save computation if necessary.

- Start with a data aided design. Assume that you know $v(t)$ or a segment of it, perhaps in a synch word, but you don't know its timing offset τ_0

  example linear modulation $v(t) = \sum_k a_k \, g_T(t - kT - \tau_0)$

  where you know the a_k and the pulse shape



plus $n_w(t)$

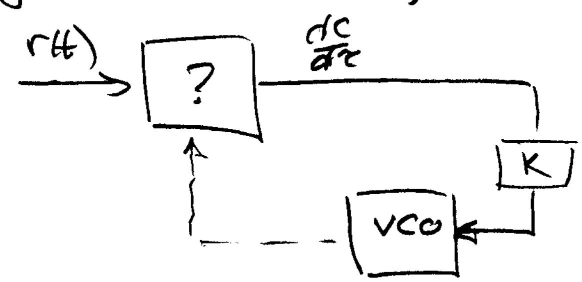
- Fundamentals again:
  - The signal space consists of all allowable translates of $v(t)$. Each choice of τ in $v(t-\tau)$ "indexes" a member.
  - Project the received waveform onto the signal space

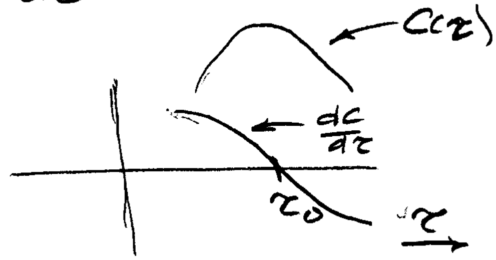
    $$C(\tau) = \int r(t)\, v(t-\tau)\, dt$$

  - If all possible values of τ are equally likely a priori, then MAP estimation of τ becomes ML, no bias term.
  - Result:

    $$\hat{\tau}_0 = \underset{\tau}{\operatorname{argmax}}\, C(\tau) = \underset{\tau}{\operatorname{argmax}} \int r(t)\, v(t-\tau)\, dt$$

- A circuit to implement this timing recovery:
  - Look for $\frac{dc}{d\tau} = 0$, so use $\frac{dc}{d\tau}$ as an error signal in a tracking loop

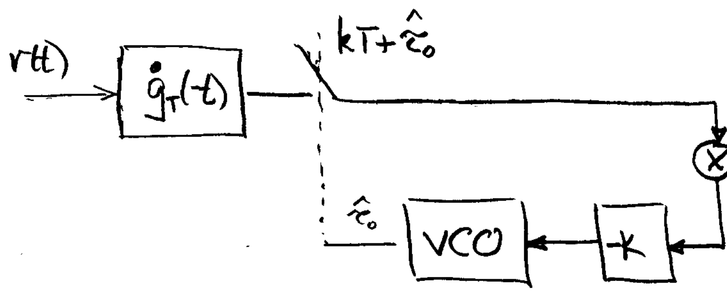




  - The derivative $\frac{dc}{d\tau} = \int r(t) \frac{d}{d\tau} v(t-\tau)\,dt = -\int r(t)\,\dot{v}(t-\tau)\,dt$

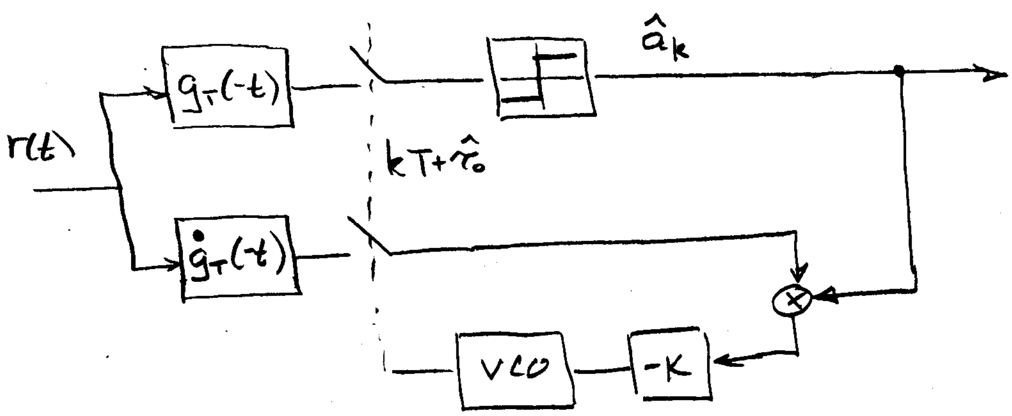
  - For our linear modulation example

$$\frac{dc}{d\tau} = -\sum_k a_k \int r(t)\,\dot{g}_T(t-kT-\tau)\,dt$$



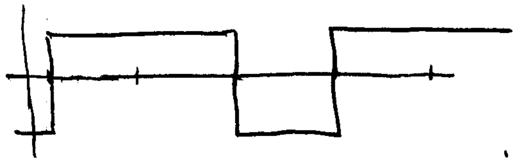
a_k assumed known (DA)

Now replace a_k with decisions:

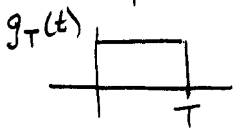


Compare with circuit on page 6.3.6

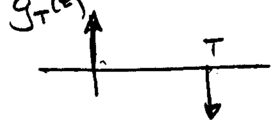
- An example – the digital transition tracker.



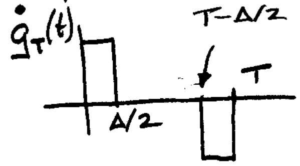
The pulse



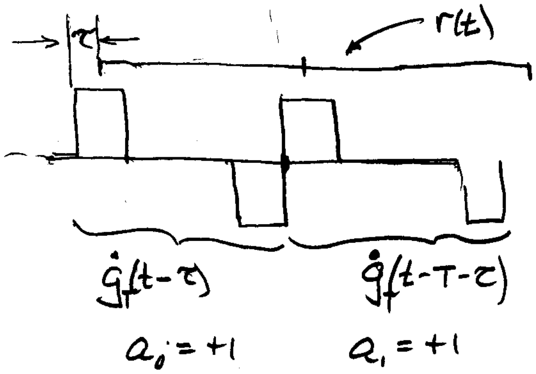
and its derivative



spread out a little



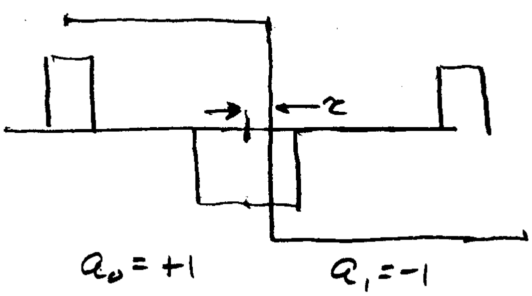
Consider what happens at bit boundaries



If no transition, then

$$\int r(t)\left(+\dot{g}_T(t-\tau) + \dot{g}_T(t-T-\tau)\right)dt$$

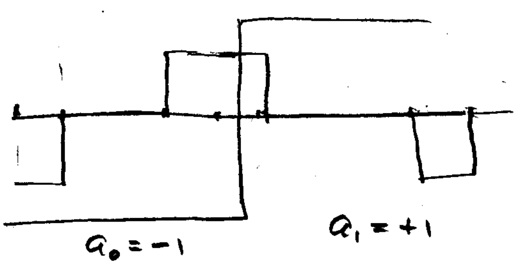
$$= 0$$



If + → – transition then

$$\int r(t)\left(+\dot{g}_T(t-\tau) - \dot{g}_T(t-T-\tau)\right)dt$$

$$= -K\tau \qquad |\tau| \le \Delta/2, \quad K > 0$$



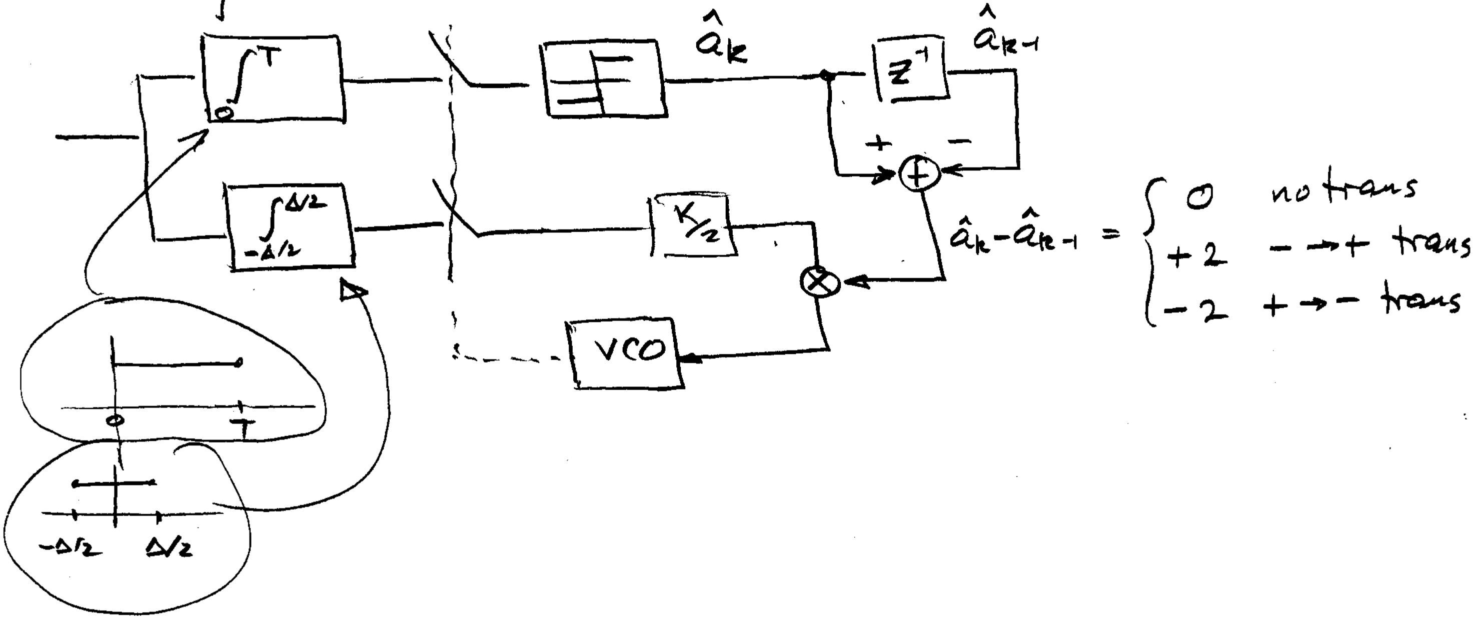
If – → + transition then

$$\int r(t)\left(-\dot{g}_T(t-\tau) + \dot{g}_T(t-T-\tau)\right)dt$$

$$= -K\tau \quad \text{again}$$

A good timing error to the VCO on every transition

Since the only action occurs at boundaries and transitions, structure it as two integrators: one for decisions, one for timing.



This one works well down to very low SNR values. Used in early space probes. Small Δ gives lower jitter, but more chance of pulling out.